

Lecture 12: Bonuses – Symbolic Math

B0B17MTB, BE0B17MTB – MATLAB

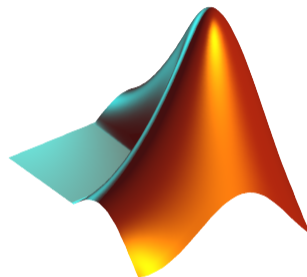
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1. Introduction
2. Polynomials
3. Limit and Derivative
4. Integration



Mathematical Analysis



- ▶ Two different attitudes are distinguished:
 - ▶ **symbolic** math,
 - ▶ **numerical** math.
- ▶ Possible classification: analytical result, in principle, enables to get result in infinite number of decimals; numerical result is encumbered with numerical errors.
- ▶ There exist wide range of techniques in MATLAB (symbolical as well as numerical) and only selected topics are discussed in this lecture.



Function Handle and Anonymous Functions — Recap

- ▶ Enables indirect function invoking.
- ▶ Reference to the function is stored in variable `hn`:

```
hn = @function_name
```

- ▶ Function handle is a data type in MATLAB (see `whos`).
- ▶ Anonymous function (exists as variable without m-file):

```
as = @(args) function_name(args)
```

- ▶ Enables to invoke a function from locations where it is not visible to MATLAB:

```
>> fxy = @(x, y) x^2 + y^2 - 5  
>> fxy(2, -2)  
  
>> fcos = @(alpha) cos(alpha)  
>> fcos(pi)
```



Polynomials I.

- Representation of polynomials in MATLAB

$$P = C_n x^n + C_{n-1} x^{n-1} + \cdots + C_1 x + C_0 = [C_n \quad C_{n-1} \quad \cdots \quad C_1 \quad C_0]. \quad (1)$$

- Roots of a polynomial: `roots`

```
>> x = roots([1 0 -1]);
>> x1 = x(1)
>> x2 = x(2)
```

- Polynomial evaluation: `polyval`

```
>> % x = 2
>> % p1 = 3*x^5 - 7*x^3 + 1/2*x^2 - 5
>> polyval([3 0 -7 1/2 0 -5], 2)
```

- Polynomial multiplication: `conv`

$$A_1 = x - 1,$$

$$A_2 = x + 1,$$

$$A_1 A_2 = (x - 1)(x + 1) = x^2 - 1$$

```
>> A1 = [1 -1]
>> A2 = [1 1]
>> conv(A1, A2)
% = [1 0 -1]
```



Polynomials II.

- ▶ Polynomial division: deconv:

$$\frac{x^2 - 1}{x + 1} = x - 1$$

```
>> deconv([1 0 -1], [1 1])
>> % = [1 -1]
```

- ▶ Some polynomial-related functions:

- ▶ residue: residue of ratio of two polynomials,
- ▶ polyfit: approximation of data with polynomial of order n ,
- ▶ polyint: polynomial integration,
- ▶ polyder: polynomial derivative.

```
>> S = [1 1];
>> T = polyint(S) % = [0.5 1 0]
>> U = polyder(T) % = S = [1 1]
>> polyder(U) % = 1
```

$$\int (x + 1) dx = \frac{1}{2}x^2 + x, \quad \frac{d\left(\frac{1}{2}x^2 + x\right)}{dx} = x + 1$$

Polynomials III.



- ▶ Polynomial multiplication (symbolic evaluation):

$$P_1 = A + Bx, \quad P_2 = 4x^2 + 2x - 4$$

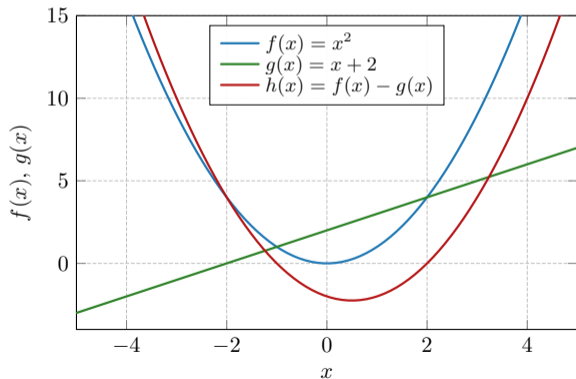
```
>> syms A B x
>> P1 = A + B*x;           % entering 1. polynomial
>> P2 = 4*x^2 + 2*x - 4; % 2. polynomial
>> P0 = P1*P2;            % multiplication
>> P = expand(P0)         % expansion
```

- ▶ Note: function `expand` requires Symbolic Math Toolbox.



Intersection of Two Functions

- ▶ Two functions are given, $f(x) = x^2$ and $g(x) = x + 2$. Find $x : f(x) = g(x)$.
- ▶ Recast as to find $x : h(x) = 0$, where $h(x) = f(x) - g(x)$:



To find the solution:

```
>> syms x;
>> f = x^2;
>> g = x + 2;
>> x0 = solve(f - g) % = 2; -1
```

To depict the functions:

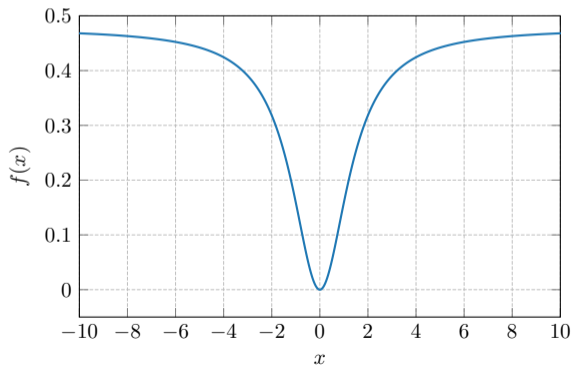
```
>> ezplot(f);
>> hold on;
>> grid on;
>> ezplot(g);
```




Limit and Derivative I.

► Find out function limit:

$$f(x) = \frac{3x^3}{2\pi x^3 + 4\pi x} = \frac{3}{2\pi} \left(\frac{x^2}{x^2 + 2} \right), \text{ L'Hôpital's rule: } \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = \frac{3}{2\pi} \approx 0.4775$$



```
>> syms x real;
>> f = 3*x^3/(2*pi*x^3 + 4*pi*x)

>> lim1 = limit(f, x, -inf)
>> lim2 = limit(f, x, inf)

>> double(lim1) % = 0.4775
>> double(lim2) % = 0.4775

>> figure;
>> ezplot(f);
>> grid on;
```



Limit and Derivative I.

- ▶ Apply L'Hôpital's rule to previous function:
 - ▶ Function $f(x)$ contains 3rd power of x ; carry out 3rd derivative (of numerator $f_1(x)$ and denominator $f_2(x)$ separately) in x .

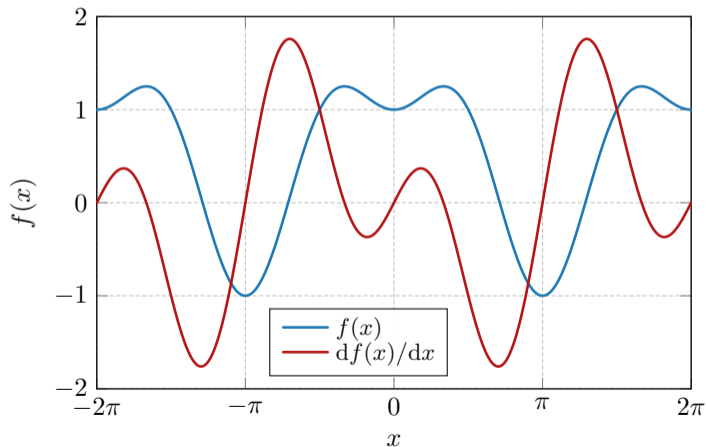
$$f(x) = \frac{3x^3}{2\pi x^3 + 4\pi x}, \quad f_1(x) = 3x^3, \quad f_2(x) = 2\pi x^3 + 4\pi x$$

```
>> f1 = 3*x^3;  
>> f2 = 2*pi*x^3 + 4*pi*x;  
>> A1 = diff(f1,3)  
>> A2 = diff(f2,3)  
>> double(A1/A2) % = 0.4775
```



Limit and Derivative I

- ▶ Carry out derivative of the following function in $f(x) = \sin^2(x) + \cos(x)$.
 - ▶ Compare results and plot them.





Integration I.

- ▶ Let's first symbolically carry out derivative of function $f(x) = \sin(x) + 2$.
- ▶ Save the second derivative of $f(x)$ and call it $g(x)$, compare the results.
- ▶ Now, integrate function $g(x)$ (once, twice), do we get the original function $f(x)$?
 - ▶ Ignore integration constants.

```
>> clear, clc;
>> x = sym('x');

>> f = sin(x) + 2
>> figure; ezplot(f);

>> fd = diff(f)
>> figure; ezplot(fd);

>> fdd = diff(f, 2)
>> figure; ezplot(fdd);
```

```
>> g = fdd;
>> gi = int(g)
>> figure; ezplot(gi);

>> gii = int(gi);
>> err = f - gii

>> figure;
>> subplot(1, 2, 1);
>> ezplot(f);
>> subplot(1, 2, 2);
>> ezplot(gii);
```

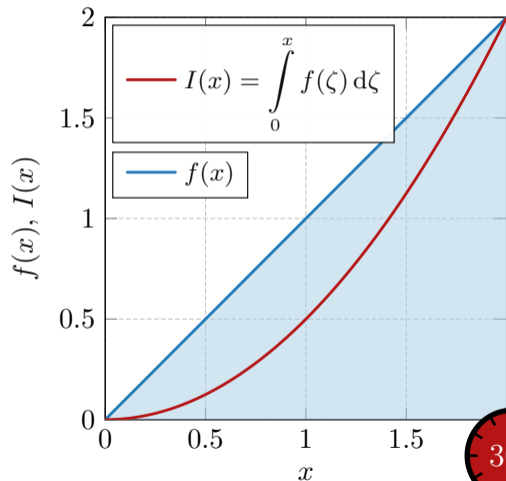


Integration I.

- ▶ Consider a function $f(x) = x$,
 - ▶ calculate indefinite integral of $f(x)$,
 - ▶ calculate definite integral for $x \in [0, 2]$,
use, e.g., function `int` or `integral`,

$$I = \int_0^2 f(x) dx = \left[\frac{x^2}{2} \right]_0^2 = 2.$$

```
>> fill([0 2 2], [0 0 2], 'y')
```



300



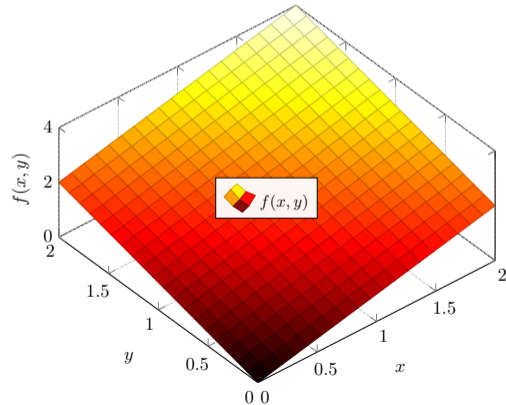
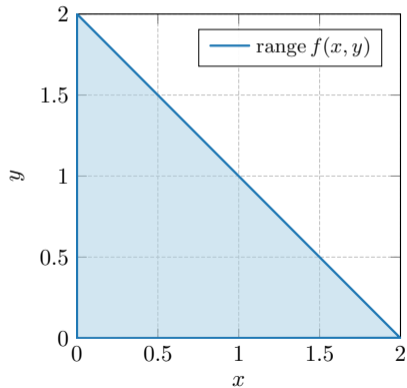
Numerical Integration I.

- ▶ Numerical approach is used whenever the closed-form (analytical) solution is not known which happens quite often in technical sciences (almost always).
- ▶ It is possible to use various numerical integration methods, see literature.
- ▶ Alternatively, MATLAB functions can be utilized:
 - ▶ `integral`, `integral2`, `integral3`, `quadgk`, and others,
 - ▶ define function to be integrated (write your own function or use function handle).



Numerical Integration II.

► Solve the following integral $I = \int_0^2 \int_0^{2-x} (x+y) \, dy \, dx = \dots = \int_0^2 \left(2 - \frac{x^2}{2}\right) \, dx \approx 2.666$



Numerical Integration III.



```
>> clear, clc;
% solution:
>> f = @(x, y) x + y
>> ymax = @(x) 2 - x
>> integral2(f, 0, 2, 0, ymax)

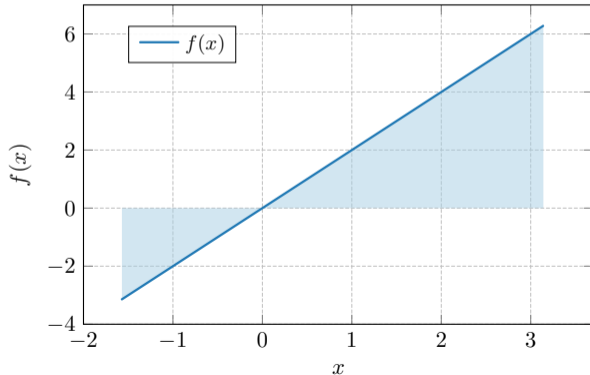
% plotting
>> t = 0:1/10:2
>> [x, y] = meshgrid(t);
>> z = x + y;
>> figure('color', 'w');
>> mesh(x, y, z);
```




Numerical Integration IV.

- It is possible to work with external scripts as well; *i.e.*, having “complex” expression that

we don't want to process as handle:
$$I = \int_{-\pi/2}^{\pi} f(x) dx = \dots = \frac{3}{4}\pi^2.$$



```
function fx = myIntFcn(x)
% function to calculate integral:
% int{2*x}

c = 2;
fx = c*x;
```

```
>> integral(@myIntFcn, -pi/2, pi)
```



Closing Notes

- ▶ General problem of derivative (it is not possible to approach zero), *i.e.*,

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- ▶ Various sophisticated numerical methods of various complexity are used.
- ▶ Web pages to solve this problem in a complex way:
 - ▶ <http://www.matrixlab-examples.com/derivative.html>
- ▶ In the case there is a lot of symbolic calculations or when approaching MATLAB limits, try another mathematical tool (*e.g.*, Mathematica, Maple).
- ▶ Nevertheless, MATLAB is a perfect choice for numerical computing (although both Mathematica's symbolic and numerical kernels are excellent).

Questions?

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