

# Extensive-Form Games

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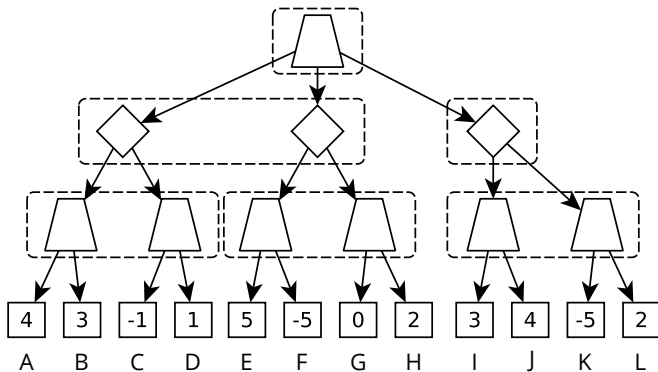
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Previously ... on computational game theory  
(tutorials and lectures).

- 1 Different representations:
  - Normal-Form Games (game matrixes)
  - Extensive-Form Games (game trees)
- 2 Algorithms for computing different solution concepts:
  - Nash equilibrium
  - Correlated equilibrium
  - Stackelberg equilibrium

Task 1: Consider following game. Write down a normal-form representation it.



Task 2: Formulate the following game of a small “poker” as an EFG.

- there is an ante of \$1
- there is a limited deck of cards  $\{J, J, Q, Q\}$
- each player receives a card
- player 1 either folds or bets \$2
- player 2 either calls or folds
- player with the higher card wins

Task 3: Formulate a game on the following map as an EFG.





- There are two players: one Terrorist (T) and one Counter-Terrorist (CT).
- The players decide which sites (A or B) to attack/defend. Once they make a choice they cannot change it.
- First, CT goes to defend either site A or B.
- The T waits to see if CT goes to B: he can be detected by looking through the door at MID, with  $p_{\text{see}}$ .

- When T sees CT through the door, it snipes at CT, and is successful with  $p_{\text{snipe}}$ .
- When CT neutralizes/gets killed by the T, it receives  $\pm 1$  point.
- If T knows CT goes to B, T goes to site A to gain advantage. Otherwise, T must make a choice between A and B.
- Once CT arrives at a site, it camps at locations (L or R) or (U or D) respectively.
- If T and CT pick the same site, they engage in combat. When T knows which site CT picked, it always kills CT, otherwise CT wins with  $p_{\text{combat}}$ .
- When T picks an undefended site, it plants the bomb.
- As the CT is informed the bomb has been planted, it runs to the other site, and kills T with  $p_{\text{strike}}$ . Then it tries to defuse the bomb.
- L is closer to B than R, and similarly, D is closer to A than U.
- If CT had to run from a more distant location, it fails to defuse the bomb and is penalized with  $-1/2$  point.

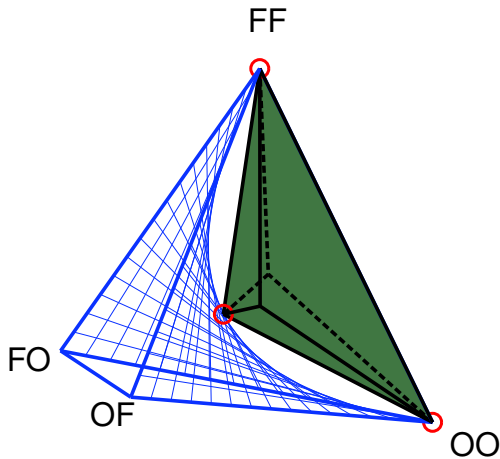
Discussion. How would the game change if ...

- there are teams of T and CT?
- the action space is continuous?



Task 4: Prove or disprove that all convex combinations of (two) NE are CE.

Geometry of NE and CE <sup>1</sup> for the game battle of sexes:



<sup>1</sup>From <https://www.cs.cmu.edu/~ggordon/CE/>

Optional homework:

Write down EFG representation of Kuhn poker (see wikipedia for rules).

Don't look at the picture there, only to verify your solution.

Optional homework:

Implement linear program for computing Nash equilibrium for player (I) and (II) for the previously mentioned zero-sum game:

	<b>L</b>	<b>M</b>	<b>R</b>
<b>U</b>	1	4	6
<b>D</b>	3	2	5