Normal-Form Games

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Formulating a game

Consider a problem with 4 targets (denoted $T_1, \ldots, T_4$) where the defender decides which target they should protect while the attacker decides which target to attack. If the attacker chooses to attack a protected target, the defender gets utility of 1 while the attacker gets the utility of -1. Otherwise, the attack is successful, the defender gets utility 0 and the attacker gets a utility as follows: for T_1, the attacker gets 3, $T_2 \to 7$, $T_3 \to 1$, and $T_4 \to 5$.

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Task 1: Formalize the game.

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Task 2: Let's assume that the defender can not only protect selected target but also, to some extent, neighbor targets – i.e., if the defender chooses to protect T_i then if the attackers chooses to attack target T_{i+1} or T_{i-1} (if they are defined), there is 50% that the defender will catch the attacker in those targets. Formalize the game.

Solution Concepts

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Consider the following game

	L	**M**	**R**
U	$1, 3$	$4, 2$	$-1, 2$
C	$1, 0$	$2, -2$	$0, -1$
D	$1, 2$	$-1, 1$	$3, 3$

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Task 3: Can you find all pareto optimal outcomes?

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Task 4: Can you find all pure Nash equilibria?

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Task 3: Can you find all pareto optimal outcomes?

Task 4: Can you find all pure Nash equilibria?

Task 5: Can you find all (weakly) dominated pure strategies?
Apply iterative removal of (weakly) dominated pure strategies.

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Task 6: Design a game where a pure Nash equilibrium outcome will be removed in a process of iterative removal of (weakly) dominated pure strategies.

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Task 7: Is every Nash equilibrium also a correlated equilibrium? Justify your answer (provide an example if not).

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Task 6: Design a game where a pure Nash equilibrium outcome will be removed in a process of iterative removal of (weakly) dominated pure strategies.

Task 7: Is every Nash equilibrium also a correlated equilibrium? Justify your answer (provide an example if not).

Task 8: Is every correlated equilibrium also a Nash equilibrium? Justify your answer (provide an example if not).