Ambiguity in the reconstruction from 2 calibrated views

$$\zeta_1 \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} = \underbrace{\begin{bmatrix} \mathtt{R}_1 & -\mathtt{R}_1 \vec{C}_{1\delta} \end{bmatrix}}_{\mathtt{P}_{1\gamma_1}} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix}$$

$$\zeta_2 \mathtt{K}_2^{-1} \vec{x}_{2\beta_2} = \underbrace{\begin{bmatrix} \mathtt{R}_2 & -\mathtt{R}_2 \vec{C}_{2\delta} \end{bmatrix}}_{\mathtt{P}_{2\gamma_2}} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix}$$

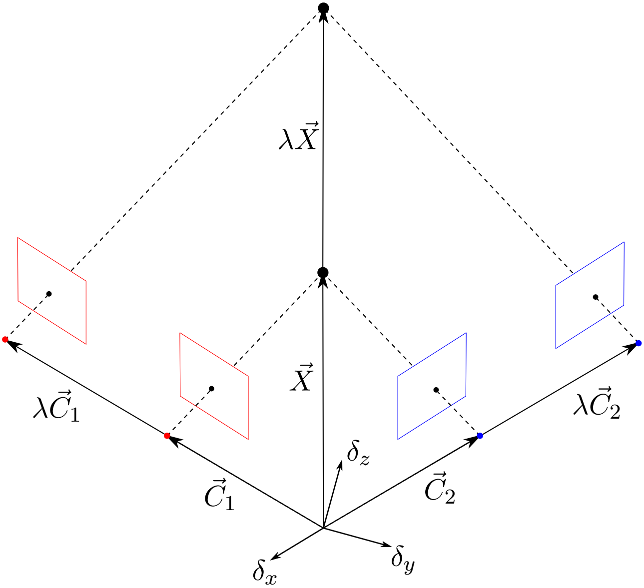
The solution set S is infinite since

$$\left(\mathtt{P}_{1\gamma_1}, \mathtt{P}_{2\gamma_2}, \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \right) \in S \Rightarrow \left(\mathtt{P}_{1\gamma_1} \mathtt{H}^{-1}, \mathtt{P}_{2\gamma_2} \mathtt{H}^{-1}, \mathtt{H} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \right) \in S$$

for

$$\mathtt{H} = \begin{bmatrix} \mathsf{R} & \mathsf{c} \\ \mathsf{0}^\top & \lambda \end{bmatrix}$$

Ambiguity in the reconstruction from 2 calibrated views



Ambiguity in the reconstruction from 2 calibrated views

Our aim is to find just 1 solution. We start from the unknown solution

$$\left(\mathtt{R}_1, \vec{C}_{1\delta}, \mathtt{R}_2, \vec{C}_{2\delta}, \vec{X}_\delta\right)$$

and transform it by

$$\mathtt{H}^{-1} = \begin{bmatrix} \mathtt{R}_1^\top & \vec{C}_{1\delta} \\ \mathtt{0}^\top & 1 \end{bmatrix}$$

to get a partially known solution

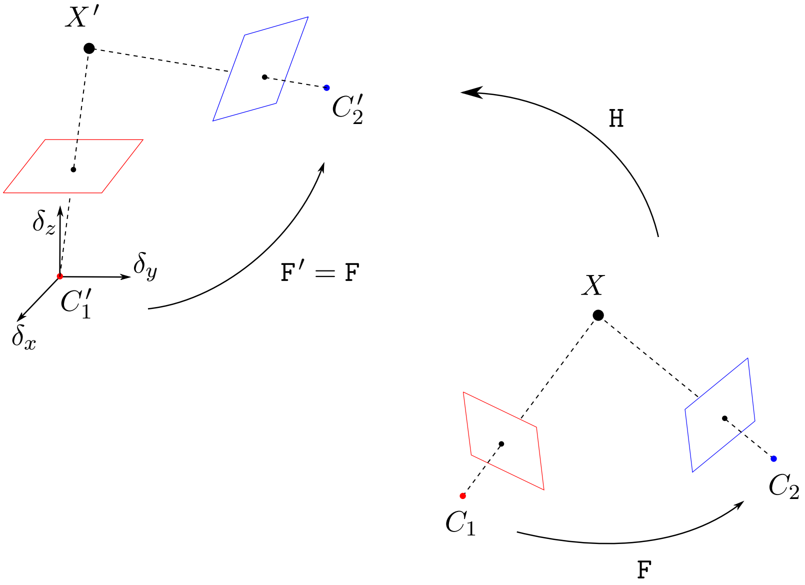
$$\left(\mathtt{R}_1', \vec{C}_{1\delta}', \mathtt{R}_2', \vec{C}_{2\delta}', \vec{X}_\delta'\right) = \left(\mathtt{I}, \vec{\mathtt{0}}, \mathtt{R}_2\mathtt{R}_1^\top, \mathtt{R}_1(\vec{C}_{2\delta} - \vec{C}_{1\delta}), \mathtt{R}_1\vec{X}_\delta - \vec{C}_{1\delta}\right)$$

with the fundamental matrix

$$\mathtt{F}' = \mathtt{K}_2^{-\top}\mathtt{R}_2' \left[\vec{C}_{2\delta}' - \vec{C}_{1\delta}'\right]_\times \mathtt{R}_1'^\top \mathtt{K}_1^{-1} = \mathtt{K}_2^{-\top}\mathtt{R}_2' \left[\vec{C}_{2\delta}'\right]_\times \mathtt{K}_1^{-1} =$$

$$= \mathtt{K}_2^{-\top}\mathtt{R}_2\mathtt{R}_1^\top \left[\mathtt{R}_1(\vec{C}_{2\delta} - \vec{C}_{1\delta})\right]_\times \mathtt{K}_1^{-1} = \mathtt{K}_2^{-\top}\mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top \mathtt{K}_1^{-1} = \mathtt{F}$$

Ambiguity in the reconstruction from 2 calibrated views



Fundamental matrix computation

Key note: it is **not** possible to reveal the fundamental matrix as it is defined by

$$\mathtt{F} = \mathtt{K}_2^{-\top}\mathtt{R}_2\left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top\mathtt{K}_1^{-1}$$

using **only image correspondences**. We can find only its scalar multiple $\tau\mathtt{F}$.

There are 2 reasons for that: algebraic and geometric.

Algebraic reason

The equations

$$\vec{x}_{2\beta_2}\mathtt{F}\vec{x}_{1\beta_1} = 0, \quad \det \mathtt{F} = 0$$

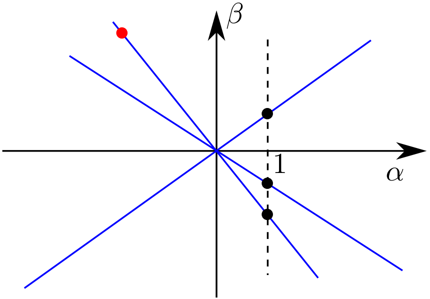
are homogeneous, i.e.

$$\mathtt{F} \text{ is a solution } \Rightarrow \tau\mathtt{F} \text{ is a solution.}$$

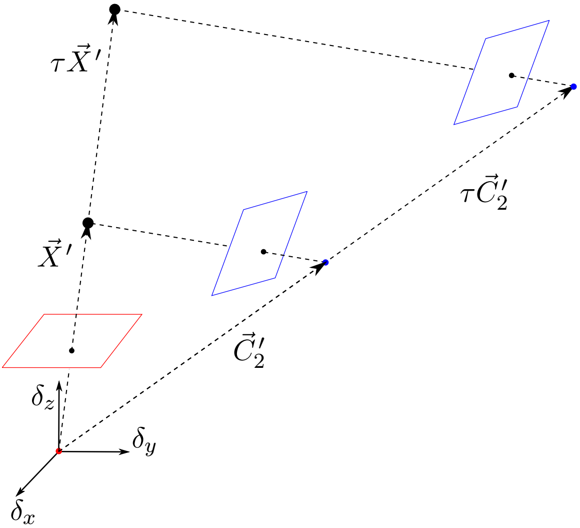
Equation

$$\det(\alpha\mathtt{G}_1 + \beta\mathtt{G}_2) = 0$$

gives the following picture of solutions:



Geometric reason



4 solutions $(\mathtt{R}, \vec{C}_\delta)$ with $\|\vec{C}_\delta\| = 1$ for a given $\mathtt{E}$

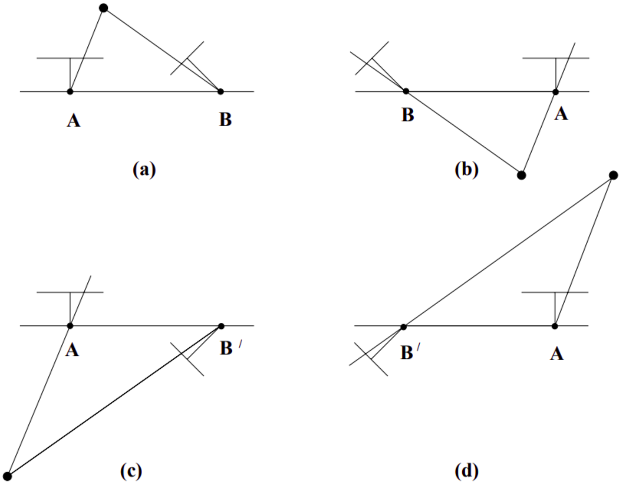


Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates* 180° *about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*