Lecture 6: Q-Learning and DQN

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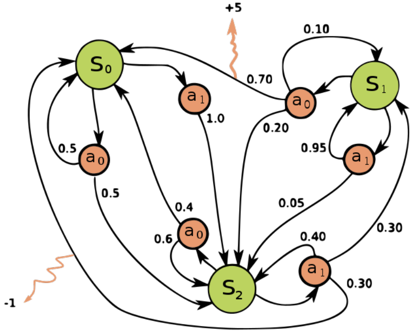
March, 2021

Plan of today's lecture

1. RL algorithms in tabular representation for unknown MDP
2. Scaling up with Neural Networks
3. DQN algorithm and its application to Atari games

Remember MDP

Standard model for Reinforcement Learning problems



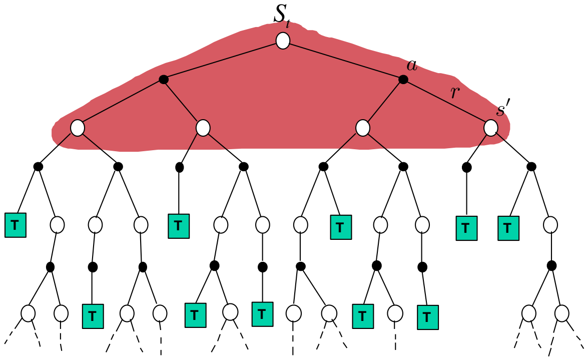
- S – states
- R – rewards
- A – actions
- Discrete steps $t = 0, 1, 2, \ldots$
- Environment *dynamics*

Source: Waldoalvarez @ wikimedia

$$p(s', r|s, a) \leftarrow Pr\{S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a\}$$

Dynamic Programming

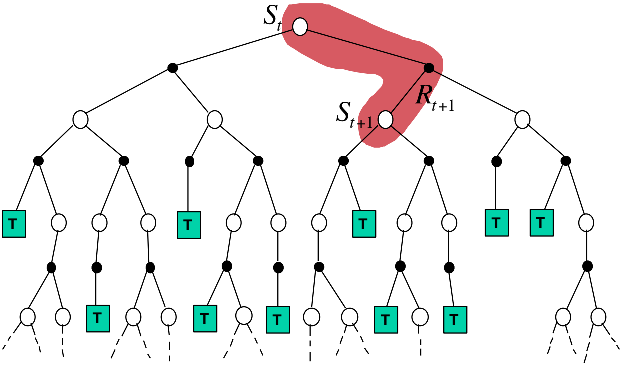
$$V(S_t) \leftarrow E_\pi\left[R_{t+1} + \gamma V(S_{t+1})\right] = \sum_a \pi(a|S_t) \sum_{s',r} p(s',r|S_t,a)[r + \gamma V(s')]$$



(Based on slides shared by R. Sutton)

Simplest Temporal Difference Method

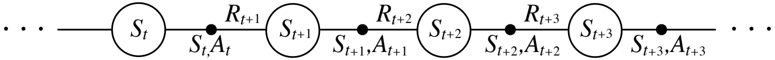
$$V(S_t) \leftarrow V(S_t) + \alpha\left[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right]$$



(Based on slides shared by R. Sutton)

Learning An Action-Value Function

Estimate q_π for the current policy π



After every transition from a nonterminal state, S_t, do this:

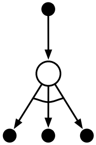
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Q-Learning: Off-Policy TD Control

One-step Q-learning:

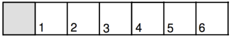
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$



Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
        $S \leftarrow S'$;
    until S is terminal

Q-Learning Example

$(2, \rightarrow) => -0.1$
$(3, \leftarrow) => -0.1$
$(2, \leftarrow) => -0.1$
$(1, \rightarrow) => -0.11$



$\alpha = 0.1$
Default: $(*, *) => 0$

$(2, \leftarrow) => -0.09$
$(1, \leftarrow) => -0.1$

$(2, \leftarrow) => -0.191$
$(1, \leftarrow) => -0.19$

Repeat (for each step of episode):
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Take action A, observe R, S'
  $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$
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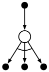
until S is terminal

Q-Learning vs. ϵ-Greedy Bandit

Q-Learning: Off-Policy TD Control

One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \Big]$$



Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
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R. S. Sutton and A. G. Barto: Reinforcement Learning: An Introduction 34

ϵ-Greedy Action Selection

In greedy action selection, you always exploit

In ϵ-greedy, you are usually greedy, but with probability ϵ you instead pick an action at random (possibly the greedy action again)

This is perhaps the simplest way to balance exploration and exploitation

Algorithm ϵ-Greedy:

Initialize, for $a = 1$ to k:
    $Q(a) \leftarrow 0$
    $N(a) \leftarrow 0$

Repeat forever:
    $A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1 - \varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$
    $R \leftarrow \textit{bandit}(A)$
    $N(A) \leftarrow N(A) + 1$
    $Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\big[R - Q(A)\big]$

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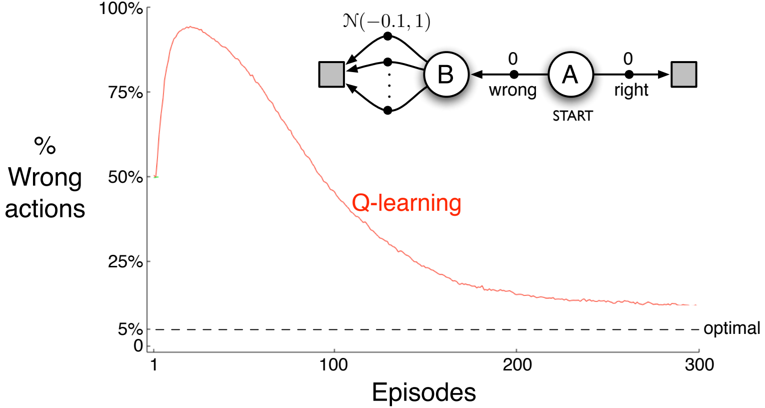


Tabular Q-learning: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \Big]$

Double Q-Learning

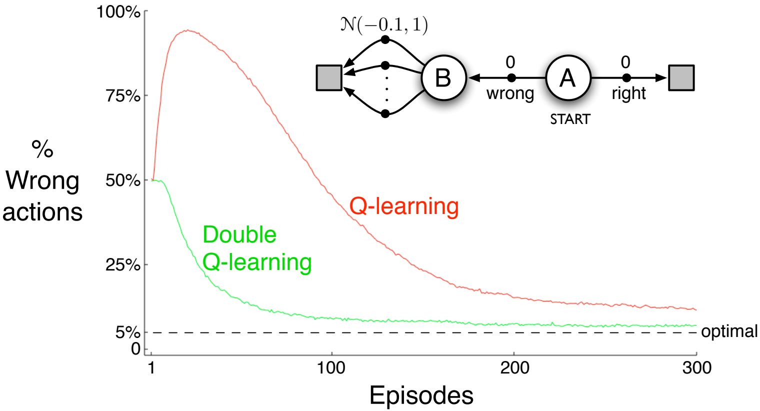
A solution to mitigate the maximization bias by van Hasselt [2010]

- Train two action-value functions Q_1 and Q_2
- Do Q-learning on both, but
  - never on the same time steps (Q_1 and Q_2 are independent)
  - pick Q_1 or Q_2 at random to be updated on each step
- If updating Q_1 use Q_2 for the value of the next state:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha \left(R_{t+1} + Q_2(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)) - Q_1(S_t, A_t) \right)$$

- Action selection can use a combination of Q_1 and Q_2

Maximization Bias Mitigated

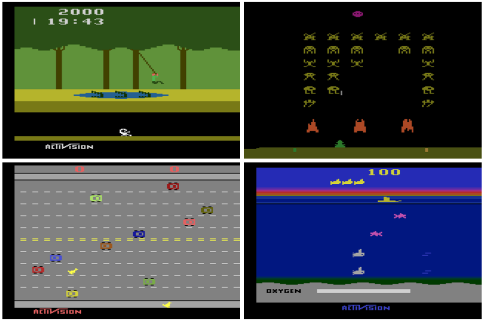


Double Q-learning:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma Q_2\big(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t)\Big]$$

Atari problem

Create a program that would learn to play any Atari game



Bellemare, M. G., Naddaf, Y., Veness, J., & Bowling, M. (2013). The arcade learning environment: An evaluation platform for general agents. *Journal of Artificial Intelligence Research*, 47, 253-279.

Atari problem solution

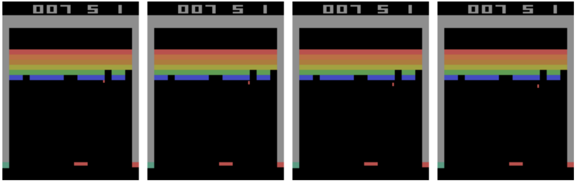
"This work bridges the divide between high-dimensional sensory inputs and actions, resulting in the first artificial agent that is capable of learning to excel at a diverse array of challenging tasks."

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller M., Fidjeland, A. K., Ostrovski, G., Petersen, S., Beattie, C., Sadik, A., Antonoglou, I., King, H., Kumaran, D., Wierstra, D., Legg, S. & Hassabis, D. (2015). Human-level control through deep reinforcement learning. Nature 518, 529-533.

Atari Games MDP Representation

States:

- Using four consecutive frames as state:



(Source: Greg Surma @ medium.com)

- Reduction of image size: $210 \times 160 \times 3 \rightarrow 84 \times 84 \times 1$

Actions:

- 2×8 directions of the joystick $+$ button

Transitions are taken directly from a game emulator

Rewards:

- Based on the game score
- Any score increase $\rightarrow +1$, any score decrease $\rightarrow -1$

How big is the MDP?

Assume we would quantise the colours to just black and white.
The number of possible states of the MDP is then:

$$2^{84x84x4=28224} \approx 10^{8496}.$$

There are estimated 10^{80} atoms in the observable universe.
Hence, a tabular representation of the q function may not work.

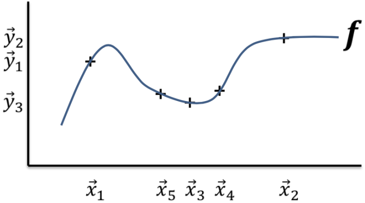
Supervised Machine Learning

A useful tool for AI, which is **not** a focus of this course

Supervised learning = fitting a (high dimensional) function

For a data set $(\vec{x}_i, \vec{y}_i)$, find a function f that minimizes:

$$\frac{1}{n}\sum_i \|f(\vec{x}_i) - \vec{y}_i\|.$$

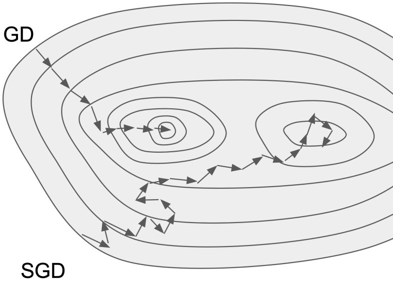


For example, $f($  $) = 2$, $f($  $) = 3$, $f($  $) = 4$, $f($  $) = 5$.

Q function is just a high-dimensional function approximable by a Neural network.

$$q(s,a) : \mathbb{R}^{28224} \times \mathcal{A} \to \mathbb{R}$$

Stochastic Gradient Descent



Dataset $D = (\vec{x_i}, \vec{y_i})$
Neural network f_w with weights $w \in \mathbb{R}^m$
Loss: $l(D, w) = \frac{1}{|D|} \sum_i \|f_w(\vec{x_i}) - \vec{y_i}\|$.
Gradient descent: $w' = w - \alpha \frac{\partial l(D,w)}{\partial w}$

Mini-batched version of the loss function:

For a uniformly selected subset of data $\tilde{D} \subset D$ called a minibatch define the approximate loss: $\hat{l}(\tilde{D}, w) = \frac{1}{|\tilde{D}|} \sum_i \|f_w(\vec{x_i}) - \vec{y_i}\|$

and update: $w' = w - \alpha \frac{\partial \hat{l}(\tilde{D},w)}{\partial w}$.

It works, because $\mathbb{E}\, \hat{l}(\tilde{D}, w) = l(D, w)$.

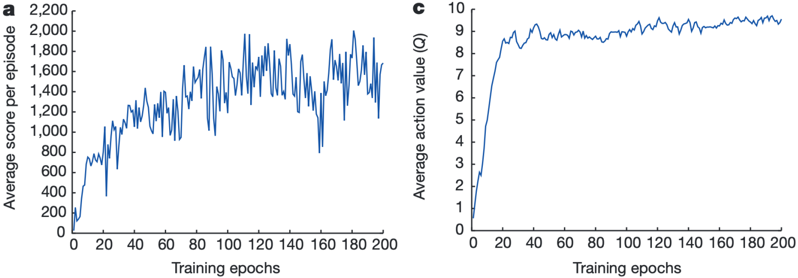
DQN algorithm

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$
For episode $= 1, M$ **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For $t = 1$,T **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^{-}) & \text{otherwise} \end{cases}$
    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For

DQN Training

Trained for each game separately, but using the same architecture and hyperparameters.



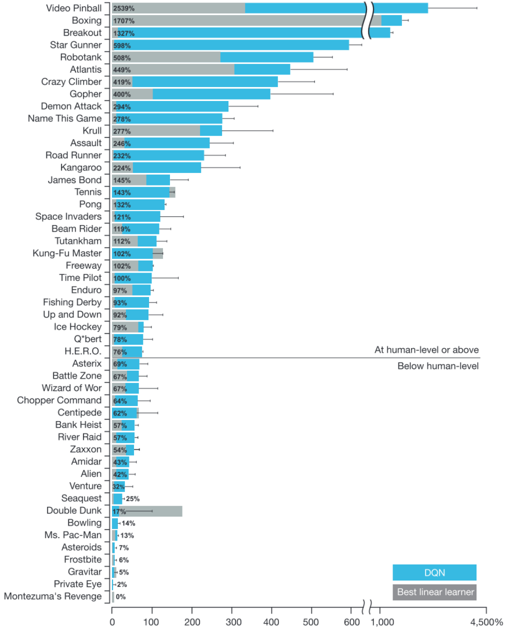
Convergence curve for "Space Invaders". One epoch is 520k frames. $\epsilon = 0.05$.

Training details: Minibatch size 32; exploration scaled from 1.0 to 0.1 over 1M frames and than fixed; overall 50M frames of training (38 days); replay buffer for 1M most recent frames. Probably 10 days of training per game and agent (not reported).

DQN Playing

Breakout
Space Invaders

Results Relative to an Expert Human



Summary

RL can solve huge MDPs without their explicit knowledge.

Key components of RL algorithms are policy evaluation and policy improvement.

Just using these steps on whole state space leads to

- policy iteration
- value iteration.

These algorithms are not super fast, but extremely versatile.

- Updates of just selected states
- Minimal / stochastic updates of policy and values
- Function approximation
- Endless modifications explored in RL literature