

Problem solving by search

Tomáš Svoboda and Matěj Hoffmann

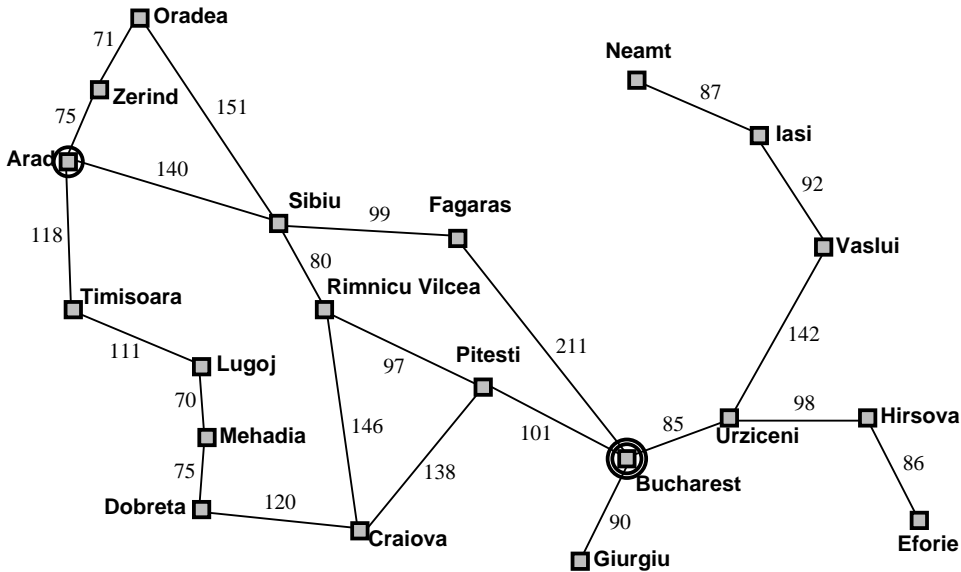
Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

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Outline

- ▶ Search problem.
- ▶ State space graphs.
- ▶ Search trees.
- ▶ Strategies: which tree branches to choose?
- ▶ Strategy/Algorithm properties.
- ▶ Programming infrastructure.

Example: Traveling in Romania



Notes

Ok, start with a simple one, almost everybody knows about the navigation - path planning problem. Waze, Garmin, ...

Can you think about more problems?

For example:

- Touring problems. Special case: Traveling salesperson problem – each city must be visited exactly once.
- Planning robot movements – mobile robot or manipulator.
- VLSI (chip) layout.
- ...

Example: Map of Romania

Goal:

be in Bucharest

Problem formulation:

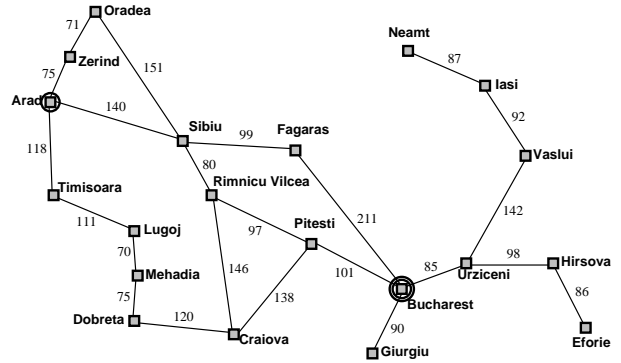
states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)

(action sequence [2])



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Classical problem from the Book [2], we use it, too.

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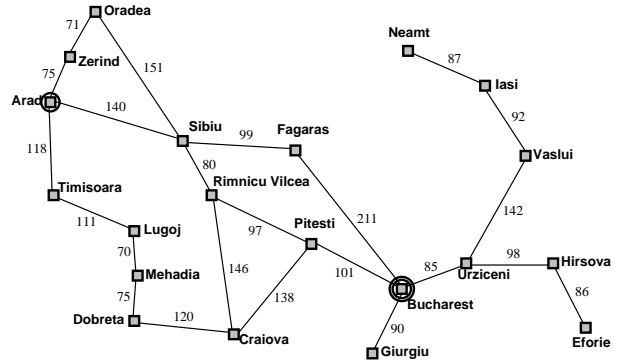
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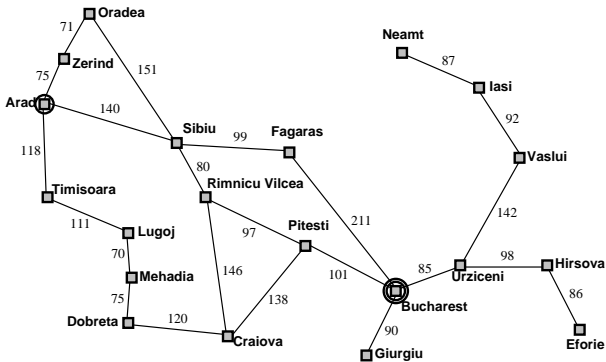
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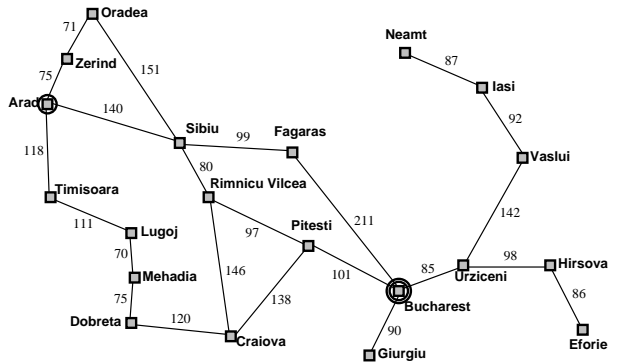
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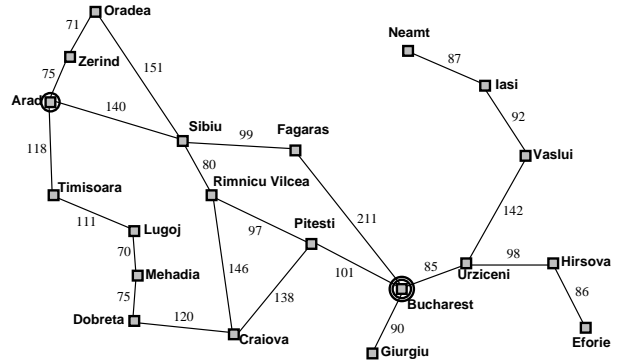
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Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states?
actions?
solution?
cost?

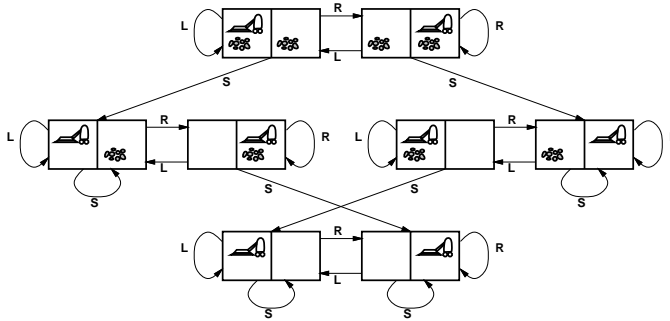
Notes

Also known as $n - 1$ puzzle.

- States: Location of each of the 8 tiles and the blank.
- Number of states: $9!$
- Initial state: any state. (Note that any given goal state can be reached from exactly half of the initial states.)
- Actions: Movements of the blank space: Left, Right, Up, Down (or a subset of these)
- Solution / goal test: Check whether state matches the goal configuration.
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

Example: Vacuum cleaner



states?
actions?
solution?
cost?

Notes

- States: Determined by agent location and dirt location. The agent is in one of two locations, each of which may or may not contain dirt.
- Number of states: 2×2^2 (two possible choices for agent location; for every location, choice dirt vs. no dirt). For n locations: $n \times 2^n$
- Initial state: any state
- Actions: Left, Right, Suck (larger envs. can have also Up and Down)
- Solution / goal test: Are all squares clean?
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

A Search Problem

- ▶ **State space** (including Start/Initial state): position, board configuration,
- ▶ Actions : drive to, Up, Down, Left ...
- ▶ Transition model : Given state and action return state (and cost)
- ▶ Goal test : Are we done?

Notes

We will use the terminology through the next 5-6 lectures; also for Markov (Sequential) Decision Processes, Reinforcement Learning

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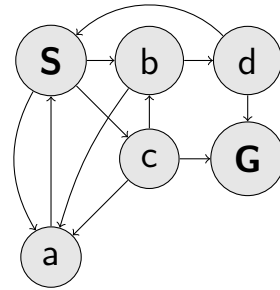
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State Space Graphs

State space graph: a representation of a search problem

- ▶ Graph Nodes – states – are abstracted world configurations
- ▶ Arcs represent action results
- ▶ Goal test – a set of goal nodes

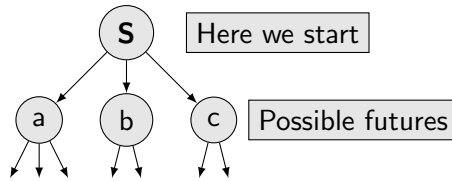
Each state occurs only *once* in a state (search) space.



Notes

Formalizing a real world problem – (creating) a state space graph – could be a problem in itself. I put creating into brackets as it may be also infinite.

Search Trees

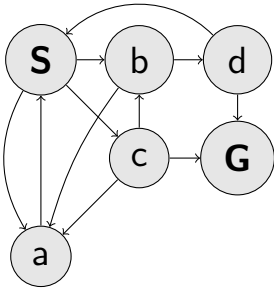


- ▶ A “what if” tree of plans and their outcomes
- ▶ Start node is the root
- ▶ Children are successors
- ▶ Nodes show/contains states, but correspond to *plans* that achieve those states

Notes

- What if decision about an action, repeats ...
- Nodes in the search tree are not the same as the nodes in the state space graph.

State Space Graphs vs. Search Trees

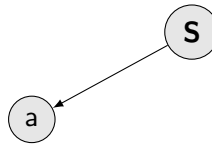
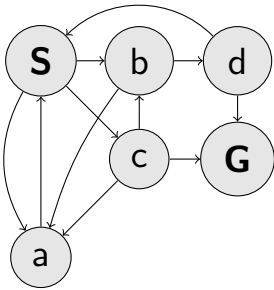


How big is the search tree?

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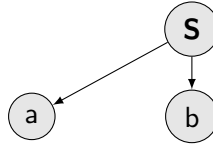
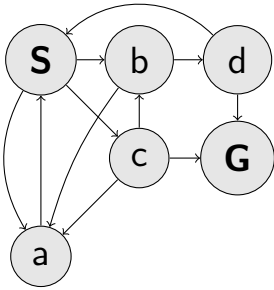


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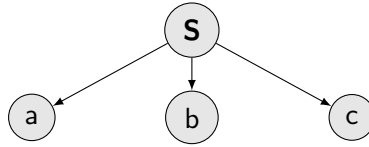
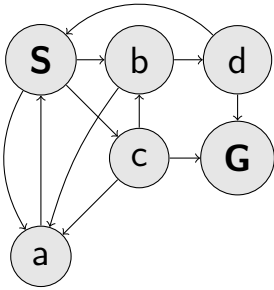


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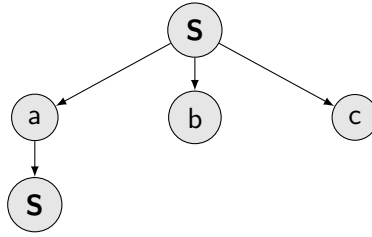
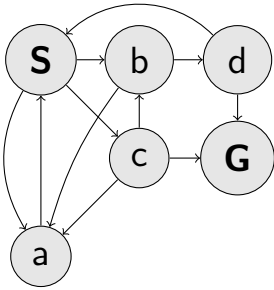


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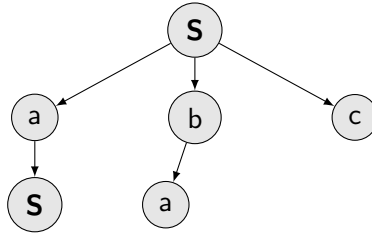
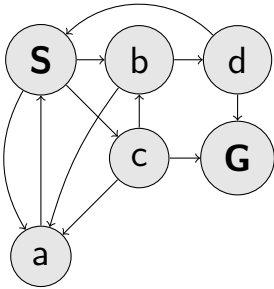


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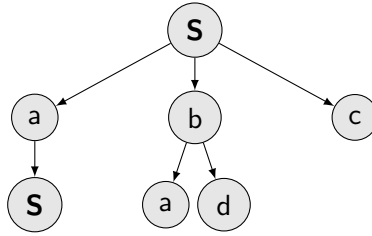
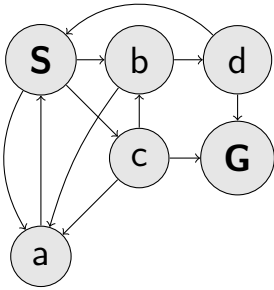


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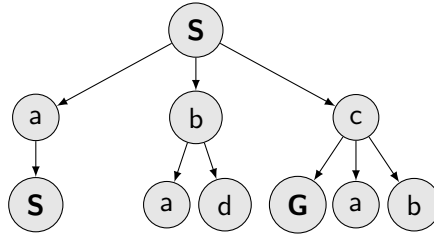
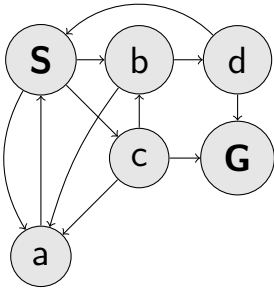


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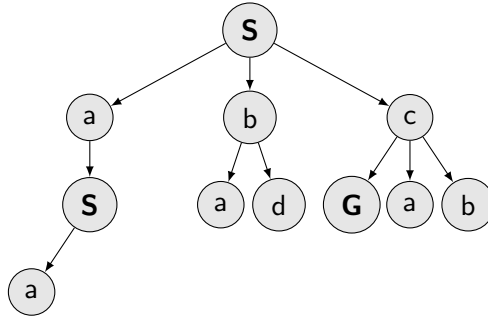
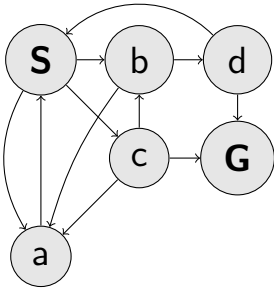


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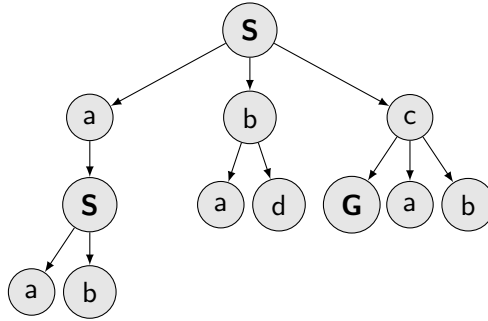
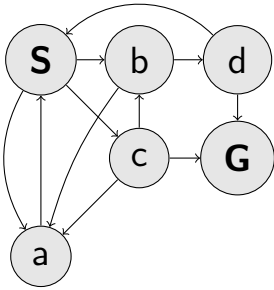


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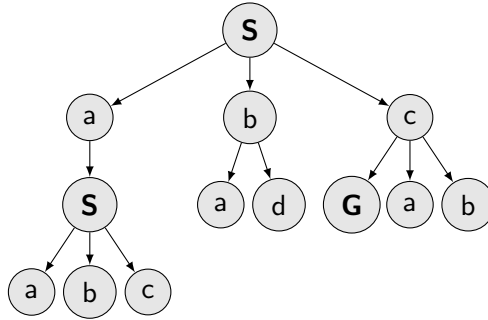
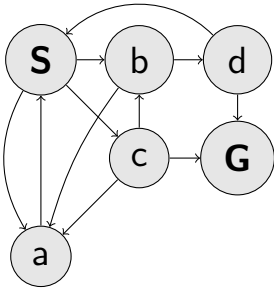


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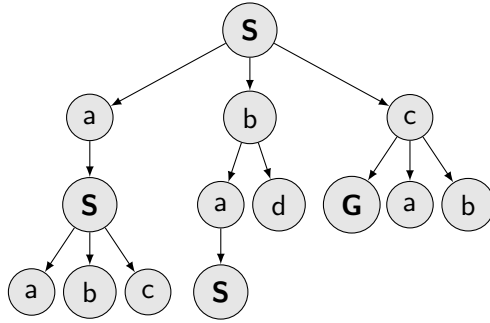
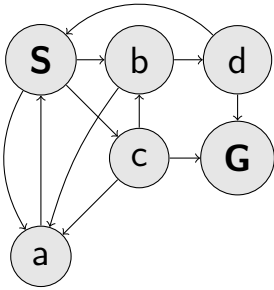


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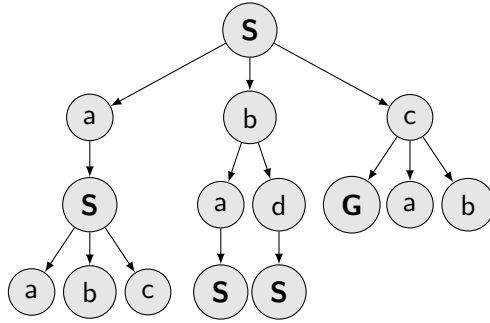
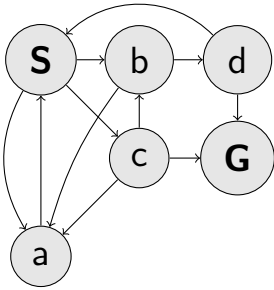


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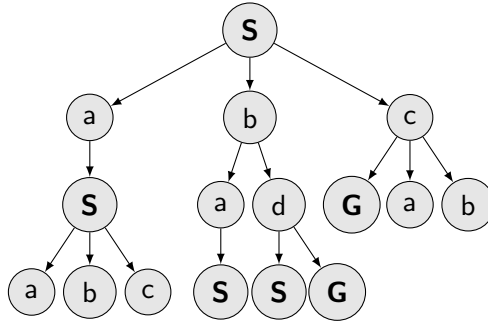
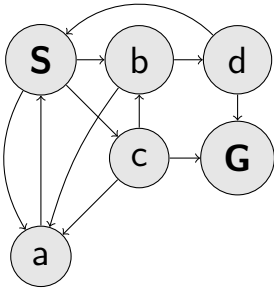


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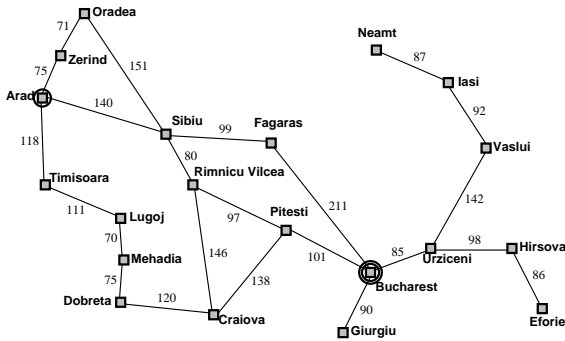


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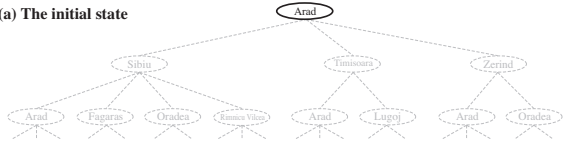
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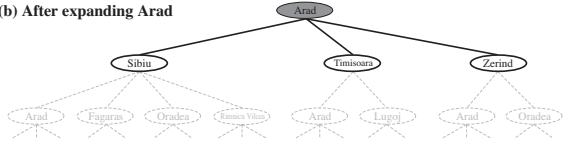
From problem/transition graph to search tree (Romania)



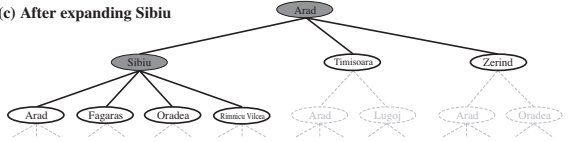
(a) The initial state



(b) After expanding Arad



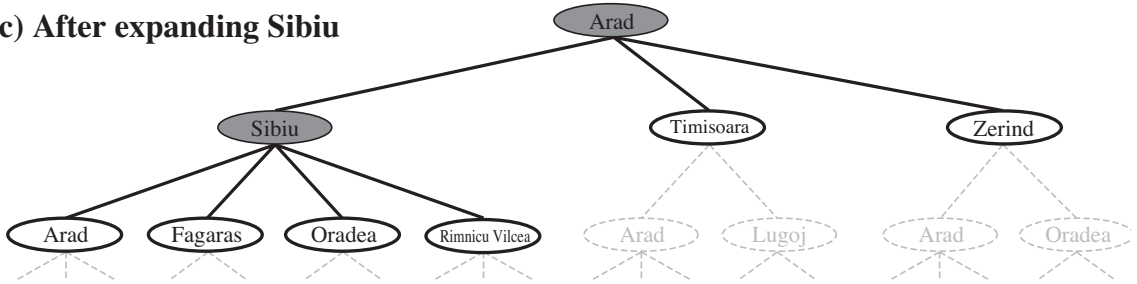
(c) After expanding Sibiu



Problem/transition graph is revealed incrementally.
The revealing strategy can be visualized as a search tree.

Notes

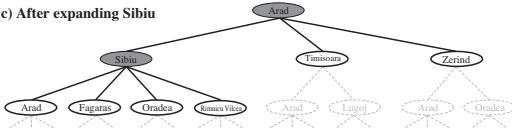
(c) After expanding Sibiu



- ▶ Expand **plans** - possible ways (**tree nodes**).
- ▶ Manage/Maintain **fringe** (or **frontier**) of plans under consideration.
- ▶ Expand new nodes *wisely(?)*.

Tree search algorithm

(c) After expanding Sibiu



function TREE_SEARCH(problem) **return** a solution or failure

initialize by using the initial state of the problem

loop

if no candidates for expansion **then return** failure

else choose a leaf node for expansion

end if

if the node contains a goal state **then return** the solution

end if

Expand the node and add the resulting nodes to the tree

end loop

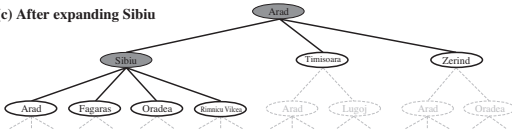
end function

Notes

A *general* tree search algorithm. Individual search algorithms vary primarily in how they choose which state to expand next – the “search strategy”.

Tree search algorithm

(c) After expanding Sibiu



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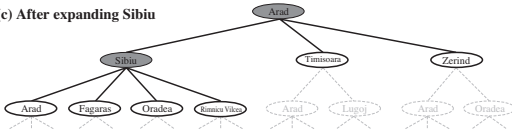
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Tree search algorithm

(c) After expanding Sibiu



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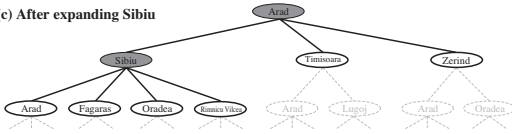
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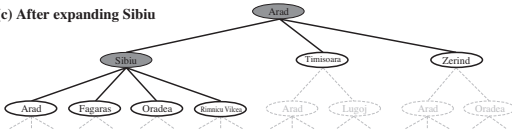
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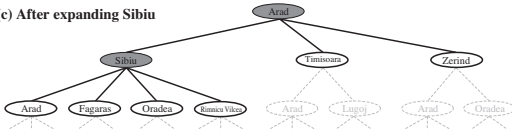
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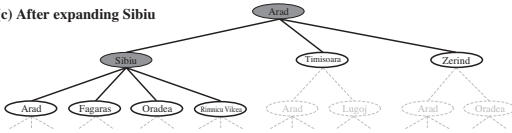
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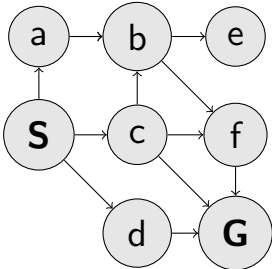
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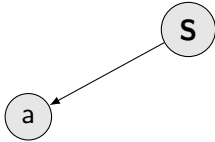
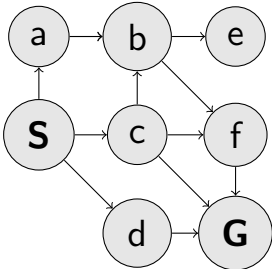
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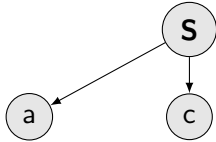
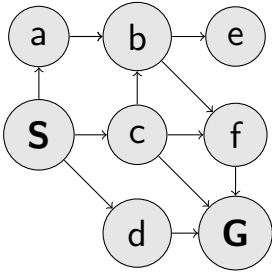
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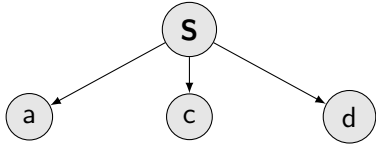
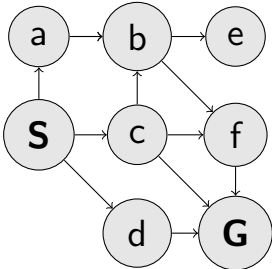
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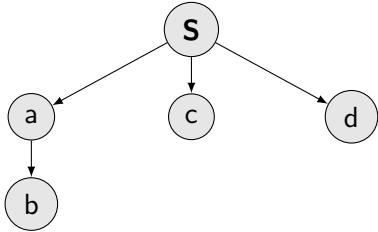
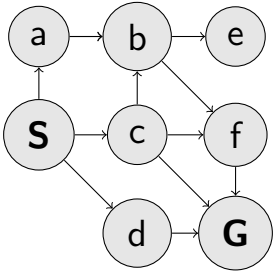
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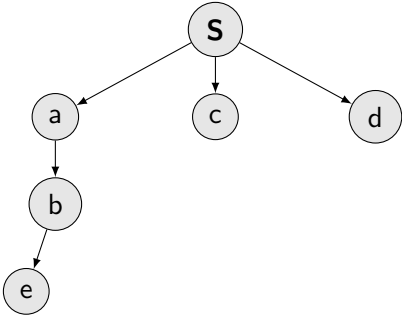
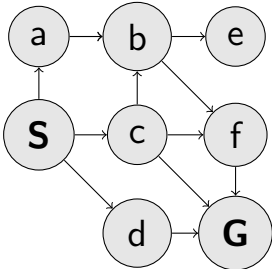
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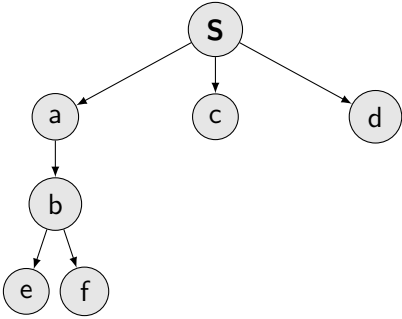
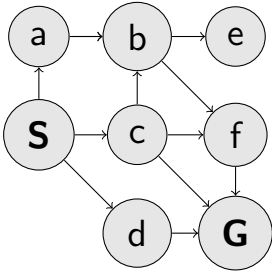
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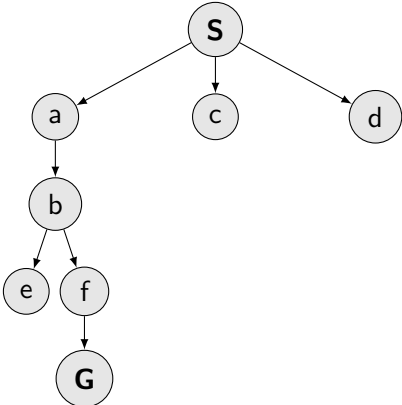
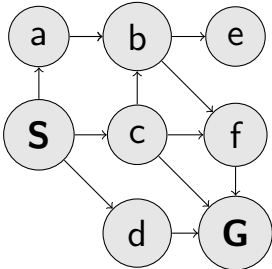
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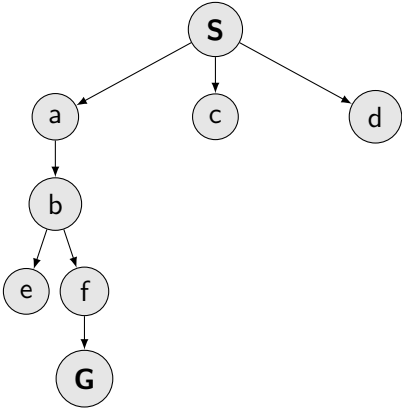
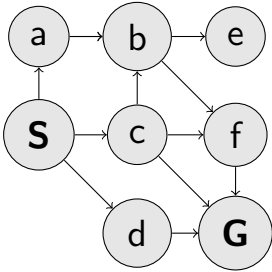
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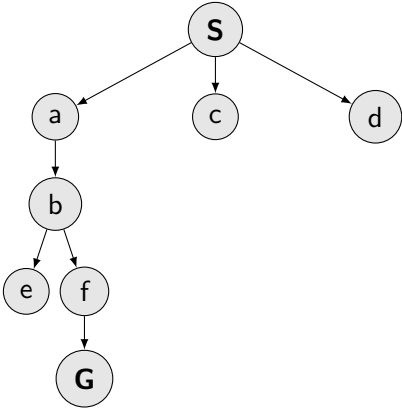
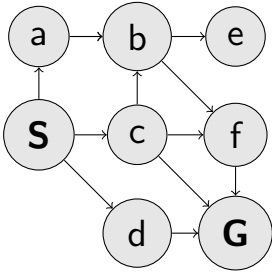
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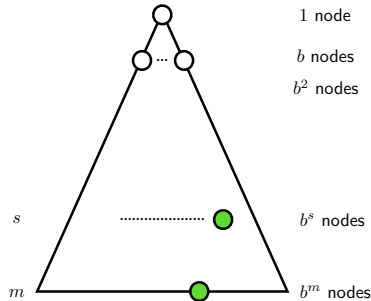
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How many nodes in a (search) tree? What are tree parameters?

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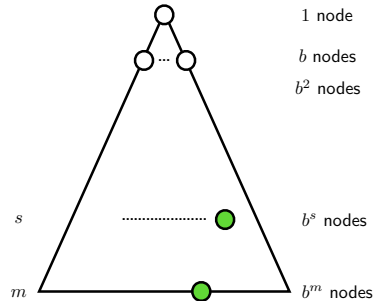
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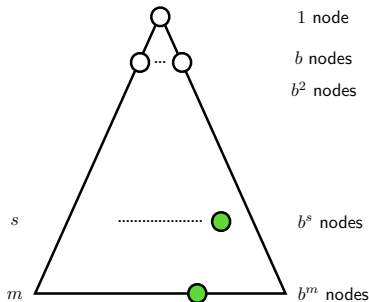
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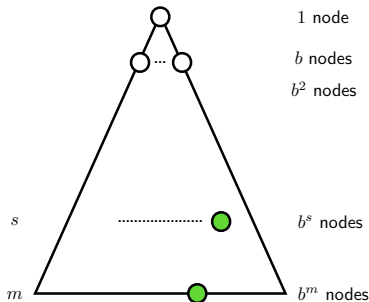
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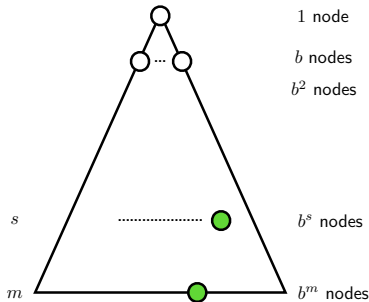
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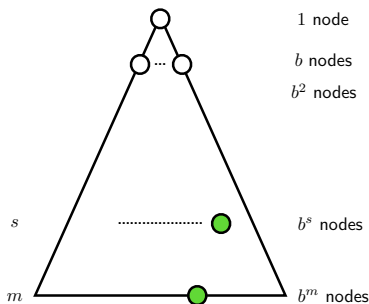
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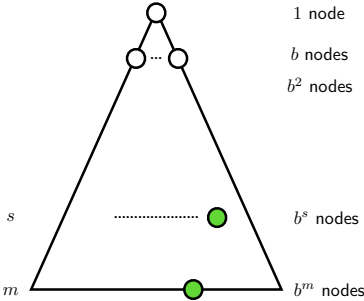
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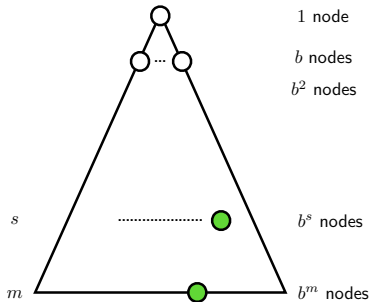
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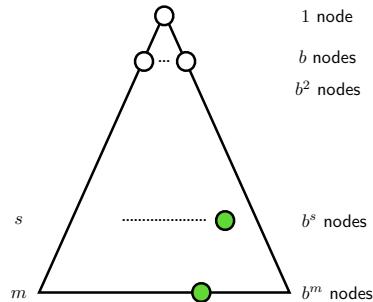
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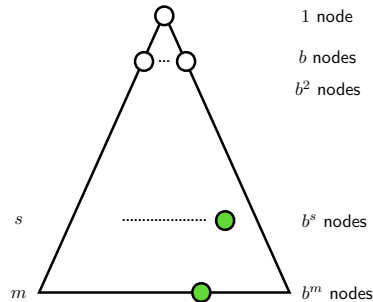
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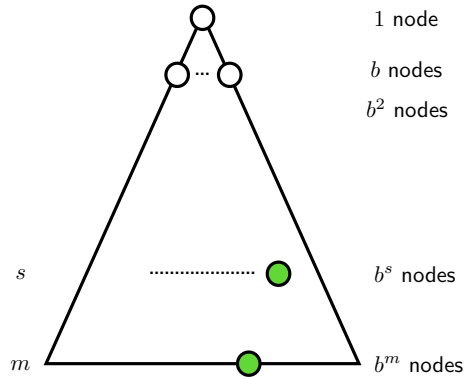
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Strategies

How to traverse/build a search tree?

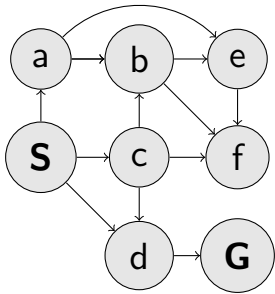
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Notes

It is perhaps worth to remember that the search tree is built as the algorithm goes. Or better said, the tree is a human friendly representation of the machine run.

Depth-First Search (DFS)



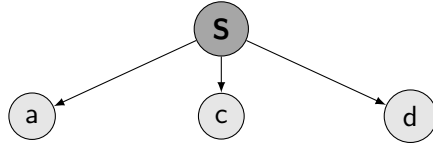
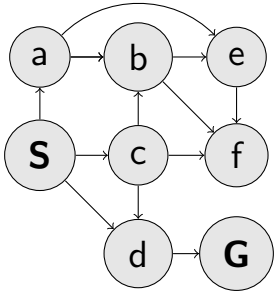
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What are the DFS properties (complete, optimal, time, space)?

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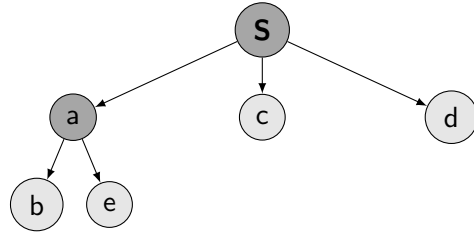
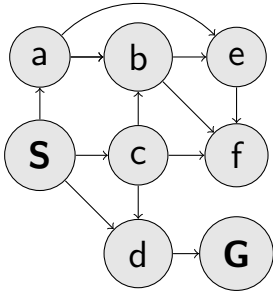
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17 / 35

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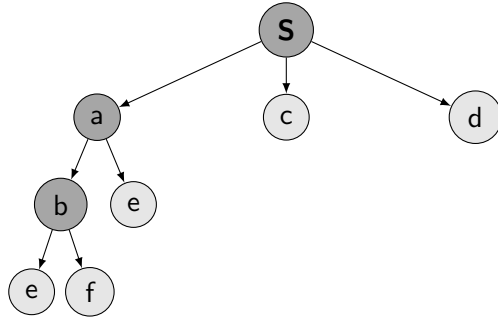
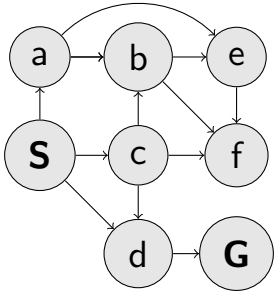


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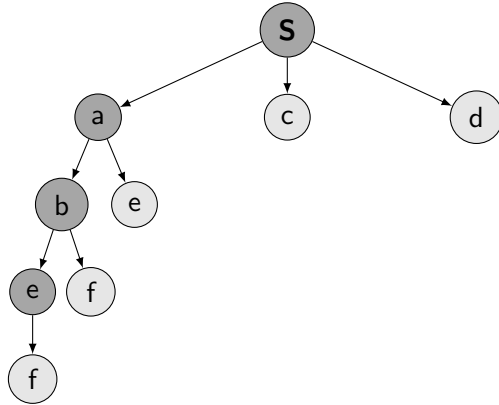
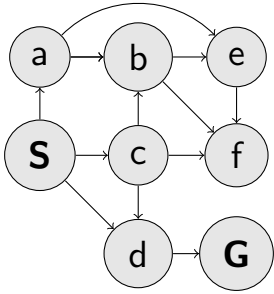


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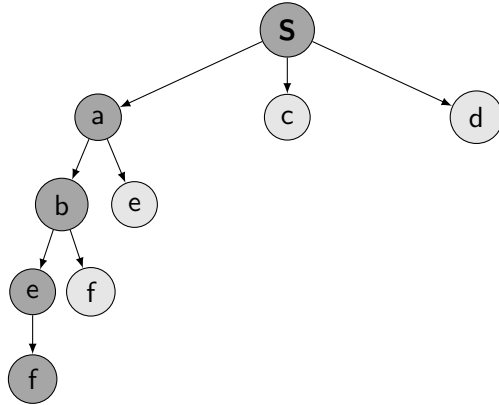
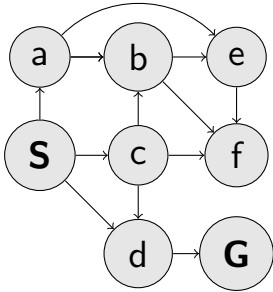


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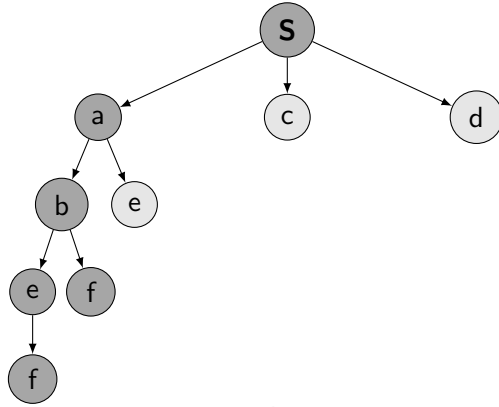
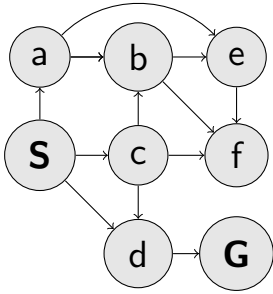


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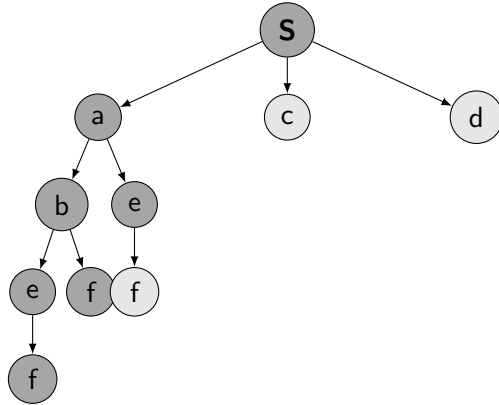
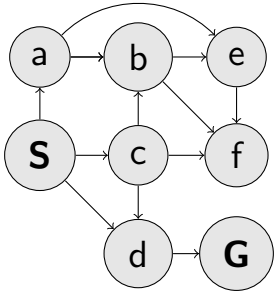


What are the DFS properties (complete, optimal, time, space)?

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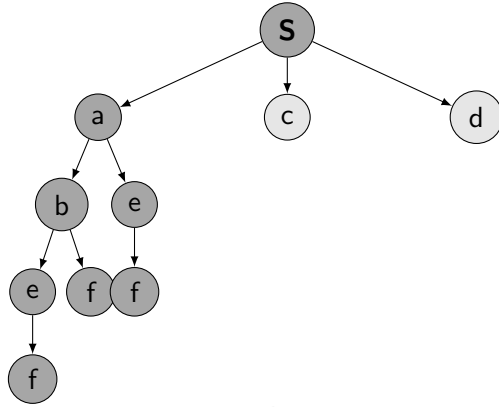
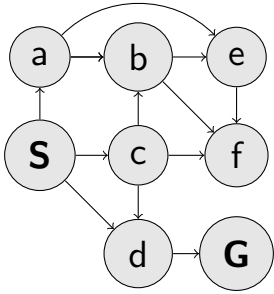
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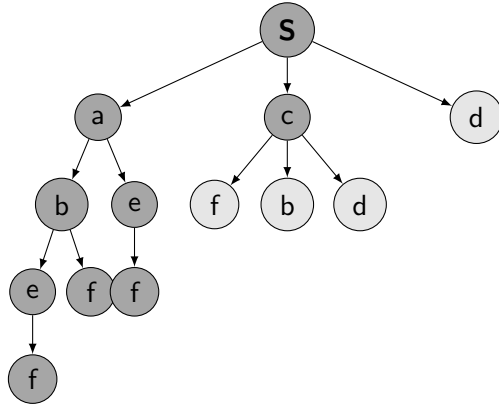
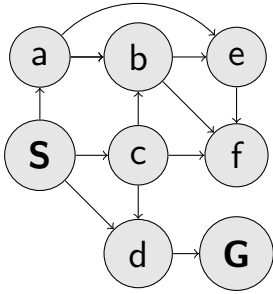
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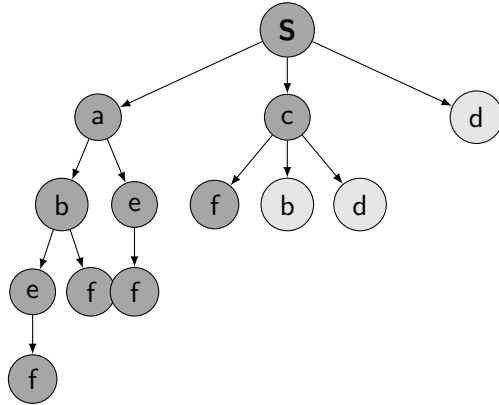
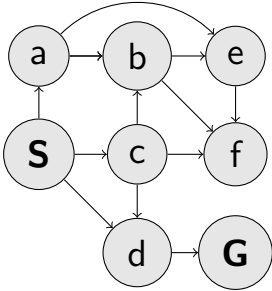
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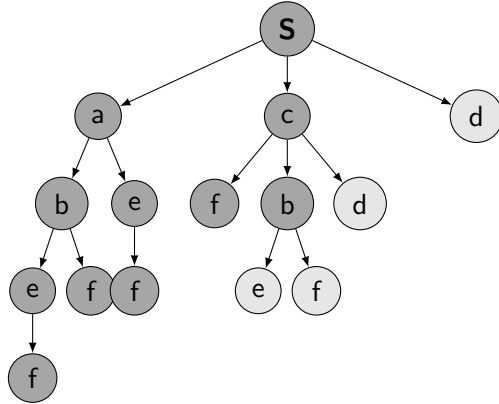
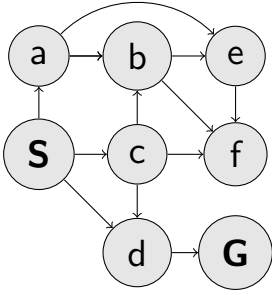


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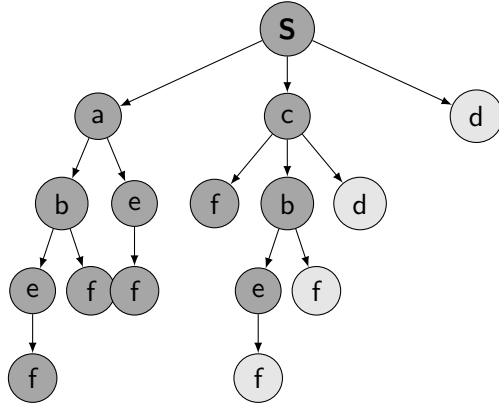
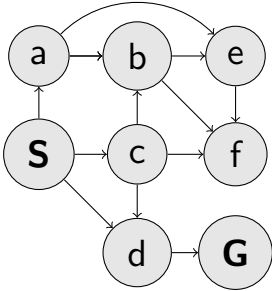


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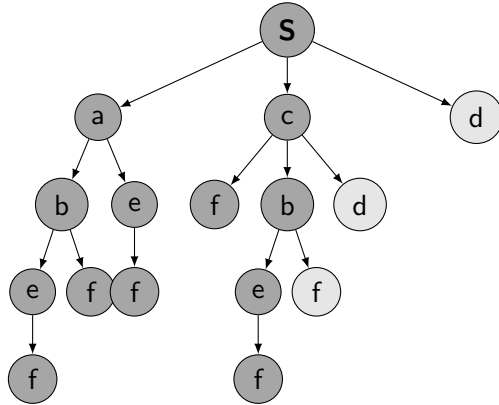
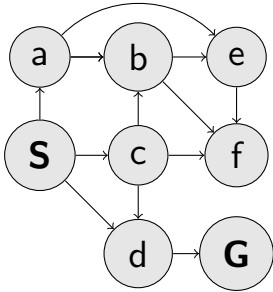


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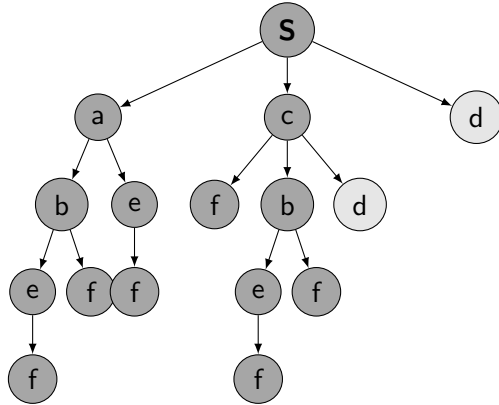
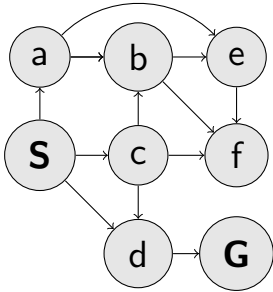
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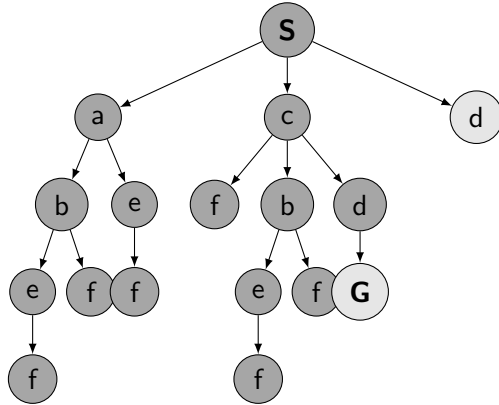
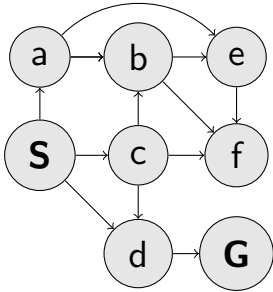
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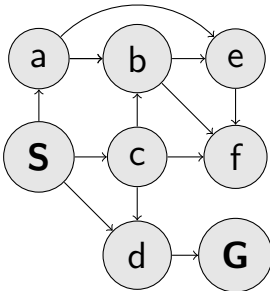
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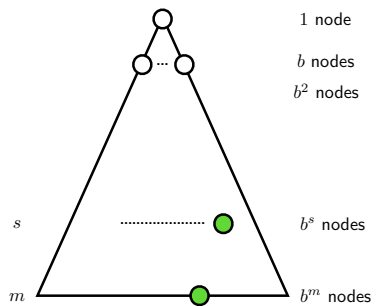
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- ▶ Space complexity?
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Notes

- Time, can process the whole tree: b^m
- Space, only the path so far: bm (a path from root to leaf (m), plus siblings on the path are also on the frontier ($b \times m$))
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- Optimality: No! It just takes the first solution found.

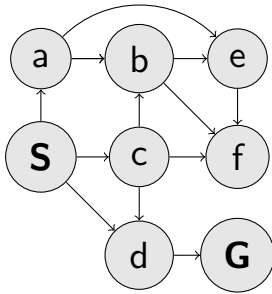


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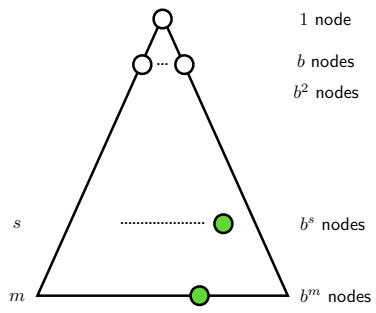
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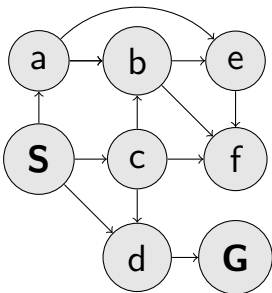


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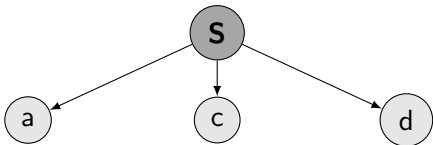
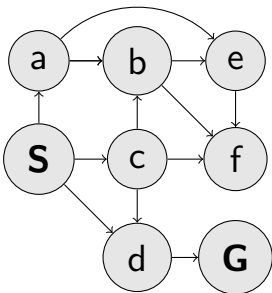


Breadth-First Search (BFS)



What are the BFS properties?

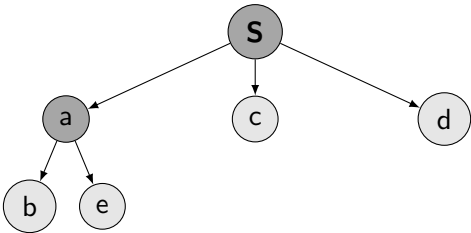
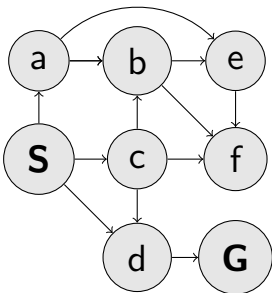
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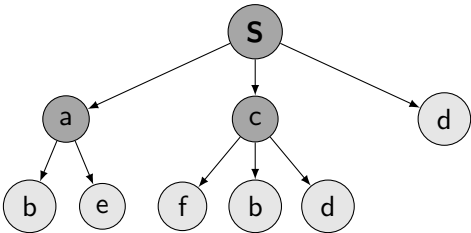
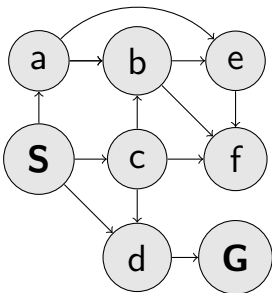
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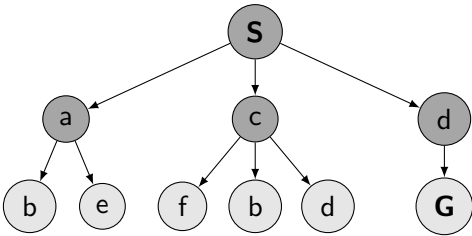
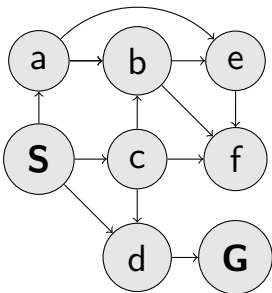
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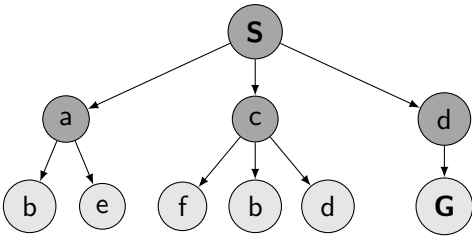
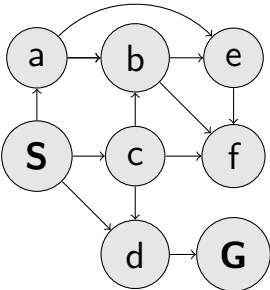
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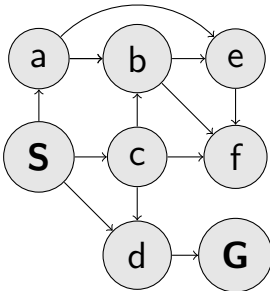
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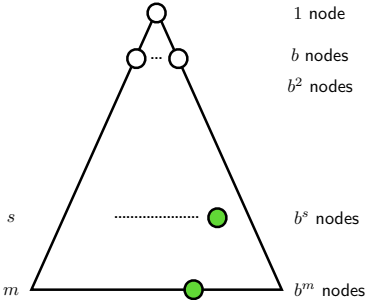
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- Time, can process the whole tree until $s: b^s$, well actually $b + b^2 + b^3 + \dots + b^s$ but the last layer vastly dominates. Try some calculations for various b .
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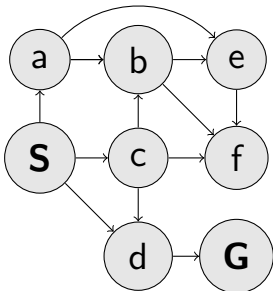


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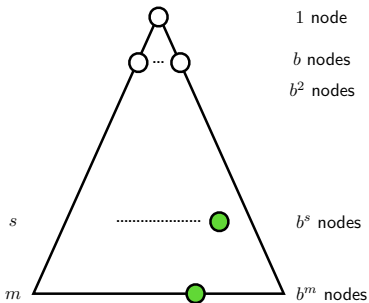
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What are (dis)advantages of the individual strategies?



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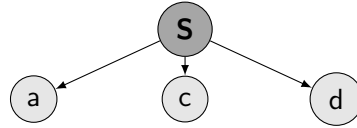
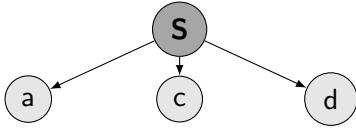
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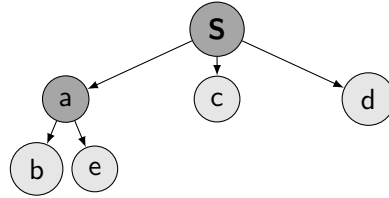
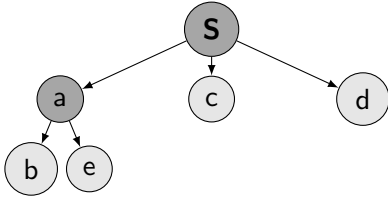
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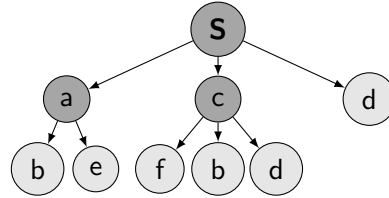
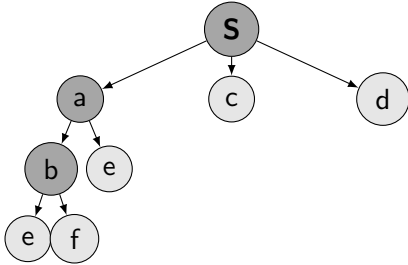
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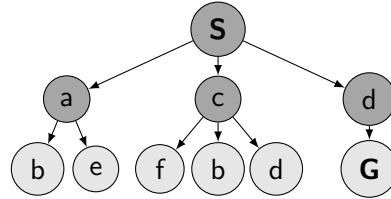
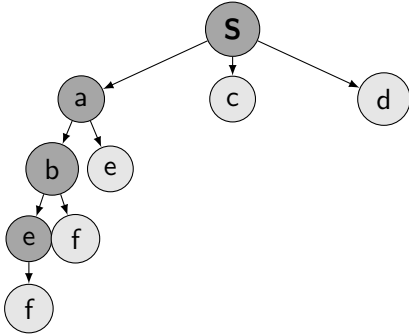
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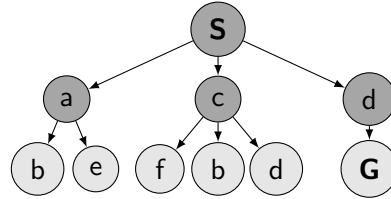
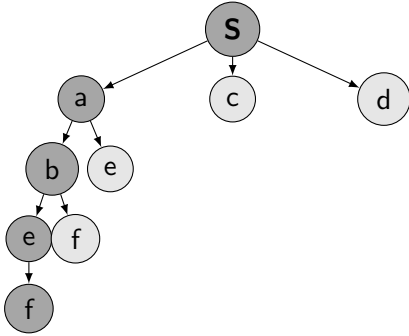
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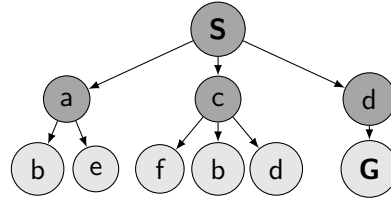
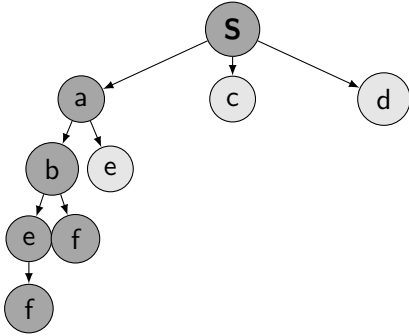
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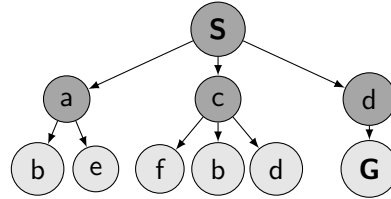
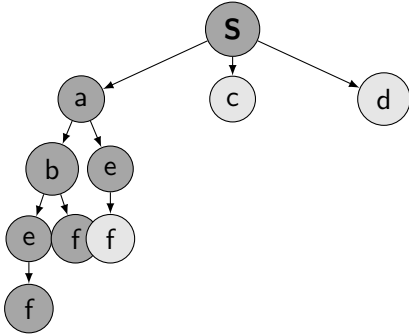
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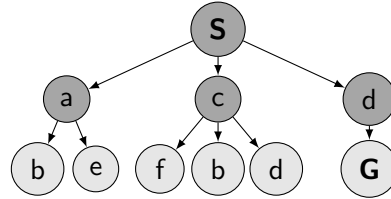
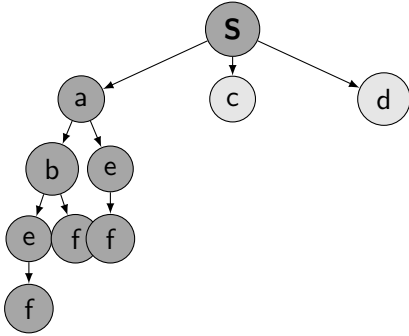
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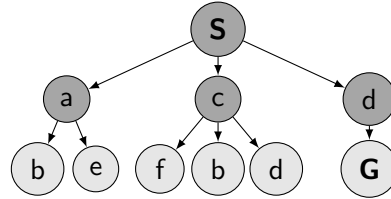
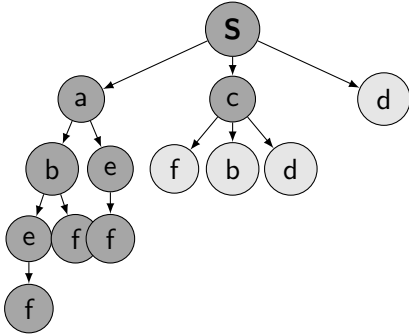
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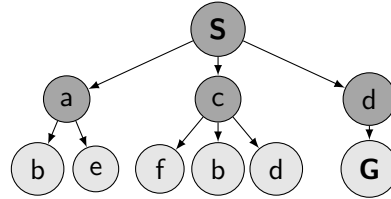
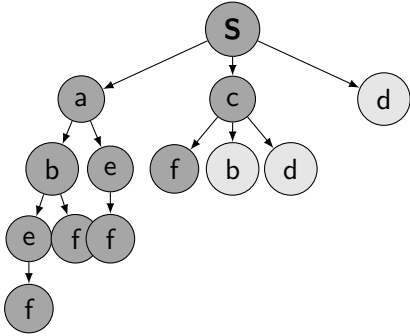
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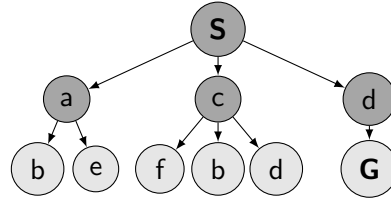
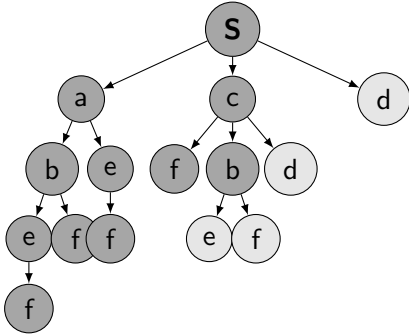
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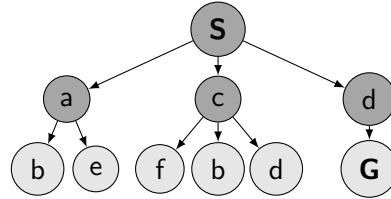
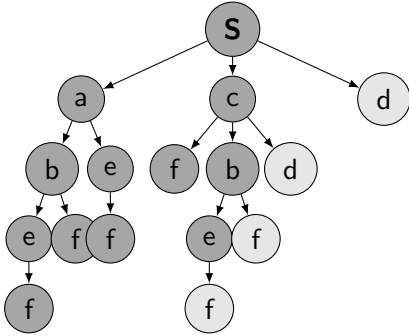
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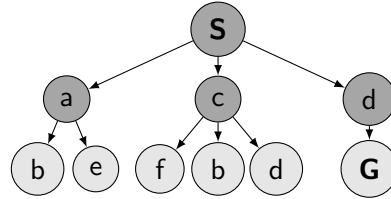
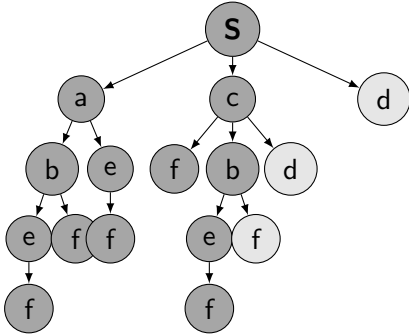
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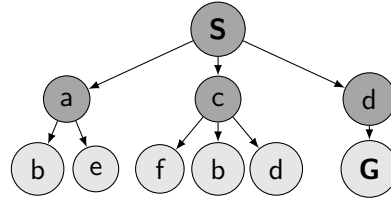
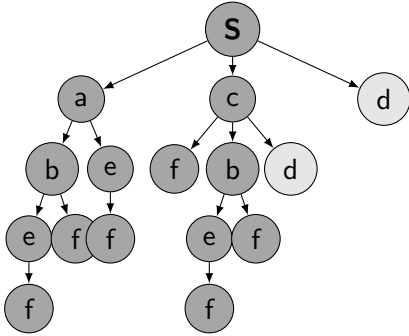
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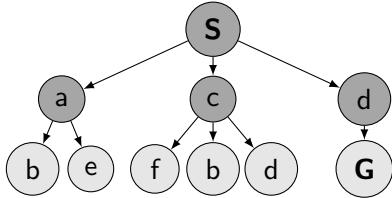
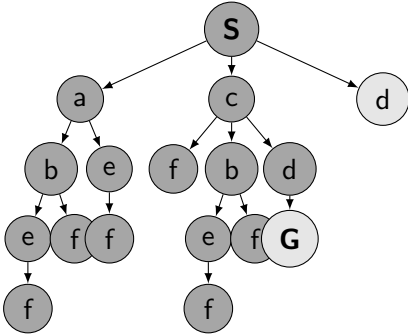
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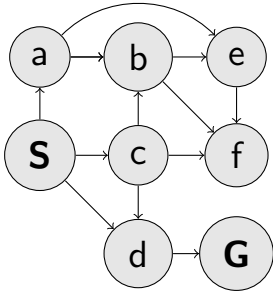
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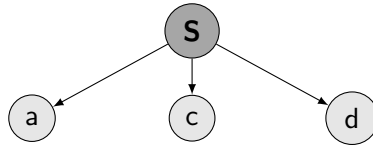
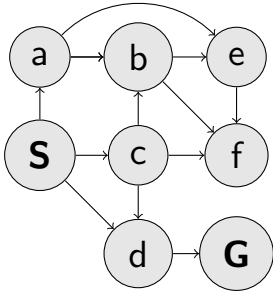


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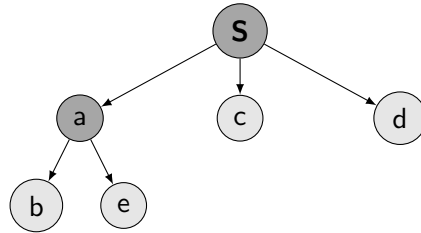
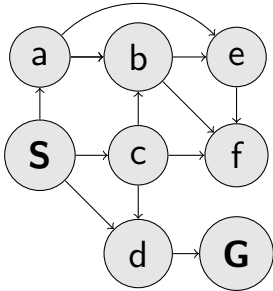


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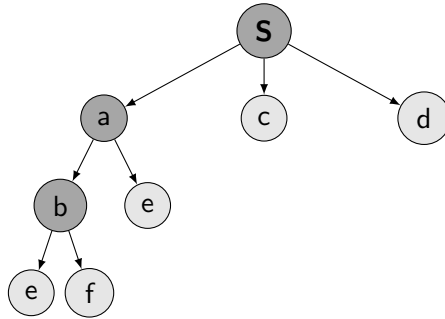
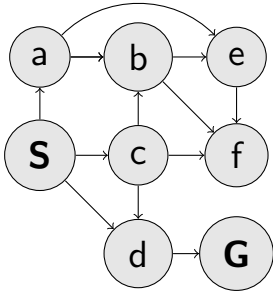


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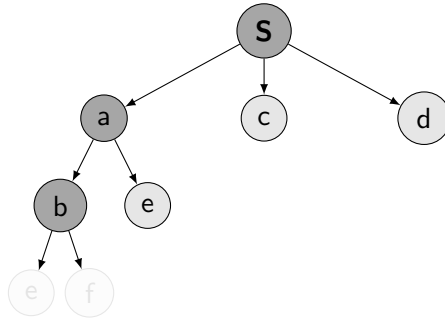
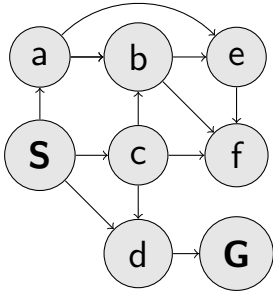


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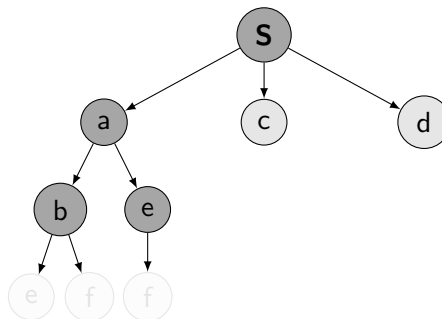
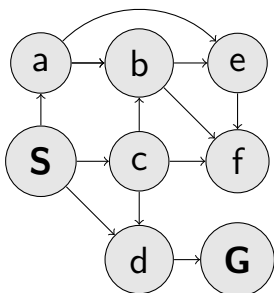


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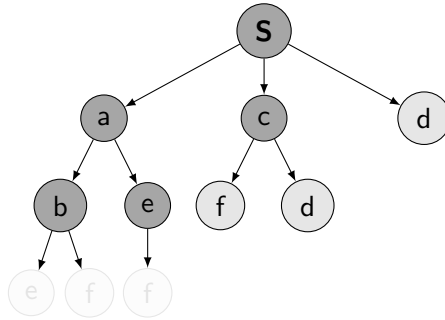
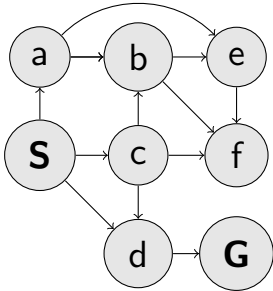


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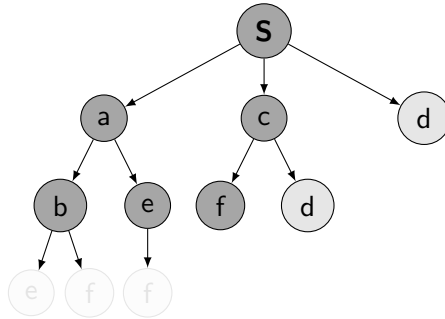
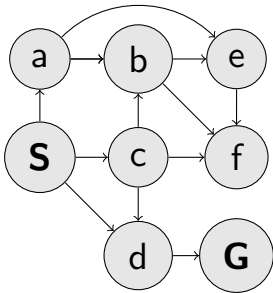


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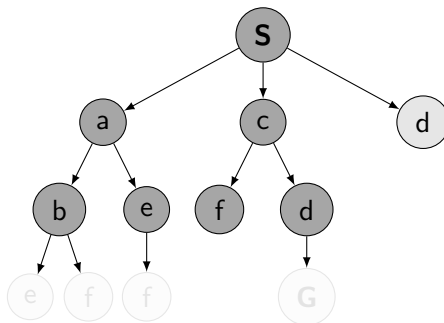
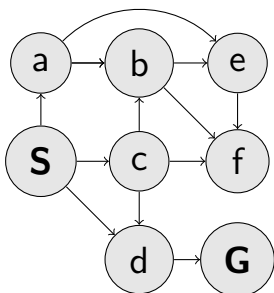


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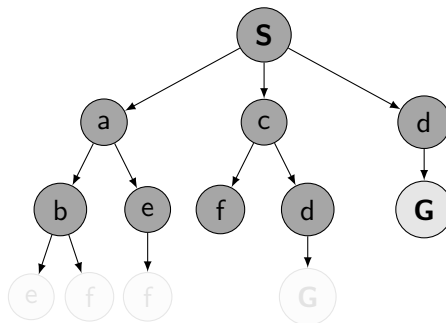
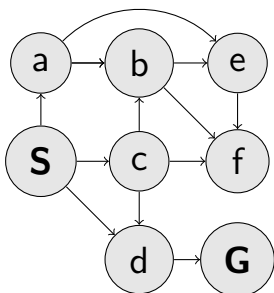


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Iterative deepening DFS (ID-DFS)

► Start with `maxdepth = 1`

► Perform DFS with limited depth. Report success or failure.

► If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

25 / 35

Notes

Really, how much do we repeat/waste? The “upper levels”, close to the root, are repeated many times. However, in a tree, most nodes are the bottom levels and nr. nodes traversed is what counts. More specifically, for a solution at depth s , the nodes on the bottom level are generated only once, those on the next-to-bottom level $2x$... children of the root are generated $s \times$. Compare the number of nodes generated ID-DFS vs. BFS:

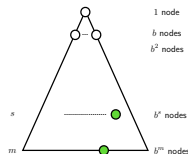
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Try some calculations for various s and b . For $b = 10$ and $d = 5$:

$$N(\text{ID-DFS}) = 50 + 400 + 3000 + 20000 + 100000 = 123450$$

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(Example from [2].)

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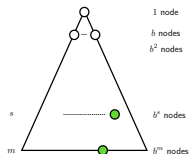
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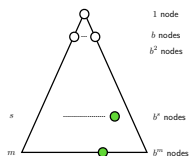
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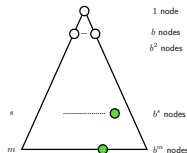
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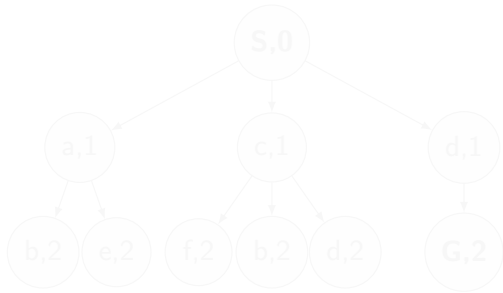
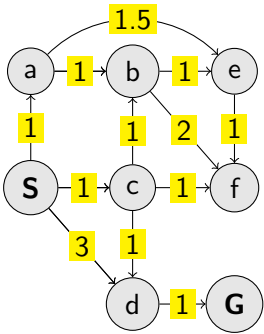
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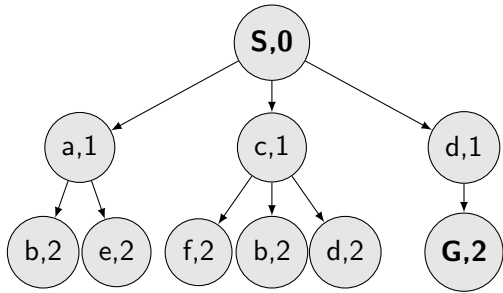
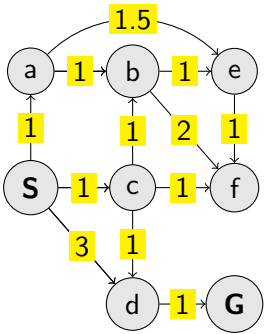
Cost sensitive search



- ▶ In BFS, DFS, node ±depth was the node-value.
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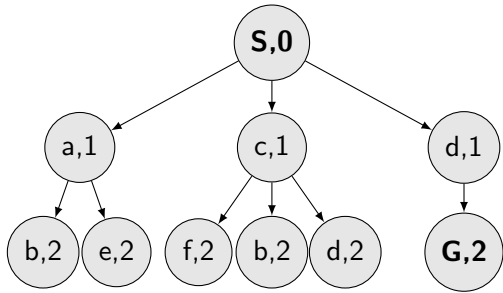
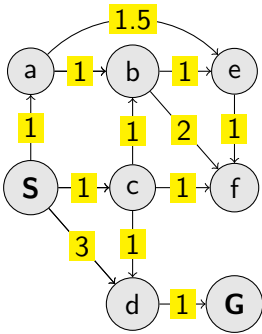
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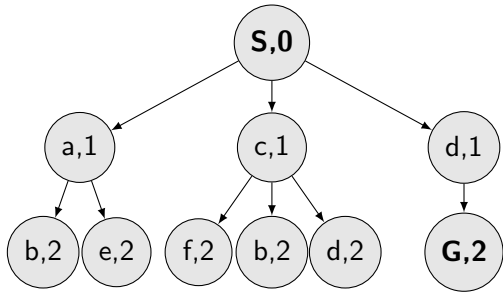
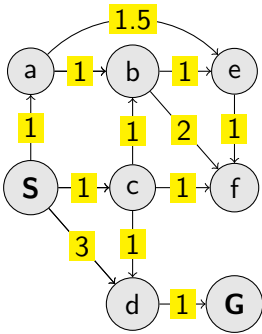
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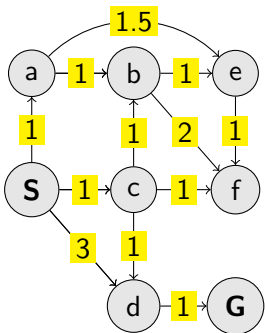
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Uniform Cost Search (UCS)



When to check the goal (and stop) the search? When visiting or expanding the node?

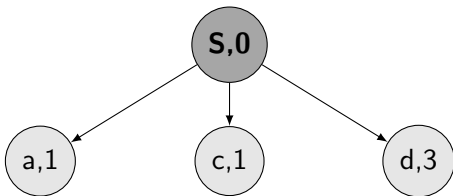
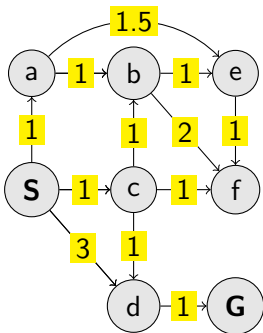
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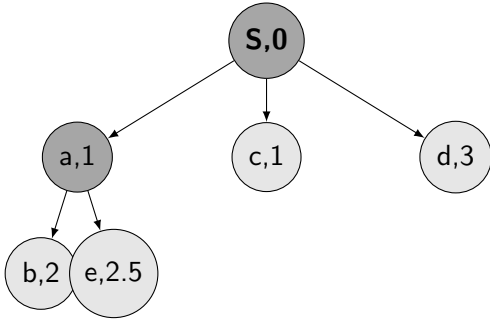
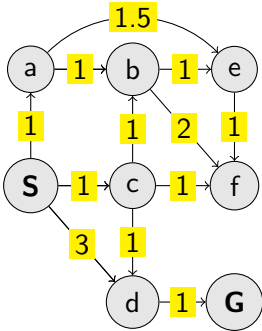
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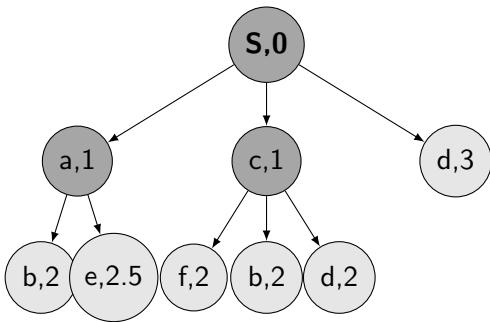
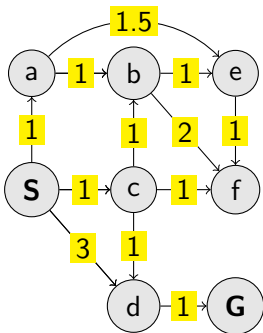
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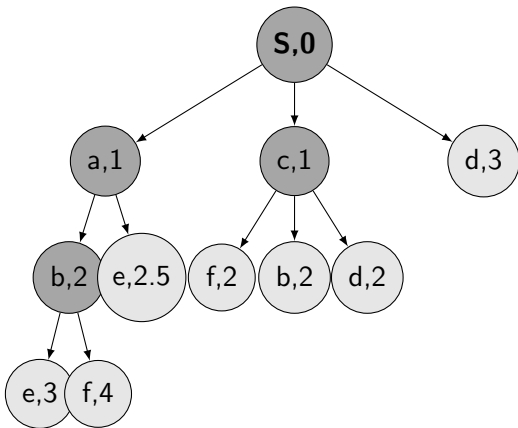
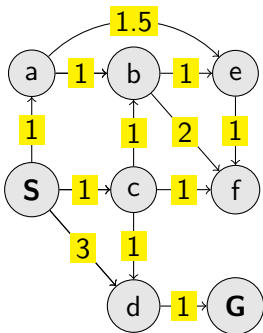
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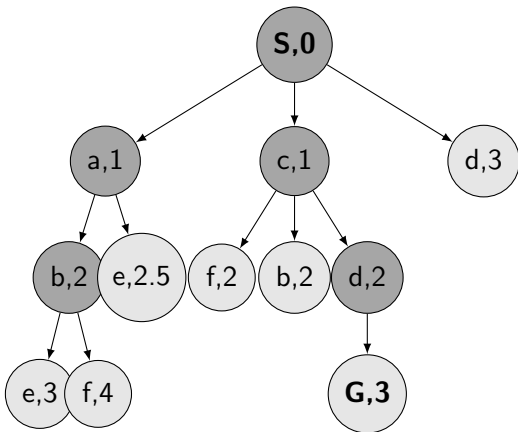
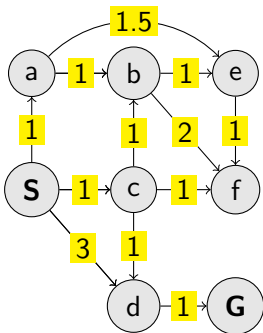
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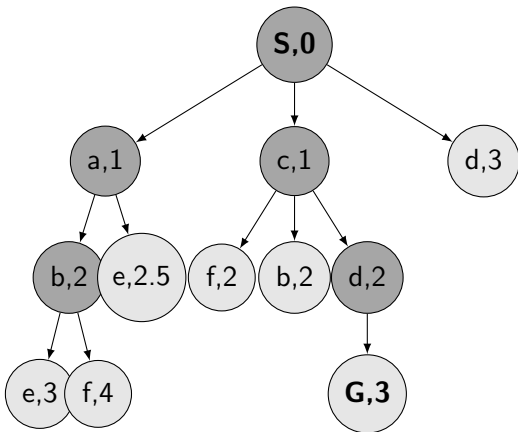
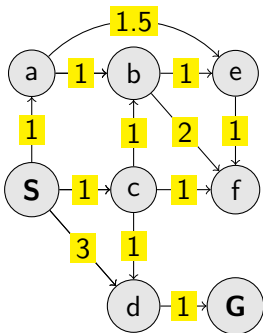
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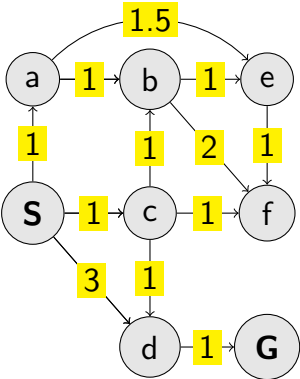
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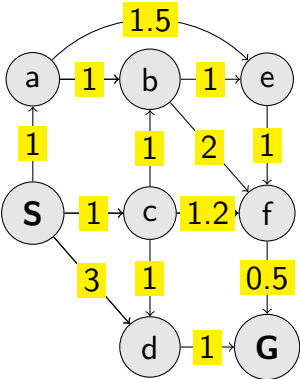
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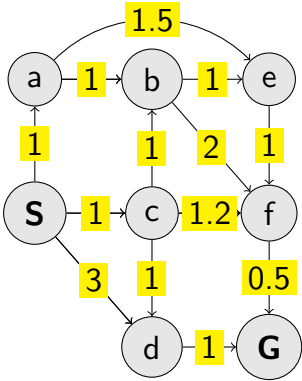
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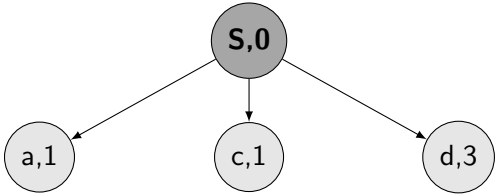
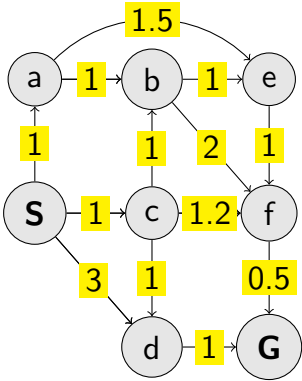
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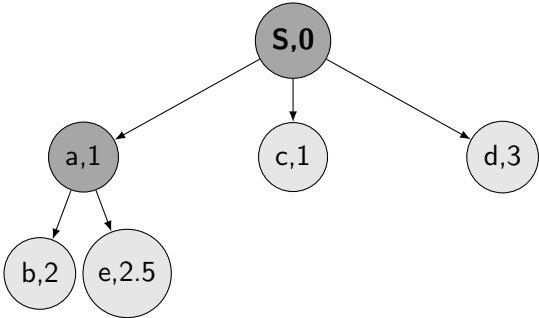
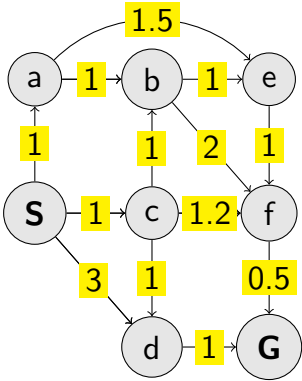
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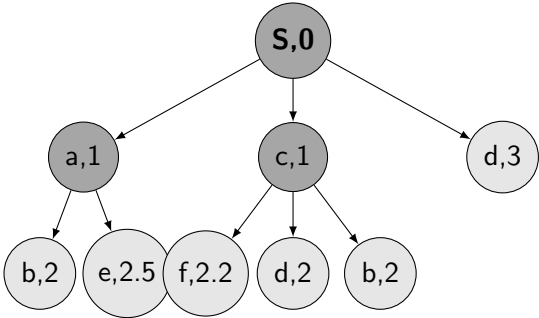
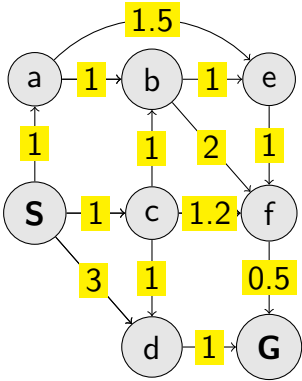
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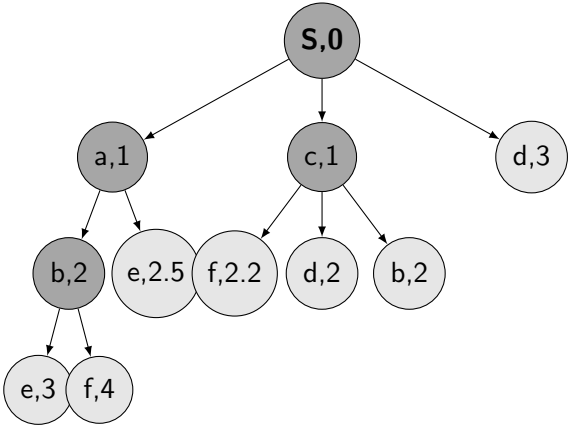
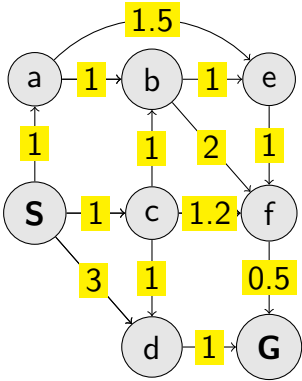
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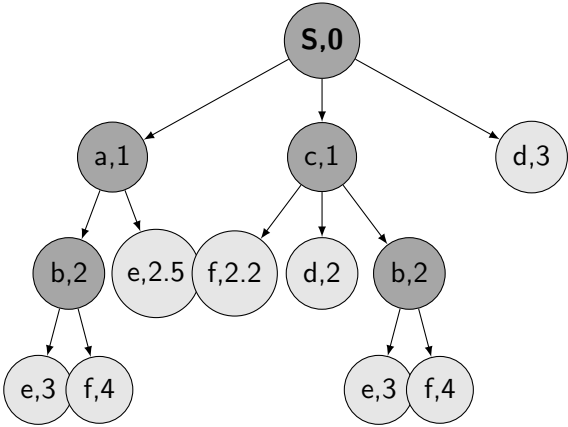
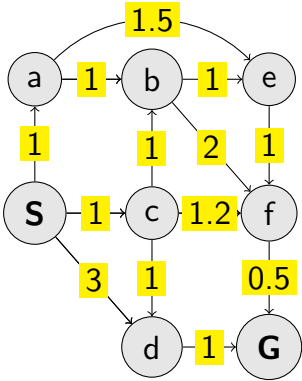
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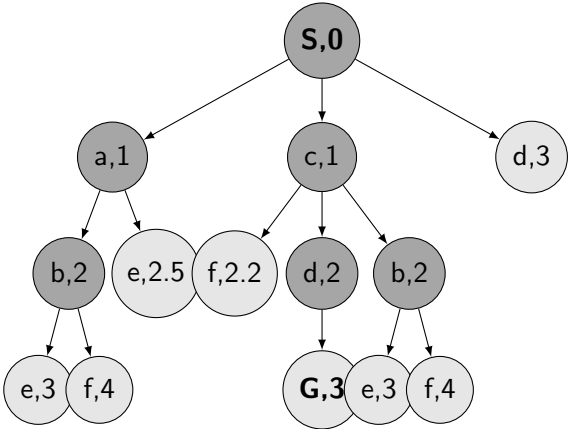
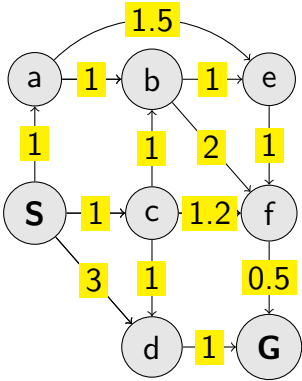
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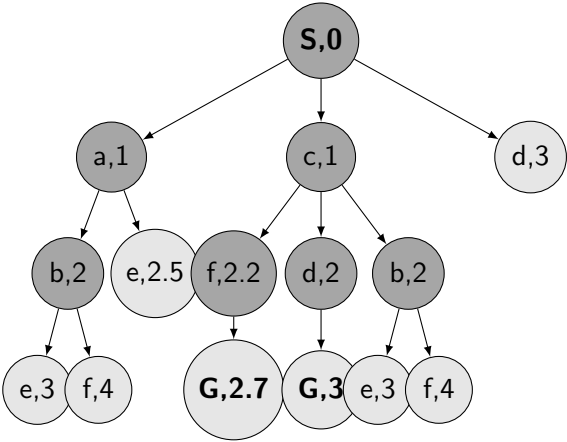
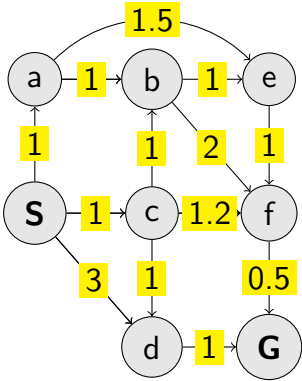
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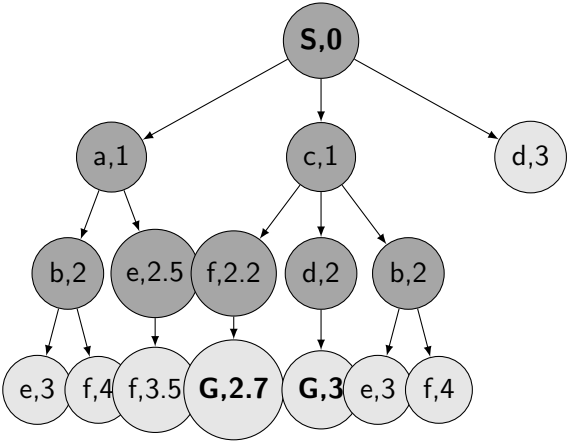
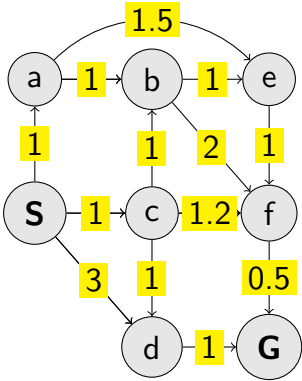
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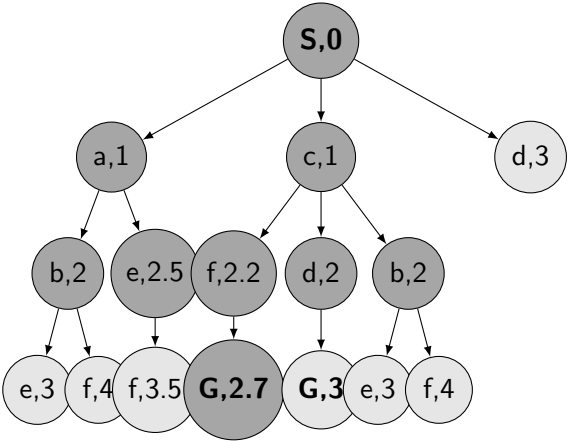
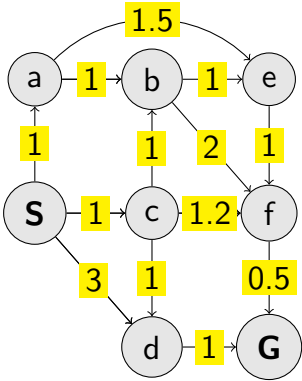
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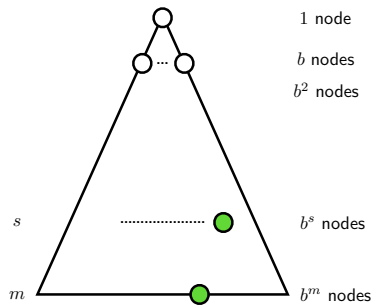
UCS properties

- ▶ Time complexity?
- ▶ Space complexity?
- ▶ Complete?
- ▶ Optimal?

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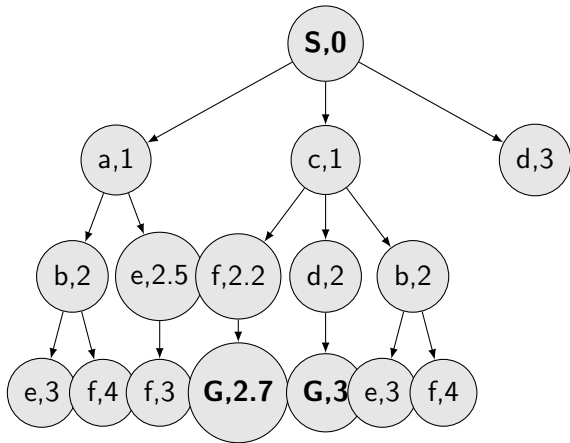
Solution cost C^* , transition cost at least ϵ . Effective depth, roughly C^*/ϵ .

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- Space: $b^{C^*/\epsilon}$
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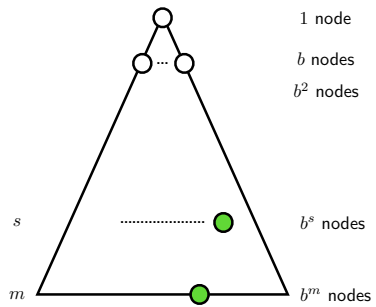
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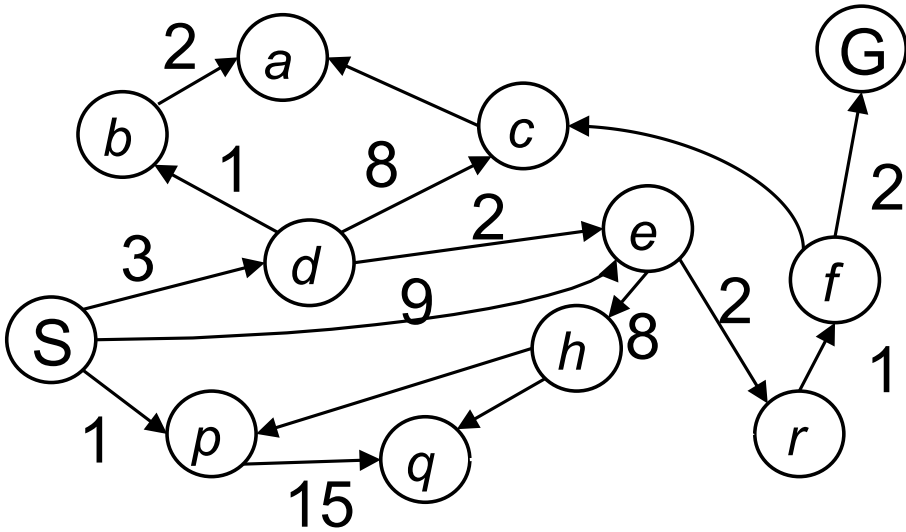
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Example: Graph with costs



Notes

Try it on paper, mark which nodes are in frontier, mark lines of equal cost.

Infrastructure for (tree) search algorithms

What should a **tree node** `n` know?

- ▶ `n.state`
- ▶ `n.parent`
- ▶ `n.pathcost`

Perhaps we may add something later, if needed ...

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How to organize nodes?

The Python examples are just suggestions, ...

- ▶ A dynamically linked structure (`list()`).
- ▶ Add a node (`list.insert(node)`).
- ▶ Take a node and remove from the structure (`node=list.pop()`).
- ▶ Check the Python modules `heapq`¹ and `queue`² for inspiration.

¹<https://docs.python.org/3.5/library/heapq.html>

²<https://docs.python.org/3.5/library/queue.html>

What is the solution?

- ▶ We stop when **Goal** is reached.
- ▶ How do we construct the **path**?

References, further reading

Some figures if from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.

Planning Algorithms.

Cambridge, 1st edition, 2006.

Online version available at: <http://planning.cs.uiuc.edu>.

[2] Stuart Russell and Peter Norvig.

Artificial Intelligence: A Modern Approach.

Prentice Hall, 3rd edition, 2010.

<http://aima.cs.berkeley.edu/>.