Temporal network example

and extensions

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PUI Tutorial
Week 11

Outline

- Quick recap of Simple Temporal Networks
- Simple Temporal Network example

Motivation

Many planners don't work with time explicitly, STNs and their extentions can be used to:

- check time consistency of a plan under time constraints,
- if consistent, determine temporal schedule,
- and manage real-time execution of a plan and new constraints.

Simple Temporal Networks [1]

Def: Simple Temporal Network

A *Simple Temporal Network* (STN) is a pair $S = (T, C)$ where:

- T is a set of *time-points*, real valued variables
- C a set of constraints of the form:

$$Y - X \leq \delta$$

for $X, Y \in T$ and $\delta \in \mathbb{R}$

[1] Slides based mostly on AIMA, these slides and this example, definition by [Dechter et al., 1991]

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We map STNs to graphs. How?

- Variables $\rightarrow$ nodes
- Constraints $\rightarrow$ edges

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Example

I have a plan for getting to the PDV exam:

- Take a train from Kolín to Prague-Libeň
- Walk from Prague-Libeň to Vysočanská (Yellow - B line)
- Take metro from Vysočanská to Karlovo náměstí

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Regardless whether there are other possible (better) plans, we want to check whether this one is consistent (i.e. feasible) under following constraints:

- You will get to the station in Kolin at 8:00
- Train ride takes *at least* 50 minutes
- Walking takes 10 to 20 minutes
- Ride on the metro takes *at most* 20 minutes
- You have to be at the PDV exam by 9:30

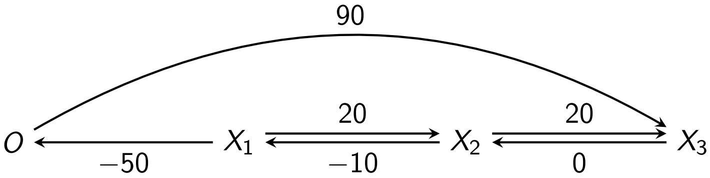
Example

$$O \text{ –Train– } X_1 \text{ – (walk) – } X_2 \text{ –Metro– } X_3$$

Given the constraints, we have $S = (T, C)$:
(We introduce a special reference variable (node), $O = 0$ as a starting point.)

$$T = \{O, X_1, X_2, X_3\}, \text{where } O \text{ maps to 8:00}$$

$$C = \begin{cases} O - X_1 \leq -50 & \text{train} \\ X_2 - X_1 \leq 20 & \text{walk} \\ X_1 - X_2 \leq -10 & \text{walk} \\ X_3 - X_2 \leq 20 & \text{metro} \\ X_3 - O \leq 90 & \text{exam start} \end{cases}$$



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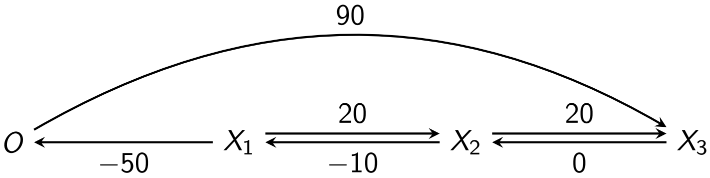

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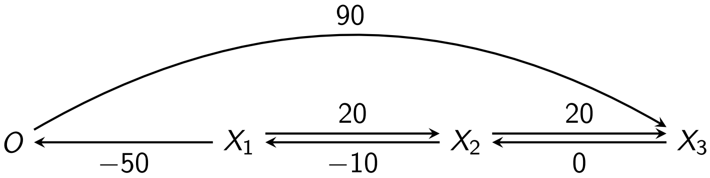

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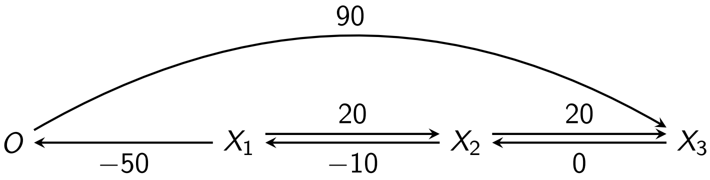
Tip: Right-to-left arrows are (+) upper bounds, left-to-right are (-) lower bounds

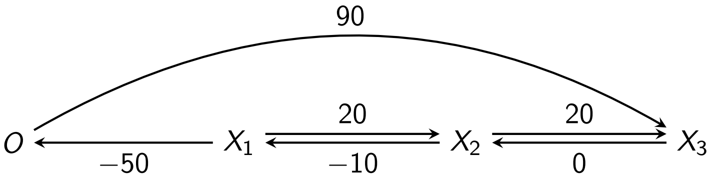
Example



Now, we can calculate shortest path lengths between all combinations of nodes:

D	O	X_1	$X2$	X_3
O	0	∞	∞	90
X_1	-50	0	20	∞
X_2	∞	-10	0	20
X_3	∞	∞	0	0

Example



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(e.g. by using Floyd-Warshall in more complex cases)

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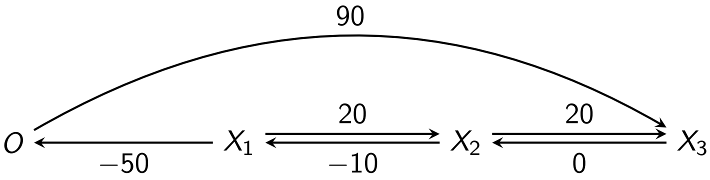

Now, we can calculate shortest path lengths between all combinations of nodes:

D	O	X_1	$X2$	X_3
O	0	80	90	90
X_1	-50	0	20	40
X_2	-60	-10	0	20
X_3	-60	-10	0	0

(e.g. by using Floyd-Warshall in more complex cases)

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Questions:

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- A: Yes.
- Q: When would it be inconsistent?
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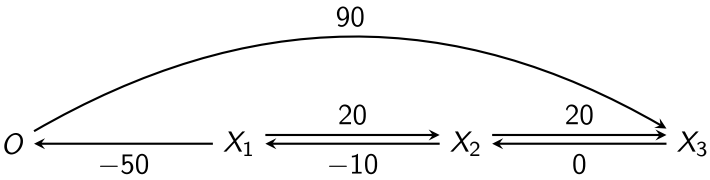
Thm: Fundamental Theorem of STNs

STN consistent $\iff$ Distance matrix has zeros on diagonal *iff* graph has no negative cycles

Summary



- Solution is an assignment of values to timepoints (nodes) that satisfies given constraints.
- If such solution exists, it solution is consistent.
- Consistency can be checked by checking the distance matrix.

Thank you for participating in the tutorials :-)

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