LP-based Heuristics

h^{flow}, h^{pot}

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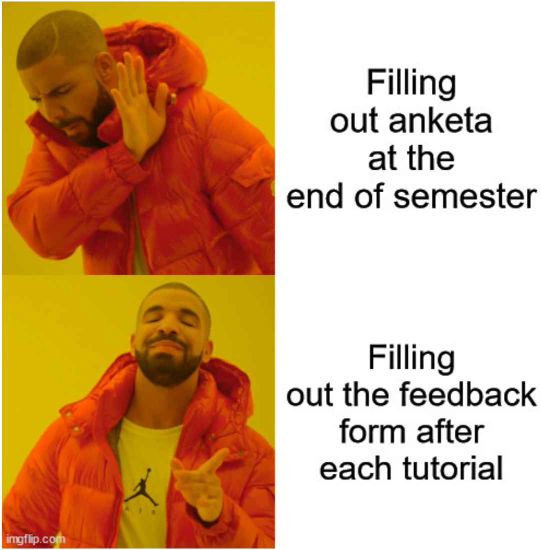
PUI Tutorial
Week 5

Lecture check

- Any questions regarding the lecture?

Feedback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LP-based heuristics

Linear program

Linear program (LP) consists of:

- a finite set of real-valued variables **V**
- a finite set of linear **constraints over V**
- an **objective function** *(linear combination of V)*

Integer linear program (ILP) is the same thing with integer-valued variables.

LP-based heuristics

- LP - solution in **polynomial time**
- ILP - finding solution is **NP-complete**
- We can approximate ILP solution with corresponding LP
- Sounds familiar? **Relaxation**
- Flow heuristic - h^{flow}
- Potential heuristic - h^{pot}

Running example

FDR problem example

FDR planning task $P = \langle \mathbf{V}, O, s_{init}, s_{goal}, c \rangle$

- $\mathbf{V} = \{A, B, C\}$
- $D_A = \{D, E\}$; $D_B = \{F, G\}$; $D_C = \{H, J, K\}$
- $s_{init} = \{A = D, B = F, C = H\}$
- $s_{goal} = \{A = D, C = K\}$
- $O = \{o_1, o_2, o_3, o_4, o_5\}$

	pre	eff	c
o_1	$\{A = D, C = H\}$	$\{A = E, C = J\}$	2
o_2	$\{A = D\}$	$\{B = G\}$	1
o_3	$\{B = G, C = J\}$	$\{C = K\}$	1
o_4	$\{A = E\}$	$\{A = D\}$	2
o_5	$\{C = H\}$	$\{C = J\}$	5

Producing and consuming

For every variable $V \in \mathbf{V}$ and every value $v \in D_V$ we define

- a set of operators **producing** $\langle V, v\rangle$:
  $prod(\langle V, v\rangle) = \{o | o \in O, V \in vars(eff(o)), eff(o)[V] = v\}$
- a set of operators **consuming** $\langle consuming\rangle$:
  $cons(\langle V, v\rangle) = \{o | o \in O, V \in vars(pre(o)) \cap vars(eff(o)), pre(o)[V] = v, pre(o)[V] \neq eff(o)[V]\}$

- FDR planning task $P = \langle \mathbf{V}, O, s_{init}, s_{goal}, c \rangle$
- state s reachable from s_{init}
- real-valued variable x_o for each $o \in O$

LP formulation

$$\text{minimize} \sum_{o \in O} c(o) x_o$$

$$\text{subject to } LB_{V,v} \leq \sum_{o \in prod(\langle V,v \rangle)} x_o - \sum_{o \in cons(\langle V,v \rangle)} x_o, \forall V \in \mathbf{V}, \forall v \in D_V$$

$$\text{where } LB_{V,v} = \begin{cases} 0 & \text{if } V \in vars(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in vars(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin vars(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin vars(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

LP formulation

$$\text{minimize} \sum_{o \in O} c(o) x_o$$

$$\text{subject to } LB_{V,v} \leq \sum_{o \in prod(\langle V,v \rangle)} x_o - \sum_{o \in cons(\langle V,v \rangle)} x_o, \forall V \in \mathbf{V}, \forall v \in D_V$$

$$\text{where } LB_{V,v} = \begin{cases} 0 & \text{if } V \in vars(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in vars(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin vars(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin vars(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

The **value of h^{flow} heuristic** for the state s is

$$\mathrm{h}^{flow}(s) = \begin{cases} \lceil \sum_{o \in O} c(o) x_o \rceil & \text{if the solution is \textbf{feasible}} \\ \infty & \text{if the solution is \textbf{not feasible}} \end{cases}$$

Long story short

- Define variable x_o for each operator *(operator "counters")*
- Create *prod* and *cons* sets for each operator $o \in O$
- Write constraints with $LB_{V,v}$ constants on the side
- Compute constants $LB_{V,v}$ based on the 4 rules
- Put it in a solver
- ???
- PROFIT!

h^{pot}

- FDR planning task $P = \langle \mathbf{V}, O, s_{init}, s_{goal}, c \rangle$
- state s reachable from s_{init}
- real-valued variable $P_{V,v}$ for each variable $V \in \mathbf{V}$ and each value $v \in D_V$
  - **potential** corresponding to $\langle V, v \rangle$
- real-valued variable M_V for each variable $V \in \mathbf{V}$
  - **upper bound** on the potentials of variable V
  - used in situations where we don't know the value $\rightarrow$ prepare for the worst case
  - just like variable B in the example goal
- unknows values can be solved in two ways
  - adding separate constraint for each $v \in D_V$ *(can be exponential in the worst case)*
  - adding M_B to the goal-awareness constraint and add auxiliary constraints *(linear number of constraints)*

h_{pot}

Goal-awareness constraint: $P_{A,D} + P_{C,K} \leq 0$ ...what about B?

- Add each case of B
  - $P_{A,D} + P_{B,F} + P_{C,K} \leq 0$
  - $P_{A,D} + P_{B,G} + P_{C,K} \leq 0$
- Use the M_B bound
  - $P_{A,D} + M_B + P_{C,K} \leq 0$
  - $P_{B,F} \leq M_B$
  - $P_{B,G} \leq M_B$

LP formulation

$$\text{maximize} \sum_{V \in \mathbf{V}} P_{V, s_{init}[V]}$$

$$\text{subject to } P_{V,v} \leq M_V, \forall V \in \mathbf{V}, \forall v \in D_V$$

$$\sum_{V \in \mathbf{V}} maxpot(V, s_{goal}) \leq 0$$

$$\sum_{V \in vars(eff(o))} (maxpot(V, pre(o)) - P_{V, eff(o)[V]}) \leq c(o), \forall o \in O$$

$$\text{where } maxpot(V, p) = \begin{cases} P_{V, p[V]} & \text{if } V \in vars(p), \\ M_V & otherwise. \end{cases}$$

The **value of h^{pot} heuristic** for the state s is

$$h^{pot}(s) = \begin{cases} \sum_{V \in \mathbf{V}} P_{V, s[V]} & \text{if the solution is } \textbf{feasible} \\ \infty & \text{if the solution is } \textbf{not feasible} \end{cases}$$

h^{pot}

Long story short

- Define potential $P_{V,v}$ for each variable and its possible value
- Define potential upper bound for each variable $V \in \mathbf{V}$
- When computing $h^{pot}(s)$ we want to maximize sum of potentials of $\langle V, v \rangle$ pairs in s
- define goal-awareness constraints
- define consistency constraints with respect to operator costs
- Solve $\rightarrow$ get the potentials

Recap

- Know definition of h^{flow} and h^{pot} heuristics
- Know how to define them and compute them

The End

Feedback form link