Summer 2016 Jméno:

A4B33ZUI – Základy umělé inteligence – 2. 6. 2016

O1	O2	O3	O4	O5	Total (50)

Instructions: You have 150 minutes to complete the test. You can use your own notes in a form of a single hand-written A4 sheet. The usage of any electronic device is strictly forbidden. In TRUE/FALSE questions circle one of the possibilities. If you are not sure about the answer, try to explain your reasoning.

Question 1 *(1 point for a correct answer, -1 point for a wrong answer) TRUE/FALSE statements*

(a) (TRUE/FALSE) Negamax algorithm improves the Alpha-Beta pruning by manipulation of the upper and lower bound values (i.e., alpha and beta values).

(b) (TRUE/FALSE) Breadth first search (BFS) algorithm is complete and always finds an optimal solution in finite search trees.

(c) (TRUE/FALSE) No informed search algorithm needs to reevaluate some node from the closed list in order to find an optimal solution.

(d) (TRUE/FALSE) Iterative-deepening A* (IDA*) is the enhancement of A* which reduces the time complexity of the algorithm.

(e) (TRUE/FALSE) Backtracking search algorithm is a form of depth-first search algorithm.

(f) (TRUE/FALSE) Every search problem can be posed as MDP with the identical optimal solution.

(g) (TRUE/FALSE) Markov decision process (MDP) is defined as a quadruple $\{S, A, P, R\}$, where S is a finite set of observable states, A is a finite set of available actions, P stands for the transition probabilities and R represents the reward function.

(h) (TRUE/FALSE) An agent that has only uncertain information about the world arount it cannot be perfectly rational.

(i) (TRUE/FALSE) Let us have an agent which executes actions that influence states of its surrounding world. Such an agent can be considered rational if and only if it works with a numeric function that gives a utility for all the available states of the world and then it maximizes its expected value.

(j) (TRUE/FALSE) Since the value of information is non-negative, the outcome of acting on more information will always be at least as good as the outcome of acting on less information.

Question 2 *(10 points) CSP*

Crypto-arithmetic is a known problem where it is required to assign to different letters different numbers in a way that after the assignment the constraints for the arithmetic operations are valid. An example is $SEND + MORE = MONEY$, which is valid if we assign the following numbers to the letters:

S	E	N	D	M	O	R	Y
9	5	6	7	1	0	8	2

(a) (4 points) Formalize the general problem of the crypto-arithmetic for adding up two numbers of unlimited length with a given result (i.e., with the structure demonstrated above) as a CSP problem. Propose and **formally** write the most effective representation of variables, their domains, and constraints.

(b) (3 points) Describe a basic scheme of algorithm solving a CSP and show in which steps the algorithm can use sub-algorithm of edge-consistency and heuristics for variable selection (such as MRV).

(c) (2 points) Name all other heuristics which can be used in a CSP. Propose which could be effective for the crypto-arithmetic problem and **justify your answer**.

(d) (1 point) Draw a part of the search space for a specific task given below in a way in which it is iterated by the CSP algorithm which is using arc consistency, forward checking and some heuristics from previous point. Choose and draw a part of the space in which the algorithm will prune some states.

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PUZZLE
ZUIZUI
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UNEASY
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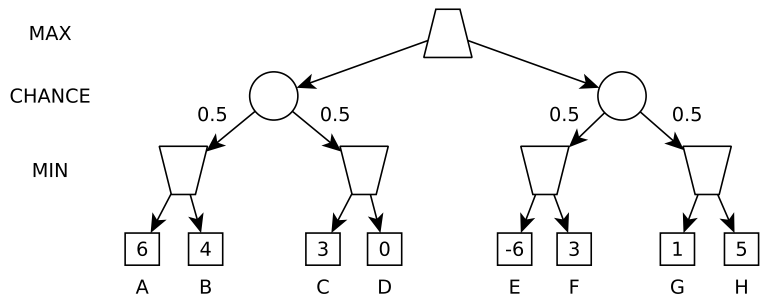
Question 3 *(10 points) Two-Player Games*



Consider a two-player game depicted in Figure that includes stochastic events (visualized as ellipsoid nodes in the game tree; the probability with which the actions are taken is written over the edges).

(a) (2 points) Compute the expected value of the game if both players play optimally.

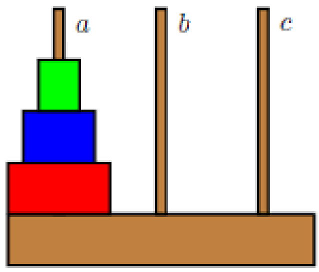
(b) (4 points) Consider a modified variant of Alpha-Beta pruning that is able to work with chance nodes. **Formally describe** necessary modifications to the standard Alpha-Beta algorithm (focus on computation of upper and lower bounds – i.e., values alpha and beta, respectively). Assume that the utility values are assigned from some interval $[u_{min}, u_{max}]$.

(c) (2 points) List terminal states that will be visited by such modified Alpha-Beta pruning algorithm.

(d) (2 points) Consider a situation where we restrict the interval of possible utility values in the terminal states. Can the modified Alpha-Beta prune more nodes? Give a specific example of such an interval and terminal states that the modified Alpha-Beta will visit.

Question 4 *(10 points) Tower of Hanoi – situation calculus*

Consider the well-known Tower of Hanoi puzzle. There are three rods denoted a, b, c and several discs of different sizes. The puzzle starts with all the discs in a neat stack in ascending order of size (smaller discs lie on larger ones) on the rod a. The objective of the puzzle is to move the entire stack to the rod b using the auxiliary rod c, obeying the following simple rules: 1) only one disc can be moved at a time, 2) each move consists of taking the upper disc from one of the stacks and placing it on top of another stack, i.e. a disc can only be moved if it is the uppermost disc on a stack, 3) no disc may be placed on top of a smaller disc.



Your goal is to verify the existence of a solution to the puzzle with situation calculus. You are allowed to employ the following simplifications: A) the closed world assumption, B) arithmetic available, C) inequality can directly be used to distinguish the objects, D) the puzzle initialization is correct, no need to check it.

Work with the only fluent predicate *loc* and the only action defined by the term *move*:

$loc(X, Y, P, S)$	the disc of size X is on the rod Y, there are P other discs above it, S is the current state of the world,
$move(X, Y1, Y2)$	move the disc of size X form the rod $Y1$ to the rod $Y2$.

The puzzle is initialized as follows ($infty$ is the symbol used for ground, it facilitates the direct testing of the number of discs lying on a rod):

loc(infty,a,3,s0).   loc(3,a,2,s0).   loc(2,a,1,s0).   loc(1,a,0,s0).   loc(infty,b,0,s0).   loc(infty,c,0,s0).

(a) (2 points) Write down an effect axiom for the *loc* predicate and the *move* action (the axiom defines the changes caused by the action on the relationship captured by the predicate):

(b) (2 points) Write down a frame axiom for the *loc* predicate and the *move* action (the axiom defines invariabilty of the relationship captured by the predicate with respect to the application of the action):

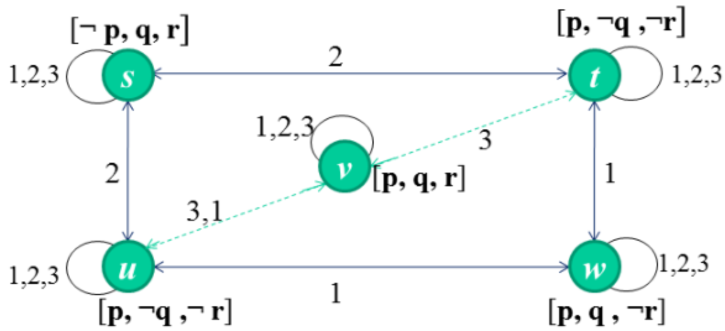
(c) (2 points) Write down the successor-state axiom attached to the predicate *loc* (an overall axiom for a single fluent predicate and all the actions, the axiom will be redundant wrt to the previous axioms):

(d) (1 point) Write down a formula that verifies whether the puzzle objective can be reached (all the discs placed on the rod b in the correct order):

(e) (3 points) Perform a single resolution step corresponding to a single move applied to the initial state $s0$ (be as formal as possible):

Question 5 *(10 points) Kripke Structures*

(4 points) Consider a Kripke structure for 3 agents with states $\{\mathbf{s}, \mathbf{t}, \mathbf{u}, \mathbf{v}, \mathbf{w}\}$ and primitive formulas $\{\mathbf{p}, \mathbf{q}, \mathbf{r}\}$: For each of the following formulas either state that the formula is true in all states of this Kripke structure,



or specify the states in which this formula is not true:

(a) $K_1\,(p \vee \neg r)$

(b) $\neg K_1 \neg q$

(c) $(p \wedge r) \rightarrow K_2\,(p \wedge r)$

(d) $C_{\{1,2,3\}}\,(p \vee r)$

(3 points) Consider formal systems $\mathbf{K_n}$ with axioms:

A1 All propositional tautologies

A2 $(K_i\alpha \wedge K_i(\alpha \rightarrow \beta)) \rightarrow K_i(\beta)$

and derivation rules:

R1 From α and $\alpha \rightarrow \beta$ infer β (Modus Ponens)

R2 Form α infer $K_i\alpha$

Formal system that created from $\mathbf{K_n}$ by adding knowledge axiom $A3 : K_i\alpha \rightarrow \alpha$ is again a consistent formal system.

(e) Write down a proof of formula $\neg K_i \mathit{false}$ in this extended formal system.

(3 points) Let α be a primitive formula (formula without any temporal operators) LTL and $\boldsymbol{M}$ is a Kripke structure with states $m_0, m_1, m_2, \ldots$ for LTL. Write down the evaluation of formula α in all states $\boldsymbol{M}$ so that both of the following conditions hold:

- $\mathbf{M_2}(\Diamond\neg\alpha \wedge \Diamond\alpha) = \mathit{true}$
- Formula $(\alpha \rightarrow \circ\Box\alpha)$ holds in all states of $\boldsymbol{M}$.