Extensive-Form Games

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Previously … on multi-agent systems (tutorials and lectures).

1 Extensive-Form Games (game trees)

Reminder from lectures:

$$\max_{s,U} \quad U \tag{1}$$

$$\text{s.t.} \quad \sum_{a_1 \in \mathcal{A}_1} s(a_1) u_1(a_1, a_2) \geq U \qquad \forall a_2 \in \mathcal{A}_2 \tag{2}$$

$$\sum_{a_1 \in \mathcal{A}_1} s(a_1) = 1 \tag{3}$$

$$s(a_1) \geq 0 \qquad \forall a_1 \in \mathcal{A}_1 \tag{4}$$

Sequence-form LP

Reminder from lectures:

$$\max_{r_1, v} v(root) \tag{5}$$

$$\text{s.t.} \quad r_1(\emptyset) = 1 \tag{6}$$

$$0 \leq r_1(\sigma_1) \leq 1 \qquad \forall \sigma_1 \in \Sigma_1 \tag{7}$$

$$\sum_{a \in \mathcal{A}(I_1)} r_1(\sigma_1 a) = r_1(\sigma_1) \quad \forall I_1 \in \mathcal{I}_1, \sigma_1 = \mathsf{seq}_1(I_1) \tag{8}$$

$$\sum_{I' \in \mathcal{I}_2 : \sigma_2 a = \mathsf{seq}_2(I')} v(I') + \sum_{\sigma_1 \in \Sigma_1} g(\sigma_1, \sigma_2 a) r_1(\sigma_1) \geq v(I) \qquad \forall I \in \mathcal{I}_2, \sigma_2 = \mathsf{seq}_2(I), \forall a \in \mathcal{A}(I) \tag{9}$$

- $\mathsf{seq}_i(I)$ is a sequence of player i to information set,
- $I \in \mathcal{I}_i$, v_I is an expected utility in an information set,
- $\inf_i(\sigma_i)$ is an information set, where the last action of σ_i has been executed,
- $\sigma_i a$ denotes an extension of a sequence σ_i with action a

Task 1: Consider the following games. Write down a
sequence-form linear program for both players:
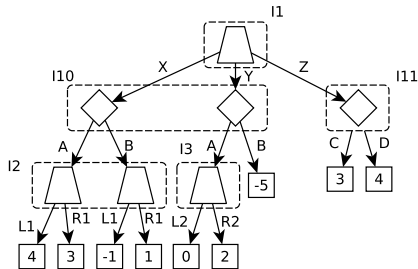
Task 2: Consider the following games. Write down a
sequence-form linear program for both players:
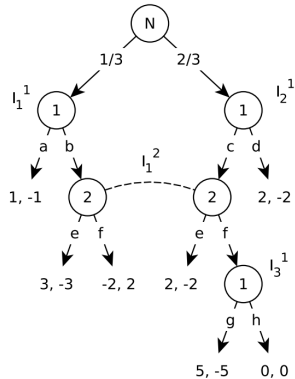
Task 3: Write down a sequence-form linear program for both players for the following game of a small "poker":

- there is an ante of 1$
- there is a limited deck of cards $\{J, J, Q, Q\}$
- each player receives a card
- player 1 either folds or bets 2$
- player 2 either calls or folds
- player with the higher card wins