

Lecture 1: MATLAB Environment, Basic Math Operators

B0B17MTB – Matlab

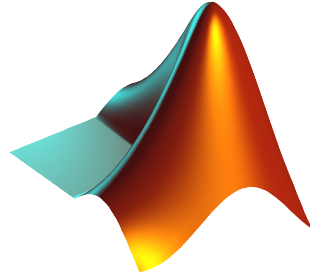
Miloslav Čapek, Viktor Adler, Michal Mašek, and Vít Losenický

Department of Electromagnetic Field
Czech Technical University in Prague
Czech Republic
matlab@fel.cvut.cz

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1. MATLAB Environment
2. Scalars, Vectors, Matrices
3. Basic Math Operations
4. Exercises





The MATLAB Environment

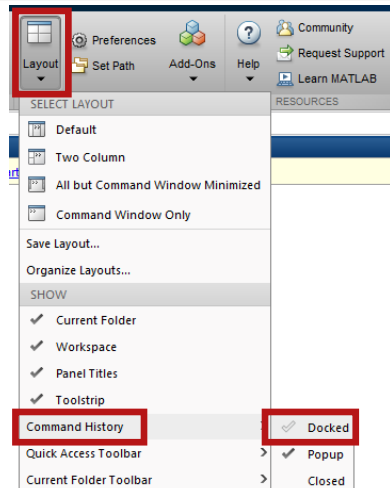
The screenshot shows the MATLAB R2019a academic user interface. The interface is divided into several panes:

- 1**: Command Window, where MATLAB code is entered and executed.
- 2**: Workspace, which displays the current variables in the workspace.
- 3**: Command History, which shows a list of previously executed commands.
- 4**: Current Folder, which shows the current directory and its contents.
- 5**: Details pane, which provides information about the selected file in the Current Folder pane.
- 6**: The File menu, which contains options for file operations.
- 7**: The status bar at the bottom of the interface.
- 8**: The search bar in the top right corner.



The MATLAB Environment – Panels

1. Command Window
2. Workspace
3. Command History – *not activated, to activate* →
4. Current Folder
5. Current Folder – Details
6. Current Working Directory
7. Status (“Busy” when MATLAB is executing your code)
8. Search in documentation





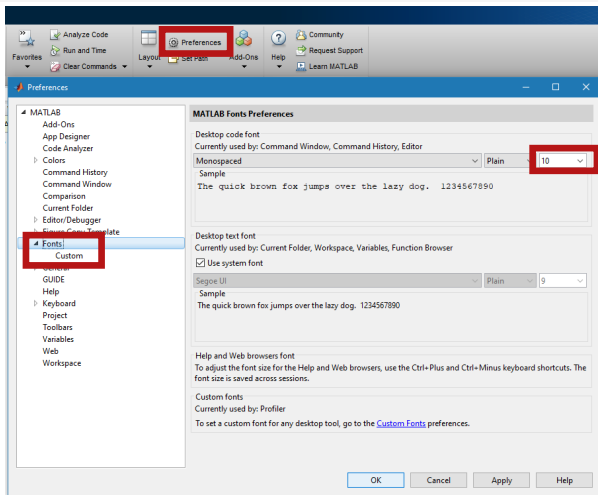
Preferences

▶ Command:

```
>> preferences
```

▶ Ribbon menu:

- ▶ Change font size.



Documentation



```
>> doc % opens documentation window
```

```
>> help % MATLAB help
```

```
>> demo % tutorials
```



The Help Structure

▶ Command:

```
>> help sin
```

▶ Output:

```
sin    Sine of argument in radians.  
      sin(X) is the sine of the elements of X.
```

```
See also asin, sind, sinpi.
```

```
Reference page for sin
```



The Documentation Structure I.

► Command:

```
>> doc sin
```

1. Documentation page
2. Search field
3. Documentation contents
4. Bookmarks of this page

The screenshot shows the MATLAB Help interface for the `sin` function. The search field at the top right is circled with a '2'. The 'sin' title is circled with a '1'. The 'CONTENTS' sidebar on the left is circled with a '3', and the 'ON THIS PAGE' section within it is circled with a '4'. The main content area includes sections for Syntax, Description, and Examples. A plot of the sine function is shown at the bottom.



The Documentation Structure II.

- ▶ Check the origin of the function.
 - ▶ Several functions with the same name may exist.
- ▶ Functions types by origin:
 - ▶ MATLAB core functions – most of them build-in, some are available for editing (not recommended!).
 - ▶ Functions from installed toolboxes.
 - ▶ User-created functions.
- ▶ Calling priority for functions will be discussed later.
- ▶ During this course, **always open a function from core installation.**

The screenshot shows a search interface with 'sin' entered in the search bar. Below the search bar, a list of functions is displayed under the heading 'Functions'. Each entry includes the function name, a brief description, and the source toolbox or package.

Function	Description	Source
<code>sin</code>	Sine of argument in radians	MATLAB
<code>sin</code>	Symbolic sine function	Symbolic Math Toolbox
<code>sin</code>	Sine of fixed-point values	Fixed-Point Designer
<code>sind</code>	Sine of argument in degrees	MATLAB
<code>sinh</code>	Hyperbolic sine of argument in radians	MATLAB

» 136 more



Workspace Browser

- ▶ List of variables.
- ▶ Deleting/modification of existing variables.
- ▶ Saving/loading.
- ▶ Values, Class and Memory information.
 - ▶ Other information can be added: size, min, max, ...
 - ▶ All information can be obtained using MATLAB functions that we learn later, *e.g.*, min, max, length.
- ▶ Fast data plotting option (in ribbon).

The screenshot displays the MATLAB Workspace Browser and the Variable Editor for variable 'A'.

Workspace Browser Table:

Name	Value	Class	Bytes
1	1	double	8
A	[-1,1;1,-2]	double	32
5	5	double	8
B	[1,2,3;4,5,6;7,8,9]	double	72
c	[1,0,0]	double	24
d	[0;1;0]	double	24

Variable Editor (Variable: A):

2x2 double

	1	2	3	4	5	6
1	-1	1				
2	1	-2				
3						
4						
5						



MATLAB Commands

- ▶ Matlab is **cAsE sEnSiTiVe!**
 - ▶ Almost entirely, with certain exceptions (properties of graphics objects, ...).
 - ▶ Pay attention to typos and variable names (see later).
 - ▶ New versions of MATLAB offer certain options.

```
>> AA = [1 1 1]
>> Aa
```

- ▶ Beware of different syntax in Mathematica.
 - ▶ Following syntax is incorrect both in MATLAB and Mathematica:

```
>> Sin(pi/2) % function names start with lower case
>> cos[pi/2] % function input is in parentheses ()
```

- ▶ Will be discussed in the next lectures.



Naming Conventions

- ▶ Names of variables can have max. 63 characters starting with letter (» `namelengthmax`)
 - ▶ Letters and numbers are allowed, other symbols (colon “:”, hyphen “-” and others) are not.
 - ▶ Underscore is allowed in the variable name “_” (not at the beginning, though!).
- ▶ Lowercase letters in the names of scalars and variables (`a = 17.59;`).
- ▶ Matrix names usually start with a capital letter (`A = [..];`).
- ▶ Iteration variables, variables used in `for` cycles usually named `m`, `n`, `k`, etc.
 - ▶ It is advisable to avoid `i` and `j` (complex unit).
- ▶ Chose the names to correspond to the purpose of the variable.
- ▶ Avoid, if possible, standalone letter “`l`” (to be confused with one “`1`”) and predefined variables in MATLAB environment (see later).
- ▶ Choose names corresponding to the meaning of each particular variable.
- ▶ Avoid using names of existing functions or scripts (overloading can occur).
- ▶ The same conventions are valid for names of functions and scripts.



Variable Names

- ▶ Examples of valid variable names:

```
a, A, b, c, x1, x2, M_12, test1, matrix_A, fx, fX
```

- ▶ Examples of invalid variable names:

```
1var      % starts with a number (not possible in MATLAB)
matrix A  % contains space
coef.a    % possible only if coef is of type 'struct'
Test-1    % algebraic expressing: ans = Test - 1
f(y)      % makes sense when using symbolic expressions
```

- ▶ Examples of valid numbers in MATLAB,

```
3, -66, +0.0015, .015, 1e2, 1.6025e-10, 05.1
```



Functions who, whos

- ▶ Function `who` lists all variables in MATLAB Workspace.
 - ▶ Wide variety of options.
- ▶ Functions `whos` lists the variable names + dimension, size and data type of the variables or displays content of a file.
 - ▶ Wide variety of options.

```
>> whos('-file', 'matlab.mat');
```

```
>> a = 15; b = true; c = 'test'; d = 1 + 5j;  
>> who  
>> whos  
>> Ws = whos;
```



Workspace – Output Deletion

- ▶ To clean (erase) command window:

```
>> clc
```

- ▶ To clean one (or more) variable(s):

```
>> clear          % whole Workspace is deleted  
>> clear XX      % variable XX is deleted  
>> clear XX YY   % variables XX and YY are deleted  
>> clear z*      % everything starting with 'z' is deleted
```

- ▶ clear has a number of other options (graphics, I/O)



Command History Window

- ▶ Command History window stores all commands from the Command Window.
- ▶ Command History is accessible through \uparrow or \downarrow .
- ▶ it is possible to filter out past commands by, *e.g.*:
 - » `A = [+ ↑`.
- ▶ It is possible to copy-and-paste entire Command History:
`SHIFT / CTRL / CTRL + A → CTRL + C`.



Matrices in MATLAB

- ▶ Matrix is a basic data structure in MATLAB.
- ▶ There are following variables types depending on size:
 - ▶ scalar: 1×1
 - ▶ vector: $M \times 1$ or $1 \times N$
 - ▶ matrix: $M \times N$
 - ▶ array (multidimensional matrices):
 $M \times N \times P \times Q \times R \times \dots$
- ▶ Matrices can be complex.
- ▶ It can contain text as well (beware the length).

- ▶ M -by- N matrix:

$$\begin{array}{c}
 a_{i,j} \\
 \downarrow \\
 M \text{ rows} \\
 i \text{ changes}
 \end{array}
 \begin{array}{c}
 \xrightarrow{N \text{ columns}} \\
 j \text{ changes}
 \end{array}
 \begin{bmatrix}
 a_{1,1} & a_{1,2} & a_{1,3} & \dots \\
 a_{2,1} & a_{2,2} & a_{2,3} & \dots \\
 a_{3,1} & a_{3,2} & a_{3,3} & \dots \\
 a_{4,1} & a_{4,2} & a_{4,3} & \dots \\
 \vdots & \vdots & \vdots & \ddots
 \end{bmatrix}$$



Matrix Creation

- ▶ Following techniques are available:
 - ▶ element-by-element entering (suitable for small matrices only),
 - ▶ colon notation “:” to define elements of series,
 - ▶ generation by built-in functions,
 - ▶ generation of matrices in m-files,
 - ▶ import and export from/to external files(.mat, .txt, .xls, ...).



Matrix Construction Element-by-element I.

- ▶ Test following commands to construct matrices by element enumeration.
 - ▶ Suitable for small matrices only.

```
>> a1 = -1
```

```
>> v1 = [-1 0 1]
>> v2 = [-1; 0; 1]
```

```
>> M1 = [-1 0 1; -2 0 2]
>> M2 = [-1 -2; 0 0 ; 1 2]
>> M3 = [[-1 -2]; [0 0]] % inner brackets are redundant
```

$$a_1 = a_2 = -1$$

$$\mathbf{v}_1 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{M}_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\mathbf{M}_2 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{M}_3 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}$$



Matrix Construction Element-by-element II.

- ▶ Construct following matrices:
 - ▶ Matrix values are defined inside square brackets [],
 - ▶ semicolon “;” separates individual rows of a matrix.

$$\mathbf{A} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$





Matrix Construction

- ▶ Semicolon placed at the end of a command suppresses display of the output in Command Window.

```
>> a = 1  
>> b = 5;
```

- ▶ When there is more than one command on the same line, comma is used to separate each of the commands.

```
>> a = 1, b = 5  
>> a = 1; b = 5;
```

- ▶ Note: it is possible to copy and paste code including “»”

- ▶ Row vs column vector:

```
>> c = [1 0 0]  
>> d = [0; 0; 1]
```



Basic Math Operators I.

- ▶ Operator types:
 - ▶ arithmetics:
 - ▶ matrix,
 - ▶ vector,
 - ▶ relational,
 - ▶ logical and other (to be mentioned later ...).
- ▶ Other operations using MATLAB functions:
 - ▶ complex conjugate,
 - ▶ sum, determinant, square root,
 - ▶ and hundreds of other functions ...

+	addition
-	subtraction
*	multiplication
^	power
.'	transpose
\	left matrix division
/	right matrix division
.	dot notation



Operator Precedence in MATLAB

► According to the following table:

- see MATLAB → Language Fundamentals → Operators and Elementary Operations → Arithmetic

1	parentheses	()					
2	transpose, power	'	.'	^	.'	^	
3	unary plus, unary minus, logical negation	+	-	~			
4	multiplication, division	*	.*	/	\	./	.\
5	addition, subtraction	+	-				
6	colon operator	:					
7	relation operators	<	>	<=	>=	==	~=
8	logical AND (element-wise)	&					
9	logical OR (element wise)						
10	logical AND (short-circuit)	&&					
11	logical OR (short-circuit)						



Basic Math Operators II.

- ▶ Type in following commands:

- ▶ Zero can be omitted with a decimal number beginning with zero (not recommended).

```
>> a3 = -2/4  
>> a4 = -0.5  
>> a5 = -.5
```

- ▶ What is the difference between a_3 , a_4 and a_5 ?
- ▶ Beware the precedence of operators (we see in the next slides):

```
>> 3*5*6  
>> a1 = 15  
>> a2 = 10;  
>> a2/a3  
>> a2/a3*a4  
>> a2/(a3*a4)
```

- ▶ Explain the difference between a_2/a_3*a_4 and $a_2/(a_3/a_4)$.
- ▶ Verify the rules of operator precedence from the previous slide.



Lengthy commands in MATLAB

- ▶ It is suitable to structure command blocks for clarity:
 - ▶ next line: SHIFT + ENTER

```
>> A = [1 1 1]; B = [2 2 2]; % SHIFT + ENTER  
C = [2 3 2];
```

- ▶ Three dots notation:
 - ▶ For continuation of the same command on the next line.
 - ▶ Compare results:

```
>> A1 = [ 1 1 ...  
2 3]
```

```
>> A2 = [ 1 1  
2 3]
```



Basic Math Functions I.

- ▶ Math functions in MATLAB are generally divided in three groups:
 - ▶ **Scalar:**
 - ▶ Function operates over individual elements of a matrix,
 - ▶ *e.g.:* sin, sqrt, log, factorial.
 - ▶ **Vector:**
 - ▶ Function operates over individual rows/columns of a matrix,
 - ▶ *e.g.:* sum, max.
 - ▶ **Matrix:**
 - ▶ Function operates over a whole matrix,
 - ▶ *e.g.:* det, trace.



Basic Math Functions II.

- ▶ Using MATLAB help, calculate the following expression: $a \sin^2(\alpha) + a \cos^2(\alpha) - a$
 - ▶ Use numerical values your own choice.

- ▶ Verify following logarithmic identity: $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

- ▶ Find sum of all elements in individual rows of the following matrix:

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 7 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{bmatrix}$$





Basic Math Functions III.

- ▶ Assume following vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.

- ▶ Calculate:

$$\begin{array}{cc} \mathbf{u}\mathbf{v}^T & \mathbf{v}\mathbf{u}^T \\ \mathbf{v}^T\mathbf{u} & \mathbf{u}^T\mathbf{v} \\ \mathbf{u} \cdot \mathbf{v} & \mathbf{u} \times \mathbf{v} \end{array}$$

- ▶ Following functions are needed:
 - ▶ transpose (.'') of a matrix,
 - ▶ dot scalar product,
 - ▶ cross product.
- ▶ What is the result of the above mentioned operations?

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$





Matrix Division in MATLAB

- ▶ Two cases are distinguished:
 - ▶ **left** division (`\` - `mldivide`),
 - ▶ **right** division (`/` - `mrdivide`).
- ▶ Solution of a linear system of equations:
 - ▶ **A** is an invertible (regular) matrix,
 - ▶ **b** is a column (row) vector.

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```

$$\mathbf{xA} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{bA}^{-1}$$

```
>> x = b / A
```



Basic Math Functions IV.

- ▶ Find the sum of diagonal elements (trace of a matrix) of the matrix \mathbf{T} with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4.
- ▶ Find determinant of matrix \mathbf{U} .

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

- ▶ Solve the linear system of equations:

$$x_1 + 2x_2 + 3x_3 = 6$$

$$\mathbf{Ax} = \mathbf{b}$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$7x_1 + 8x_2 + x_3 = 16$$

```
>> T = 10 + 4*randn(7, 7);
```

```
>> U = [1 2 3; 0 2 0; ...  
0 -2 -1];
```



Predefined Values in MATLAB

- ▶ MATLAB contains several predefined values:
 - ▶ `eps` – precision of single/double numbers (Determines the shortest distance between two single/double numbers).
 - ▶ `ans` – *answer* – most recent answer.
 - ▶ `NaN` – *not a number* (every expression containing NaN is NaN)
 - ▶ NaN can be used advantageously in some cases.
 - ▶ `Inf` – *infinite number* (variable `Inf` can be used in calculation:))
 - ▶ Pay attention to `Inf` propagation throughout your code (use allowed operations only).
 - ▶ `i`, `j` – complex unit.
 - ▶ They are all basically functions (without input parameter).
 - ▶ Check results of the following expressions:

```

>> t1 = 10/0    % t1 = Inf
>> t2 = 0/0    % t2 = NaN
>> t3 = t1*5    % t3 = Inf
>> t4 = t1 + t2 % t4 = NaN
  
```

- ▶ `pi`, `intmin`, `intmax`, `realmin`, `realmax`, ... (functions)



Format of Command Line Output

- ▶ Up to now we have been using basic setup.
- ▶ MATLAB offers number of other formatting options
 - ▶ Use format `style`.
 - ▶ Output format does not change neither the computation accuracy nor the accuracy of stored results (`eps`, `realmax`, `realmin`, ... still apply).

<code>style</code>	format description
<code>short</code>	fixed 4 decimal points are displayed
<code>long</code>	15 decimal points for double precision, 7 decimal points for single precision
<code>shortE</code>	floating-point format (scientific notation)
<code>longE</code>	-//-
<code>bank</code>	two decimal points only (eur – cents)
<code>rat</code>	MATLAB attempts to display the results as a fraction
<code>compact</code>	suppressed the display of blank lines
and others...	note: omitting <code>style</code> parameter restores default setup



Format of Command Line Output

- ▶ Try following output format settings:
 - ▶ Each format is suitable for different type of problems.
- ```
>> s = [-5 1/2 1/3 10*pi sqrt(2)];
>> format long; s
>> format rat; s
>> format bank; s
>> format hex; s
>> format +; s
>> format; s
```
- ▶ There exist other formats with slight differences.
    - ▶ Check doc `format`
  - ▶ Later, we will learn how to use formatted conversion into strings (commands `sprintf` and `fprintf`).



# Complex Numbers I.

- ▶ More entry options in MATLAB.

```
>> C1 = 1 + 1j % preferred
>> C2 = 1 + 5i % preferred
>> C3 = 1 + 5*i % NO!
>> C4 = sqrt(-1)
>> C5 = complex(1, 2)
>> C6 = 1e1i
>> C7 = exp(1j*pi/4)
```

- ▶ Frequently used functions:

---

|            |                                                              |
|------------|--------------------------------------------------------------|
| real, imag | real and imaginary part of a complex number                  |
| conj       | complex conjugate                                            |
| abs        | absolute value of a complex number                           |
| angle      | angle in complex plane [rad]                                 |
| complex    | constructs complex number from real and imaginary components |
| isreal     | checks if the input is a complex number (more on that later) |
| i, j       | complex unit                                                 |
| cplxpair   | sort complex numbers into complex conjugate pairs            |

---



## Complex Numbers II.

- ▶ Create complex number  $z = 1 + 1j$  and its complex conjugate  $s = z^*$ .
- ▶ Switch between Cartesian and polar form (find  $|z|$  and  $\varphi$ ).

$$z = \operatorname{Re}\{z\} + \operatorname{Im}\{z\} = a + jb$$

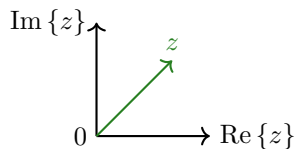
$$z = |z| e^{j\varphi}, |z| = \sqrt{a^2 + b^2}$$

$$z = |z| (\cos \varphi + j \sin \varphi)$$

- ▶ Verify Moivre's theorem:

$$z^n = (|z| e^{j\varphi})^n$$

$$z^n = |z|^n (\cos(n\varphi) + j \sin(n\varphi))$$



# Exercises

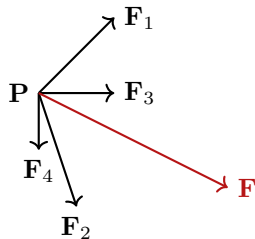


## Exercise I.

- ▶ Following forces were localized at point **P** in  $xy$  plane:

$$\begin{aligned} \mathbf{F}_1 &= [2, 2] & \mathbf{F}_3 &= [2, 0] \\ \mathbf{F}_2 &= [1, -3] & \mathbf{F}_4 &= [2, -1.5] \end{aligned}$$

- ▶ What is the direction of the resultant force **F**?
- ▶ Normalize the resulting vector.



$$\mathbf{n}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$



## Exercise II.

- ▶ Type-in following commands:

```
>> clear, clc;
>> w1 = [1 2 3 4]
>> w2 = [-2 -3 -4]
>> w3 = [-2; -3; -4]
>> w4 = w1^2, w5 = w2 - w1
```

- ▶ Compare differences.
  - ▶ What is the cause of error in calculation of w4 and w5?
- ▶ Try also:

```
>> w3*3, w1 - 3
>> w1 + [5 5 5 5]
>> w6 = 5*w1 - [3 5 6] - w2
```

- ▶ Calculate the norm (magnitude) of vector w1.
  - ▶ Try more options.
- ▶ How to modify the calculation in the case of a complex vector?



## Exercise III.

- ▶ Calculate roots of the quadratic function:

$$-2x^2 - 5x = 3.$$

- ▶ First, rearrange the terms of the function.

$$2x^2 + 5x + 3 = 0 \Rightarrow a = 2, b = 5, c = 3$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$x_1 = -1, x_2 = -\frac{3}{2}$$

- ▶ MATLAB provides particular function for calculation of roots a function, try to search it out.



## Exercise IV.

- ▶ Think over how many ways there are to calculate the length of hypotenuse when two legs of a triangle are given.
  - ▶ Make use of various MATLAB operators and functions.
  - ▶ Consider also the case where the legs are complex numbers.





## Exercise V.

- ▶ Create an arbitrary vector  $\mathbf{v}$  and rotate it around arbitrary angle  $\alpha$  in  $xz$  plane using rotation matrix  $\mathbf{R}$ .

$$\mathbf{v}' = \mathbf{R}\mathbf{v}$$
$$\mathbf{R} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$



## Exercise VI.

- ▶ Use the following code and round the resulting number to:

```
>> r = 1 + 10*rand(1)
```

- ▶ nearest integer,
  - ▶ nearest integer greater than  $r$ ,
  - ▶ nearest integer lower than  $r$ ,
  - ▶ zero,
  - ▶ zero with precision of 2 decimal digits.
- ▶ Find remainder after  $r$  is divided by 0.1.
    - ▶ *modulus* vs. *remainder after division*



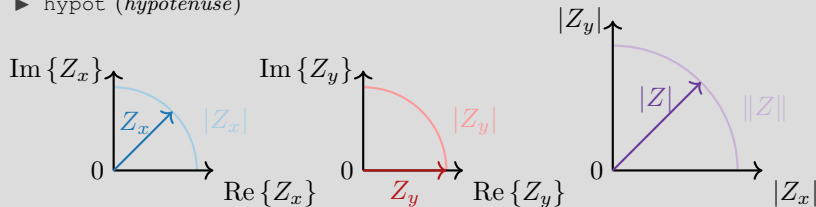
## Exercise VII.

- Find out the magnitude of a complex vector (avoid indexing).
  - Use `abs` and `sqrt`.

$$\mathbf{Z} = [ 1 + 1j \quad \sqrt{2} ]$$

$$\|\mathbf{Z}\| = ?, \quad \mathbf{Z} \in \mathbb{C}^2$$

- Alternatively, use following functions:
  - `norm`
  - `dot` (*dot product*)
  - `hypot` (*hypotenuse*)



# Questions?

B0B17MTB – Matlab  
matlab@fel.cvut.cz

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