Linear Classifiers II

Tomáš Svoboda

Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

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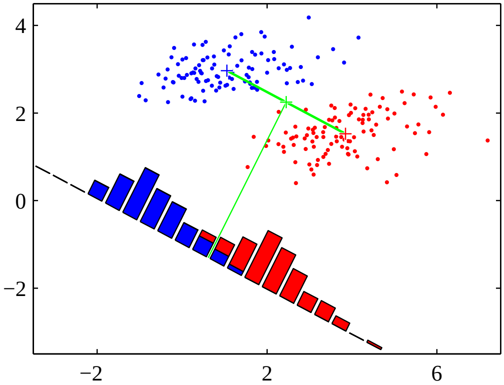
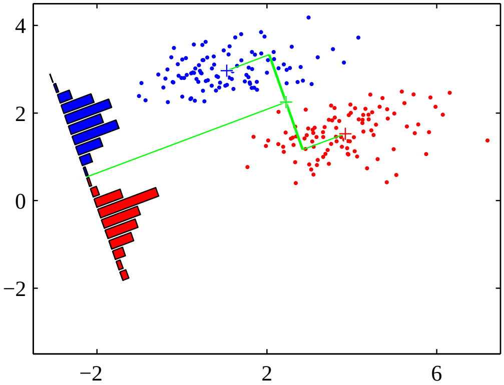
Linear Classifiers - supplement lecture

- Supplement to the lecture about learning Linear Classifiers (perceptron, ...)
- Better etalons by applying Fischer linear discriminator analysis.
- LSQ formulation of the learning task.

Fischer linear discriminant





- Dimensionality reduction
- Maximize distance between means, ...
- ...and minimize within class variance. (minimize overlap)

Figures from [1]

Projections to lower dimensions $y = \mathbf{w}^\top \mathbf{x}$

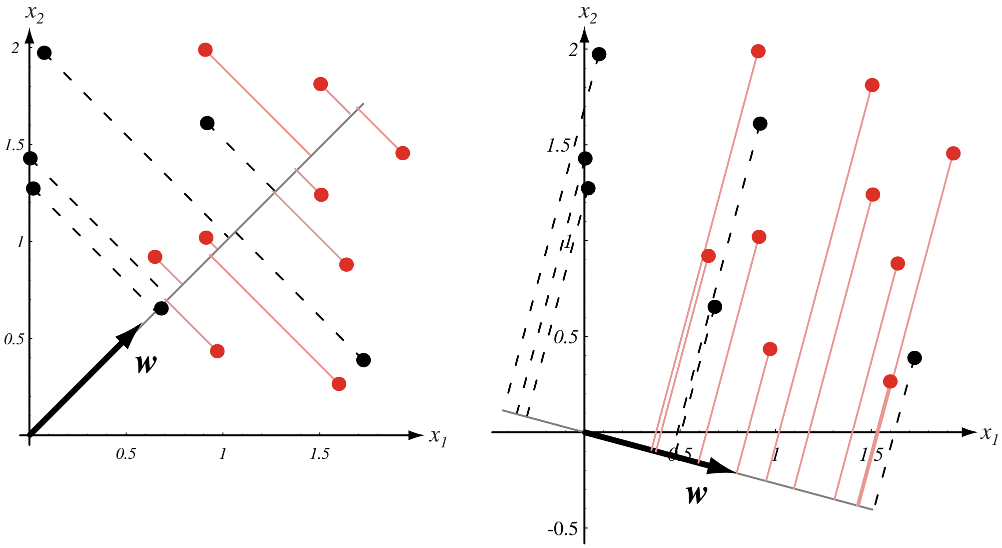


Figure from [2]

Projection to lower dimension $\mathbf{y} = \mathtt{W}^\top \mathbf{x}$

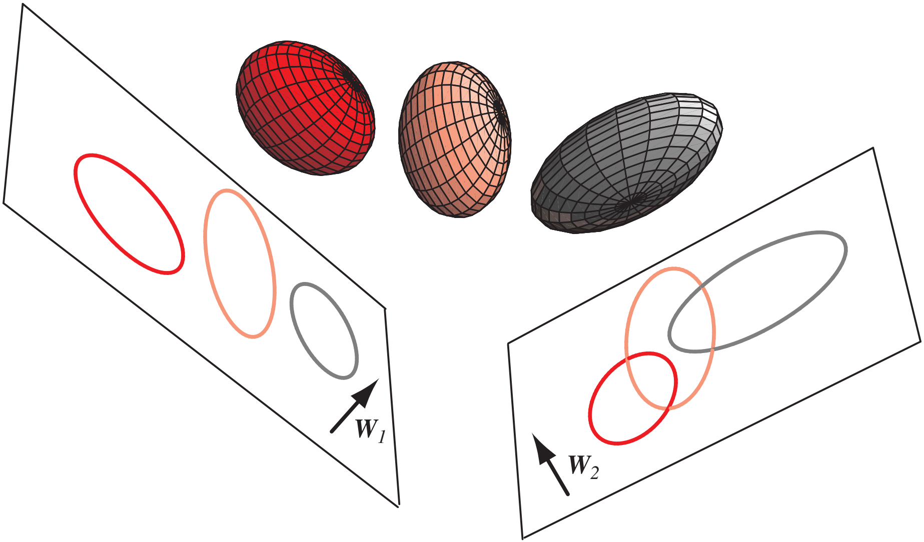
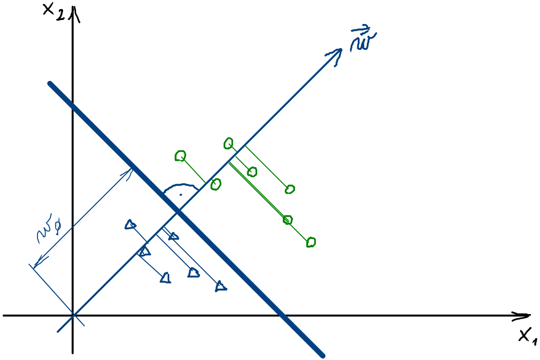
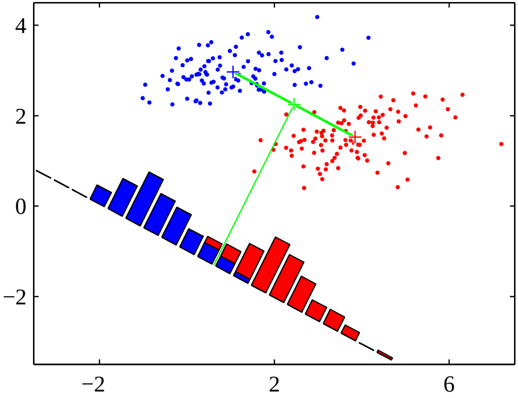


Figure from [2]

Finding the best projection $y = \mathbf{w}^\top \mathbf{x}$, $y \geq -w_0 \Rightarrow C_1$, otherwise C_2





Finding the best projection $y = \mathbf{w}^\top \mathbf{x}$, $y \geq -w_0 \Rightarrow C_1$, otherwise C_2

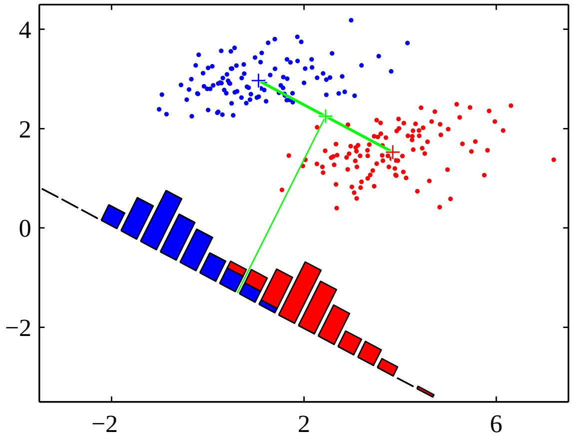
$$m_2 - m_1 = \mathbf{w}^\top(\mathbf{m}_2 - \mathbf{m}_1)$$

Within class scatter of projected samples

$$s_i^2 = \sum_{y \in C_i} (y - m_i)^2$$

Fischer criterion:

$$J(\mathbf{w}) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$$



Finding the best projection $y = \mathbf{w}^\top \mathbf{x}$, $y \geq -w_0 \Rightarrow C_1$, otherwise C_2

$$m_2 - m_1 = \mathbf{w}^\top (\mathbf{m}_2 - \mathbf{m}_1)$$

$$s_i^2 = \sum_{y \in C_i} (y - m_i)^2$$

$$J(\mathbf{w}) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$$

$$\frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} = 0$$

$$\mathrm{S}_i = \sum_{x \in C_i} (\mathbf{x} - \mathbf{m}_i)(\mathbf{x} - \mathbf{m}_i)^\top$$

$$\mathrm{S}_W = \mathrm{S}_1 + \mathrm{S}_2$$

$$\mathrm{S}_B = (\mathbf{m}_2 - \mathbf{m}_1)(\mathbf{m}_2 - \mathbf{m}_1)^\top$$

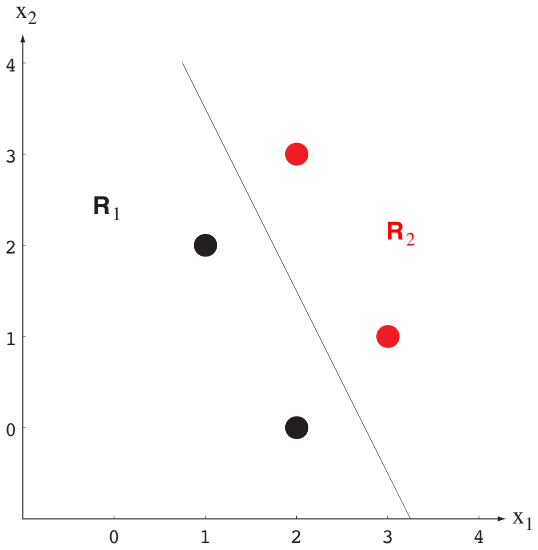
$$J(\mathbf{w}) = \frac{\mathbf{w}^\top \mathrm{S}_B \mathbf{w}}{\mathbf{w}^\top \mathrm{S}_W \mathbf{w}}$$

LSQ approach to linear classification

$$\mathbf{w} = \begin{bmatrix} w_0 \\ \mathbf{w} \end{bmatrix}$$

$$\mathtt{X}\mathbf{w} = \mathbf{b}$$

$$J(\mathbf{w}) = \|\mathtt{X}\mathbf{w} - \mathbf{b}\|^2$$



LSQ approach, better margins b?

$$\mathtt{X} = \begin{bmatrix} \mathtt{1}_1 & \mathtt{X}_1 \\ -\mathtt{1}_2 & -\mathtt{X}_2 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} \frac{n}{n_1}\mathtt{1}_1 \\ \frac{n}{n_2}\mathtt{1}_2 \end{bmatrix}$$

References I

Further reading: Chapter 4 of [1], or chapter 3 and 5 of [2].

[1] Christopher M. Bishop.
Pattern Recognition and Machine Learning.
Springer Science+Bussiness Media, New York, NY, 2006.
PDF freely downloadable.

[2] Richard O. Duda, Peter E. Hart, and David G. Stork.
Pattern Classification.
John Wiley & Sons, 2nd edition, 2001.