

Neural Networks.

Petr Pošík
petr.posik@fel.cvut.cz

Czech Technical University in Prague
Faculty of Electrical Engineering
Dept. of Cybernetics

Intro	2
Notation	3
Multiple regression	4
Logistic regression	5
Quiz	6
Gradient descent	7
Ex: Grad. for MR	8
Ex: Grad. for LR	9
Relations to NN	10
Multilayer FFN	11
MLP	12
MLP: A look inside	13
Activation functions	14
MLP: Learning	15
Quiz	16
BP	17
BP algorithm	19
BP: Example	20
BP Efficiency	21
Loss functions	22
Gradient Descent	23
Learning rate	24
Weights update	25
Momentum	26
GD improvements	27
Regularization	28
Ridge	29
Dropout	30
Other NNs	31
Beyond MLPs	32
Summary	33
Competencies	34

Notation

In *supervised learning*, we work with

- an observation described by a vector $\mathbf{x} = (x_1, \dots, x_D)$,
- the corresponding true value of the dependent variable y , and
- the prediction of a model $\hat{y} = f_w(\mathbf{x})$, where the model parameters are in vector \mathbf{w} .
- Very often, we use *homogeneous coordinates* and matrix notation, and represent the whole training data set as $T = (\mathbf{X}, \mathbf{y})$, where

$$\mathbf{X} = \begin{pmatrix} 1 & \mathbf{x}^{(1)} \\ \vdots & \vdots \\ 1 & \mathbf{x}^{(|T|)} \end{pmatrix}, \quad \text{and} \quad \mathbf{y} = \begin{pmatrix} y^{(1)} \\ \vdots \\ y^{(|T|)} \end{pmatrix}.$$

Learning then amounts to finding such model parameters \mathbf{w}^* which minimize certain loss (or energy) function:

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} J(\mathbf{w}, T)$$

Multiple linear regression

Multiple linear regression model:

$$\hat{y} = f_w(\mathbf{x}) = w_1x_1 + w_2x_2 + \dots + w_Dx_D = \mathbf{x}\mathbf{w}^T$$

The minimum of

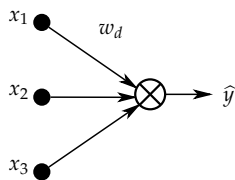
$$J_{MSE}(\mathbf{w}) = \frac{1}{|T|} \sum_{i=1}^{|T|} (y^{(i)} - \hat{y}^{(i)})^2,$$

is given by

$$\mathbf{w}^* = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y},$$

or found by numerical optimization.

Multiple regression as a **linear neuron**:



Logistic regression

Logistic regression model:

$$\hat{y} = f(w, x) = g(xw^T),$$

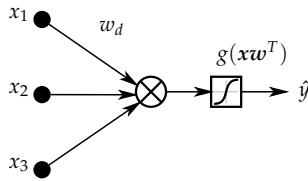
where

$$g(z) = \frac{1}{1 + e^{-z}}$$

is the **sigmoid** (a.k.a **logistic**) function.

- No explicit equation for the optimal weights.
- The only option is to find the optimum numerically, usually by some form of gradient descent.

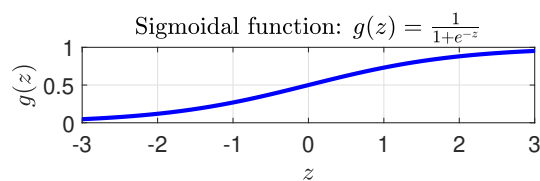
Logistic regression as a **non-linear neuron**:



Quiz

Logistic regression uses sigmoid function to transform the result of the linear combination of inputs:

$$g(z) = \frac{1}{1 + e^{-z}}$$



What is the value of the derivative of $g(z)$ at $z = 0$?

- A $g'(0) = -\frac{1}{2}$
- B $g'(0) = 0$
- C $g'(0) = \frac{1}{4}$
- D $g'(0) = \frac{1}{2}$

Gradient descent algorithm

- Given a function $J(\mathbf{w})$ that should be minimized,
- start with a guess of \mathbf{w} , and change it so that $J(\mathbf{w})$ decreases, i.e.
- update our current guess of \mathbf{w} by taking a step in the direction opposite to the gradient:

$$\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla J(\mathbf{w}), \text{ i.e.}$$
$$w_d \leftarrow w_d - \eta \frac{\partial}{\partial w_d} J(\mathbf{w}),$$

where all w_d s are updated simultaneously and η is a **learning rate** (step size).

- For cost functions given as the sum across the training examples

$$J(\mathbf{w}) = \sum_{i=1}^{|\mathcal{T}|} E(\mathbf{w}, \mathbf{x}^{(i)}, y^{(i)}),$$

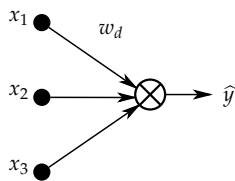
we can concentrate on a single training example because

$$\frac{\partial}{\partial w_d} J(\mathbf{w}) = \sum_{i=1}^{|\mathcal{T}|} \frac{\partial}{\partial w_d} E(\mathbf{w}, \mathbf{x}^{(i)}, y^{(i)}),$$

and we can drop the indices over training data set:

$$E = E(\mathbf{w}, \mathbf{x}, y).$$

Example: Gradient for multiple regression and squared loss



Assuming the squared error loss

$$E(\mathbf{w}, \mathbf{x}, y) = \frac{1}{2} (y - \hat{y})^2 = \frac{1}{2} (y - \mathbf{x}\mathbf{w}^T)^2,$$

we can compute the derivatives using the chain rule as

$$\frac{\partial E}{\partial w_d} = \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_d}, \text{ where}$$

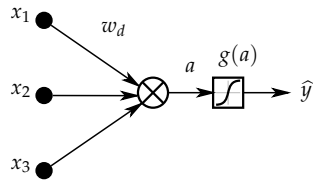
$$\frac{\partial E}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \frac{1}{2} (y - \hat{y})^2 = -(y - \hat{y}), \text{ and}$$

$$\frac{\partial \hat{y}}{\partial w_d} = \frac{\partial}{\partial w_d} \mathbf{x}\mathbf{w}^T = x_d,$$

and thus

$$\frac{\partial E}{\partial w_d} = \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_d} = -(y - \hat{y})x_d.$$

Example: Gradient for logistic regression and crossentropy loss



Nonlinear *activation* function:

$$g(a) = \frac{1}{1 + e^{-a}}$$

Note that

$$g'(a) = g(a)(1 - g(a)).$$

Assuming the crossentropy loss

$$E(\mathbf{w}, \mathbf{x}, y) = -y \log \hat{y} - (1 - y) \log(1 - \hat{y}), \quad \text{where } \hat{y} = g(a) = g(\mathbf{x}\mathbf{w}^T),$$

we can compute the derivatives using the chain rule as

$$\begin{aligned} \frac{\partial E}{\partial w_d} &= \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial a} \frac{\partial a}{\partial w_d}, \text{ where} \\ \frac{\partial E}{\partial \hat{y}} &= -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}} = -\frac{y-\hat{y}}{\hat{y}(1-\hat{y})}, \\ \frac{\partial \hat{y}}{\partial a} &= \hat{y}(1-\hat{y}), \text{ and } \frac{\partial a}{\partial w_d} = \frac{\partial}{\partial w_d} \mathbf{x}\mathbf{w}^T = x_d, \end{aligned}$$

and thus

$$\frac{\partial E}{\partial w_d} = \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial a} \frac{\partial a}{\partial w_d} = -(y - \hat{y})x_d.$$

Relations to neural networks

- Above, we derived training algorithms (based on gradient descent) for linear regression model and linear classification model.
- Note the similarity with the *perceptron algorithm* (“just add certain part of a misclassified training example to the weight vector”).
- Units like those above are used as **building blocks** for more complex/flexible models!

A more complex/flexible model:

$$\hat{y} = g^{OUT} \left(\sum_{k=1}^K w_k^{HID} g_k^{HID} \left(\sum_{d=1}^D w_{kd}^{IN} x_d \right) \right),$$

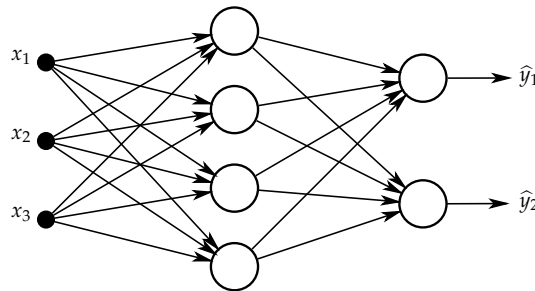
which is

- a nonlinear function of
 - a linear combination of
 - nonlinear functions of
 - linear combinations of inputs.

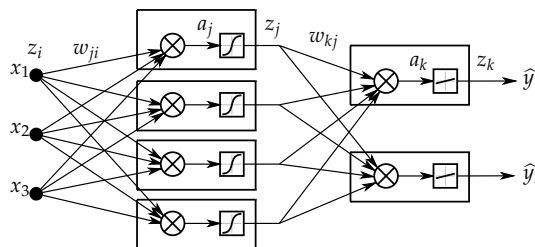
MLP

Multilayer perceptron (MLP)

- Multilayer feedforward network:
 - the „signal“ is propagated from inputs towards outputs; no feedback connections exist.
 - It realizes mapping from $\mathcal{R}^D \rightarrow \mathcal{R}^C$, where D is the number of object features, and C is the number of output variables.
 - For binary classification and regression, a single output is sufficient.
 - For classification into multiple classes, 1-of-N encoding is usually used.
 - **Universal approximation theorem:** A MLP with a single hidden layer with sufficient (but finite) number of neurons can approximate any continuous function arbitrarily well (under mild assumptions on the activation functions).



MLP: A look inside



Forward propagation:

- Given all the weights w and activation functions g , we can for a single input vector x easily compute the estimate of the output vector \hat{y} by iteratively evaluating in individual layers:

$$a_j = \sum_{i \in \text{Src}(j)} w_{ji} z_i \tag{1}$$

$$z_j = g(a_j) \tag{2}$$

- Note that
 - z_j in (1) may be the *outputs of hidden layers neurons* or the *inputs* x_i , and
 - z_j in (2) may be the *outputs of hidden layers neurons* or the *outputs* \hat{y}_k .

Activation functions

- Identity: $g(a) = a$
- Binary step: $g(a) = \begin{cases} 0 & \text{for } a < 0, \\ 1 & \text{for } a \geq 0 \end{cases}$
- Logistic (sigmoid): $g(a) = \sigma(a) = \frac{1}{1+e^{-a}}$
- Hyperbolic tangent: $g(a) = \tanh(a) = 2\sigma(a) - 1$
- Rectified Linear unit (ReLU): $g(a) = \max(0, a) = \begin{cases} 0 & \text{for } a < 0, \\ a & \text{for } a \geq 0 \end{cases}$
- Leaky ReLU: $g(a) = \begin{cases} 0.01a & \text{for } a < 0, \\ a & \text{for } a \geq 0 \end{cases}$
- ...

MLP: Learning

How to train a NN (i.e. find suitable w) given the training data set (X, y) ?

In principle, MLP can be trained in the same way as a single-layer NN using a gradient descent algorithm:

- Define the loss function to be minimized, e.g. squared error loss:

$$J(w) = \sum_{i=1}^{|T|} E(w, x^{(i)}, y^{(i)}) = \frac{1}{2} \sum_{i=1}^{|T|} \sum_{k=1}^C (y_{ik} - \hat{y}_{ik})^2, \quad \text{where}$$
$$E(w, x, y) = \frac{1}{2} \sum_{k=1}^C (y_k - \hat{y}_k)^2.$$

$|T|$ is the size of the training set, and C is the number of outputs of NN.

- Compute the gradient of the loss function w.r.t. individual weights:

$$\nabla E(w) = \left(\frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, \dots, \frac{\partial E}{\partial w_W} \right).$$

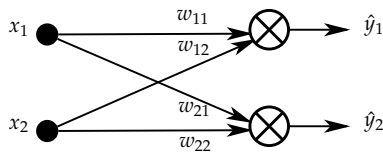
- Make a step in the direction opposite to the gradient to update the weights:

$$w_d \leftarrow w_d - \eta \frac{\partial E}{\partial w_d} \quad \text{for } d = 1, \dots, W.$$

How to compute the individual derivatives?

Quiz

Let's have a simple linear NN with 2 inputs, 2 outputs, and 4 weights (parameters):



The current values of the weights are
 $w_{11} = 1$,
 $w_{12} = 2$,
 $w_{21} = 3$, and
 $w_{22} = 4$.

We would like to update the weights of the NN using a single example from the training dataset $(\mathbf{x}, \mathbf{y}) = (x_1, x_2, y_1, y_2) = (1, 2, 1, 10)$. We use squared error loss.

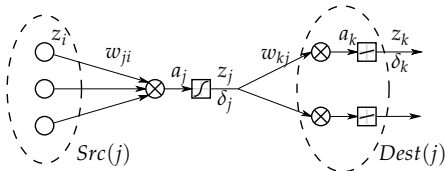
Which of the weights will change the most with this update? I.e., which of $\frac{\partial E}{\partial w_{ji}}$ is of the biggest size?

- A w_{11}
- B w_{12}
- C w_{21}
- D w_{22}

Error backpropagation

Error backpropagation (BP) is the algorithm for efficient computation of $\frac{\partial E}{\partial w_d}$.

Consider only $\frac{\partial E}{\partial w_d}$ because $\frac{\partial J}{\partial w_d} = \sum_n \frac{\partial}{\partial w_d} E(\mathbf{w}, \mathbf{x}^{(n)}, \mathbf{y}^{(n)})$.



E depends on w_{ji} only via a_j :

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial a_j} \frac{\partial a_j}{\partial w_{ji}} \quad (3)$$

Let's introduce the so called **error** δ_j :

$$\delta_j = \frac{\partial E}{\partial a_j} \quad (4)$$

From (1) we can derive:

$$\frac{\partial a_j}{\partial w_{ji}} = z_i \quad (5)$$

Substituting (4) and (5) into (3):

$$\frac{\partial E}{\partial w_{ji}} = \delta_j z_i, \quad (6)$$

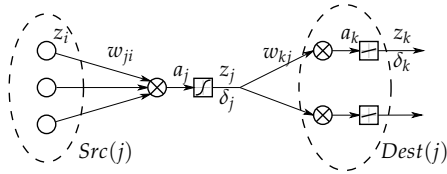
where δ_j is the error of the neuron on the output of the edge $i \rightarrow j$, and z_i is the input of the edge $i \rightarrow j$.

"The more we excite edge $i \rightarrow j$ (big z_i) and the larger is the error of the neuron on its output (large δ_j), the more sensitive is the loss function E to the change of w_{ji} ."

- All values z_i are known from forward pass,
- to compute the gradient, we need to compute all δ_j .

Error backpropagation (cont.)

We need to compute the *errors* δ_j .



For the output layer:

$$\delta_k = \frac{\partial E}{\partial a_k}$$

E depends on a_k only via $\hat{y}_k = g(a_k)$:

$$\delta_k = \frac{\partial E}{\partial a_k} = \frac{\partial E}{\partial \hat{y}_k} \frac{\partial \hat{y}_k}{\partial a_k} = g'(a_k) \frac{\partial E}{\partial \hat{y}_k} \quad (7)$$

For the hidden layers:

$$\delta_j = \frac{\partial E}{\partial a_j}$$

E depends on a_j via all a_k ,
 $k \in \text{Dest}(j)$:

$$\begin{aligned} \delta_j &= \frac{\partial E}{\partial a_j} = \sum_{k \in \text{Dest}(j)} \frac{\partial E}{\partial a_k} \frac{\partial a_k}{\partial a_j} = \\ &= \sum_{k \in \text{Dest}(j)} \delta_k \frac{\partial a_k}{\partial a_j} = \\ &= g'(a_j) \sum_{k \in \text{Dest}(j)} w_{kj} \delta_k, \end{aligned} \quad (8)$$

because

$$a_k = \sum_{j \in \text{Src}(k)} w_{kj} z_j = \sum_{j \in \text{Src}(k)} w_{kj} g(a_j),$$

$$\text{and thus } \frac{\partial a_k}{\partial a_j} = w_{kj} g'(a_j)$$

“The error δ_k is distributed to δ_j in the lower layer according to the weight w_{kj} (which is the speed of growth of the linear combination a_k) and according to the size of $g'(a_j)$ (which is the speed of growth of the activation function).”

Error backpropagation algorithm

Algorithm 1: Error Backpropagation: the computation of derivatives $\frac{\partial E}{\partial w_d}$.

1 **begin**

2 Perform a forward pass for observation x . This will result in values of all a_j and z_j for the vector x .

3 Evaluate the error δ_k for the output layer (using Eq. 7):

$$\delta_k = g'(a_k) \frac{\partial E}{\partial \hat{y}_k}$$

4 Using Eq. 8, propagate the errors δ_k back to get all the remaining δ_j :

$$\delta_j = g'(a_j) \sum_{k \in \text{Dest}(j)} w_{kj} \delta_k$$

5 Using Eq. 6, evaluate all the derivatives to get the whole gradient:

$$\frac{\partial E}{\partial w_{ji}} = \delta_j z_i$$

Error backpropagation: Example

NN with a single hidden layer:

- Squared error loss: $E = \frac{1}{2} \sum_{k=1}^C (y_k - \hat{y}_k)^2$
- Activation func. in the output layer: identity $g_k(a_k) = a_k$, $g'_k(a_k) = 1$
- Activation func. in the hidden layer: sigmoidal $g_j(a_j) = \frac{1}{1 + e^{-a_j}}$, $g'_j(a_j) = z_j(1 - z_j)$

Computing the errors δ :

- Output layer: $\delta_k = g'_k(a_k) \frac{\partial E}{\partial \hat{y}_k} = -(y_k - \hat{y}_k)$
- Hidden layer: $\delta_j = g'_j(a_j) \sum_{k \in Dest(j)} w_{kj} \delta_k = z_j(1 - z_j) \sum_{k \in Dest(j)} w_{kj} \delta_k$

Computation of all the partial derivatives:

$$\frac{\partial E}{\partial w_{ji}} = \delta_j x_i$$

$$\frac{\partial E}{\partial w_{kj}} = \delta_k z_j$$

Online learning:

$$w_{ji} \leftarrow w_{ji} - \eta \delta_j x_i$$

$$w_{kj} \leftarrow w_{kj} - \eta \delta_k z_j$$

Batch learning:

$$w_{ji} \leftarrow w_{ji} - \eta \sum_{n=1}^{|T|} \delta_j^{(n)} x_i^{(n)}$$

$$w_{kj} \leftarrow w_{kj} - \eta \sum_{n=1}^{|T|} \delta_k^{(n)} z_j^{(n)}$$

Error backpropagation efficiency

Let W be the number of weights in the network (the number of parameters being optimized).

- The evaluation of E for a single observation requires $\mathcal{O}(W)$ operations (evaluation of $w_{ji} z_i$ dominates, evaluation of $g(a_j)$ is neglected).

We need to compute W derivatives for each observation:

- Classical approach:
 - Find explicit equations for $\frac{\partial E}{\partial w_{ji}}$.
 - To compute each of them $\mathcal{O}(W)$ steps are required.
 - In total, $\mathcal{O}(W^2)$ steps for a single training example.
- Backpropagation:
 - Requires only $\mathcal{O}(W)$ steps for a single training example.

Loss functions

Task	Suggested loss function
Binary classification	Cross-entropy: $J = - \sum_{i=1}^{ T } [y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$
Multinomial classification	Multinomial cross-entropy: $J = - \sum_{i=1}^{ T } \sum_{k=1}^C I(y^{(i)} = k) \log \hat{y}_k^{(i)}$
Regression	Squared error: $J = \sum_{i=1}^{ T } (y^{(i)} - \hat{y}^{(i)})^2$
Multi-output regression	Squared error: $J = \sum_{i=1}^{ T } \sum_{k=1}^C (y_k^{(i)} - \hat{y}_k^{(i)})^2$

Note: often, mean errors are used.

- Computed as the average w.r.t. the number of training examples $|T|$.
- The optimum is in the same point, of course.

Gradient Descent

Learning rate annealing

Task: find such parameters w^* which minimize the model cost over the training set, i.e.

$$w^* = \arg \min_w J(w; X, y)$$

Gradient descent: $w^{(t+1)} = w^{(t)} - \eta^{(t)} \nabla J(w^{(t)})$,
where $\eta^{(t)} > 0$ is the **learning rate** or **step size** at iteration t .

Learning rate decay:

- Decrease the learning rate in time.
- **Step decay:** reduce the learning rate every few iterations by certain factor, e.g. $\frac{1}{2}$.
- **Exponential decay:** $\eta^{(t)} = \eta_0 e^{-kt}$
- **Hyperbolic decay:** $\eta^{(t)} = \frac{\eta_0}{1+kt}$

Weights update

When should we update the weights?

■ Batch learning:

- Compute the gradient w.r.t. all the training examples (epoch).
- Several epochs are required to train the network.
- Inefficient for redundant datasets.

■ Online learning:

- Compute the gradient w.r.t. a single training example only.
- **Stochastic Gradient Descent (SGD)**
- Converges almost surely to local minimum when $\eta^{(t)}$ decreases appropriately in time.

■ Mini-batch learning:

- Compute the gradient w.r.t. a small subset of the training examples.
- A compromise between the above 2 extremes.

Momentum

Momentum

- Perform the update in an analogy to physical systems: a particle with certain mass and velocity gets acceleration from the gradient (“force”) of the loss function:

$$\begin{aligned} \mathbf{v}^{(t+1)} &= \mu \mathbf{v}^{(t)} + \eta^{(t)} \nabla J(\mathbf{w}^{(t)}) \\ \mathbf{w}^{(t+1)} &= \mathbf{w}^{(t)} + \mathbf{v}^{(t+1)} \end{aligned}$$

- SGD with momentum tends to keep traveling in the same direction, preventing oscillations.
- It builds the velocity in directions with consistent (but possibly small) gradient.

Nesterov’s Momentum

- Slightly different update equations:

$$\begin{aligned} \mathbf{v}^{(t+1)} &= \mu \mathbf{v}^{(t)} + \eta^{(t)} \nabla J(\mathbf{w}^{(t)} + \mu \mathbf{v}^{(t)}) \\ \mathbf{w}^{(t+1)} &= \mathbf{w}^{(t)} + \mathbf{v}^{(t+1)} \end{aligned}$$

- Classic momentum corrects the velocity using gradient at $\mathbf{w}^{(t)}$; Nesterov uses gradient at $\mathbf{w}^{(t)} + \mu \mathbf{v}^{(t)}$ which is more similar to $\mathbf{w}^{(t+1)}$.
- Stronger theoretical convergence guarantees; slightly better in practice.

Further gradient descent improvements

Resilient Propagation (Rprop)

- $\frac{\partial J}{\partial w_d}$ may differ a lot for different parameters w_d .
- Rprop does not use the value, only its *sign* to adapt the step size for each weight separately.
- Often, an order of magnitude faster than basic GD.
- Does not work well for mini-batches.

Adaptive Gradient (Adagrad)

- Idea: Reduce learning rates for parameters having high values of gradient.

Root Mean Square Propagation (RMSprop)

- Similar to AdaGrad, but employs a moving average of the gradient values.
- Can be seen as a generalization of Rprop, can work also with mini-batches.

Adaptive Moment Estimation (Adam)

- Improvement of RMSprop.
- Uses moving averages of gradients and their second moments.

See also:

- <http://sebastianruder.com/optimizing-gradient-descent/>
- <http://cs231n.github.io/neural-networks-3/>
- <http://cs231n.github.io/assets/n3/opt2.gif>, <http://cs231n.github.io/assets/n3/opt1.gif>

Regularization

Overfitting and regularization

Overfitting in NN is often characterized by weight values that are very large in magnitude. How to deal with it?

- Get more data.
- Use a simpler model (less hidden layers, less neurons, different activation functions).
- Use *regularization* (penalize the model complexity).

Ridge regularization:

- Modified loss function, e.g. for squared error:

$$J'(w) = J(w) + \text{penalty} = \frac{1}{2m} \sum_{i=1}^m (y^{(i)} - \mathbf{x}^{(i)} w^T)^2 + \frac{\alpha}{m} \sum_{d=1}^D w_d^2.$$

- Modified weight update in GD:

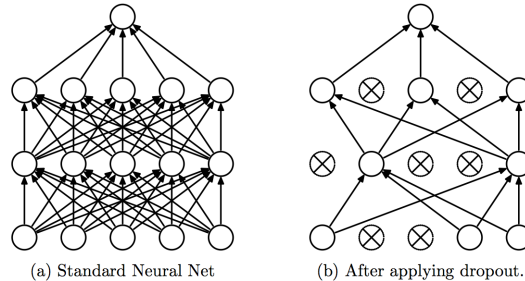
$$w_d \leftarrow w_d - \eta \frac{\partial J'}{\partial w_d} = \underbrace{\left(1 - \frac{\eta \alpha}{m}\right)}_{\text{weight decay}} w_d - \eta \frac{\partial J}{\partial w_d},$$

where η is the learning rate, α is the regularization strength, m is the number of examples in the batch.

- The biases (weights connected to constant 1) should not be regularized!

Dropout

- Idea: Average many NNs, share weights to make it computationally feasible.
- For each training example, omit each neuron with certain probability (often $p = 0.5$).
- This is like sampling from 2^N networks where N is the number of units.
- Only a small part of the 2^N networks is actually sampled.
- Prevents coadaptation of feature vectors.



(a) Standard Neural Net (b) After applying dropout.
Srivastava et al.: A Simple Way to Prevent Neural Networks from Overfitting, 2014

Other types of Neural Networks

Beyond MLPs

MLPs are only one type of neural networks. Other types of FFNNs include:

- **Radial basis functions (RBF) nets.** Neurons contain prototypes, forward propagation resembles a (smoothed) nearest neighbors method.
- **Autoencoders.** Learn a compact representation of the input data.
- **Convolutional nets.** Replace the fully-connected layer with a convolutional layer that has smaller number of weights and reuses them for many input variables. Aimed at image processing.
- ...

Recurrent nets contain also *feedback connections*.

- They preserve a kind of *state* of the network.
- Simple recurrent architectures: **Jordan**, **Elman**. Network output or state used together with input in the next iteration.
- **Hopfield net.** Used as associative memory.
- **Long short-term memory (LSTM).** Suitable for processing data sequences in time.
- ...

Other architectures:

- **Kohonen's self-organizing maps (SOM).** Used for unsupervised learning.
- **Neural gas.** Used e.g. to approximately solve the traveling salesperson problem.
- ...

Competencies

After this lecture, a student shall be able to ...

1. describe the model of a simple neuron, and explain its relation to multivariate regression and logistic regression;
2. explain how to find weights of a single neuron using gradient descent (GD) algorithm;
3. derive the update equations used in GD to optimize the weights of a single neuron for various loss functions and various activation functions;
4. describe a multilayer feedforward network and discuss its usage and characteristics;
5. compare the use of GD in case of a single neuron and in case of NN, discuss similarities and differences;
6. explain the error backpropagation (BP) algorithm — its purpose and principle;
7. implement BP algorithm for a simple NN, and suggest how the implementation should be modified to allow application for complex networks;
8. discuss the purpose of various modifications of GD algorithm (learning rate decay, weight update schedule, momentum, ...);
9. discuss the regularization options for NN (weight decay, dropout);
10. be aware of other types of NNs, not only feedforward nets.