

Classifiers

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Today two examples:

1. Recall and Precision
2. Linear classification

Recall and Precision

Confusion matrix

| | | Actual class | |
|-----------------|----------|--------------|-----------|
| | | <i>P</i> | <i>N</i> |
| Predicted class | <i>P</i> | <i>TP</i> | <i>FP</i> |
| | <i>N</i> | <i>FN</i> | <i>TN</i> |

The abbreviations have the following meaning true positive (TP), false positive (FP), false negative (FN), true negative (TN).

Recall and Precision

You get a few confusion matrices of 4 different classifiers. The abbreviations have the following meaning true positive (TP), false positive (FP), false negative (FN), true negative (TN).

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| classifier | $TP = 20$ | $FP = 3$ |
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| | | |
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| classifier | $TP = 14$ | $FP = 16$ |
| D | $FN = 4$ | $TN = 80$ |

Recall:

$$A: \frac{TP}{TP+FN}$$

$$B: \frac{TP}{TP+TN}$$

$$C: \frac{TP}{FP+FN}$$

$$D: \frac{FP}{TP+FN}$$

Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

Precision:

A: $\frac{TP}{TP+FN}$

B: $\frac{FP}{FP+TN}$

C: $\frac{TP}{TP+FP}$

D: $\frac{FP}{TP+FN}$

Recall and Precision

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A: $\frac{TP}{TP+FN}$

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Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

► Precision: $\frac{TP}{TP+FP}$

For the classifier from table A count the number of members for the two classes (from the point of view of the data/reality):

A: class1: 23, class2: 32

B: class1: 38, class2: 17

C: class1: 21, class2: 34

D: class1: 23, class2: 36

Recall and Precision

You get a few confusion matrices of 4 different classifiers. The abbreviations have the following meaning true positive (TP), false positive (FP), false negative (FN), true negative (TN).

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► Recall: $\frac{TP}{TP+FN}$

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Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

► Precision: $\frac{TP}{TP+FP}$

Let us assume that this is a table of classifiers that recognize the presence of a person in front of an autonomous car. The car stops or continues its journey.

Task: Which classifier (A/B/C/D) will unnecessarily stop the car the least amount of times?

Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

► Precision: $\frac{TP}{TP+FP}$

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Task: Which classifier (A/B/C/D) will unnecessarily stop the car the least amount of times?

A because it has the lowest $\frac{FP}{TP+FP}$

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Task: Which classifier (A/B/C/D) will unnecessarily stop the car the least amount of times?

A because it has the lowest $\frac{FP}{TP+FP}$

Which measure is more important for the decision, recall or precision?:

A: Recall

B: Precision

Recall and Precision

You get a few confusion matrices of 4 different classifiers. The abbreviations have the following meaning true positive (TP), false positive (FP), false negative (FN), true negative (TN).

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► Recall: $\frac{TP}{TP+FN}$

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Task: Which classifier (A/B/C/D) will unnecessarily stop the car the least amount of times?

A because it has the lowest $\frac{FP}{TP+FP}$

Which measure is more important for the decision, recall or precision?:

B: Precision, because $\frac{FP}{TP+FP} = 1 - Precision$. Therefore, we are looking for the maximum Precision.

Recall and Precision

You get a few confusion matrices of 4 different classifiers. The abbreviations have the following meaning true positive (TP), false positive (FP), false negative (FN), true negative (TN).

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Let us assume that this is a table of classifiers that recognize the presence of a person in front of an autonomous car. The car stops or continues its journey.

Task: Which classifier (A/B/C/D) is the safest?

Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

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Let us assume that this is a table of classifiers that recognize the presence of a person in front of an autonomous car. The car stops or continues its journey.

Task: Which classifier (A/B/C/D) is the safest?

D because it has the lowest $\frac{FN}{TP+FN}$

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► Recall: $\frac{TP}{TP+FN}$

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Task: Which classifier (A/B/C/D) is the safest?

D because it has the lowest $\frac{FN}{TP+FN}$

Which metric is important for this decision, recall or precision?:

A: Recall

B: Precision

Recall and Precision

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► Recall: $\frac{TP}{TP+FN}$

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Task: Which classifier (A/B/C/D) is the safest?

D because it has the lowest $\frac{FN}{TP+FN}$

Which metric is important for this decision, recall or precision?:

A: Recall, because $\frac{FN}{TP+FN} = 1 - \text{Recall}$. Therefore, we are looking for the maximum Recall.

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► Recall: $\frac{TP}{TP+FN}$

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Let us assume that this is a table of classifiers that recognize the presence of a person in front of an autonomous car. The car stops or continues its journey.

Discussion: Suggest the simplest and safest classifier. Use chat or microphone.

Could it be used in practice?

Summary Recall and Precision

- ▶ Recall: $\frac{TP}{TP+FN}$
- ▶ Precision: $\frac{TP}{TP+FP}$

Think about object (pedestrian) detection in images:

Recall: What is related to recall?

- A: How many (what percentage of) objects/pedestrians missed?
- B: How often are the (pedestrian) detections truly negative compared to truly positive detections?
- C: How often are the (pedestrian) detections truly positive compared to truly negative detections?
- D: How much are the (pedestrian) detections contaminated by false detections?

Summary Recall and Precision

- ▶ Recall: $\frac{TP}{TP+FN}$
- ▶ Precision: $\frac{TP}{TP+FP}$

Think about object (pedestrian) detection in images:

Recall: What is related to recall?

- ▶ How many (what percentage of) objects/pedestrians missed?

Precision: What is related to precision?

- A: How many (what percentage of) objects/pedestrians are not missed?
- B: How often are the (pedestrian) detections truly negative compared to truly positive detections?
- C: How often are the (pedestrian) detections truly positive compared to truly negative detections?
- D: How much are the (pedestrian) detections contaminated by false detections?

Summary Recall and Precision

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Precision: What is related to precision?

- D: How much are the (pedestrian) detections contaminated by false detections?

Summary Recall and Precision

- ▶ Recall: $\frac{TP}{TP+FN}$
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Think about object (pedestrian) detection in images:

Recall: What is related to recall?

- ▶ How many (what percentage of) objects/pedestrians missed?

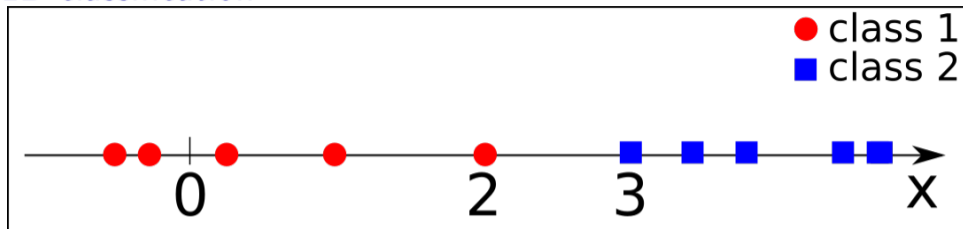
Precision: What is related to precision?

- ▶ How much are the (pedestrian) detections contaminated by false detections?

It is difficult to satisfy both.

Linear classification

1D classification



Classification is made according to:

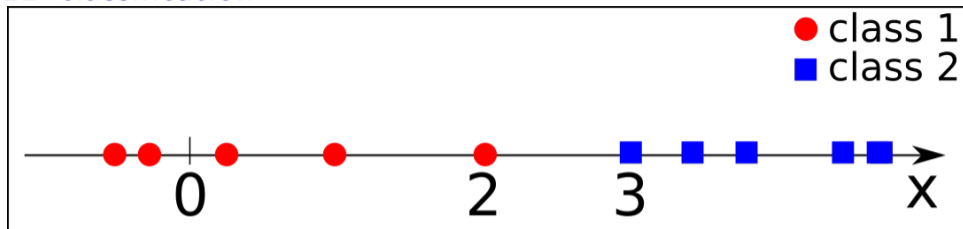
$$s^* = \arg \max_s f_s(x)$$

where $f_s(x) = w_s x + b_s$, $w_s, b_s \in \mathbb{R}$, $s \in \{1, 2\}$

Select a classifier with zero classification error on the given dataset:

- A: $w_1 = 1, b_1 = 0; w_2 = -1, b_2 = 0$
- B: $w_1 = 1, b_1 = -2.4; w_2 = -1, b_2 = 2.5$
- C: $w_1 = -1, b_1 = 2.5; w_2 = 1, b_2 = -2.4$
- D: $w_1 = -1, b_1 = 0; w_2 = 1, b_2 = 0$

1D classification



Classification is made according to:

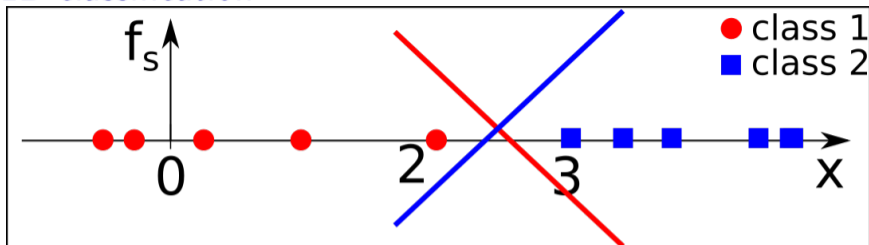
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1D classification



Classification is made according to:

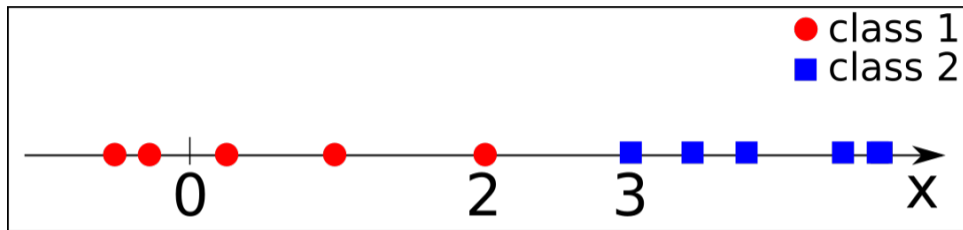
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- B: $w_1 = 1, b_1 = -2.4; w_2 = -1, b_2 = 2.5$
- C: $w_1 = -1, b_1 = 2.5; w_2 = 1, b_2 = -2.4$
- D: $w_1 = -1, b_1 = 0; w_2 = 1, b_2 = 0$

1D classification



Alternatively (only for binary classification):

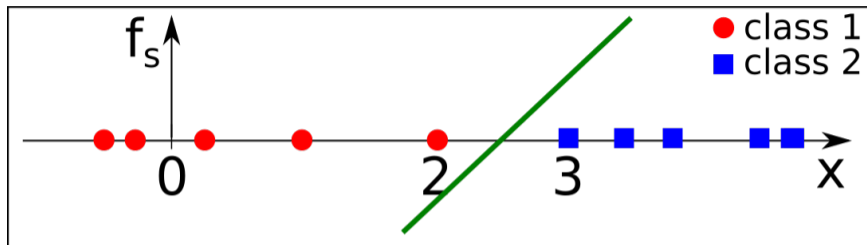
A: $s^* = (\text{sgn}(x - 2.5) + 1)/2 + 1$

B: $s^* = \text{sgn}(x - 2.5)$

C: $s^* = \max(x_i - 2.5)$

D: $s^* = \text{sgn}(x - 2.5) + 1$

1D classification



Alternatively (only for binary classification):

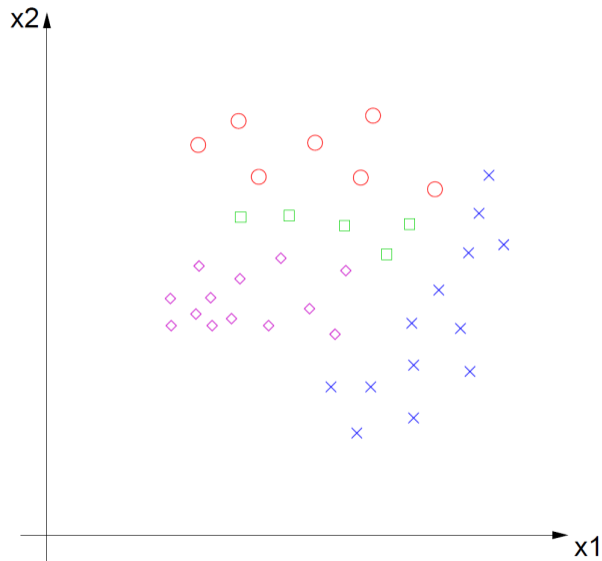
A: $s^* = (\text{sgn}(x - 2.5) + 1)/2 + 1$

B: $s^* = \text{sgn}(x - 2.5)$

C: $s^* = \max(x_i - 2.5)$

D: $s^* = \text{sgn}(x - 2.5) + 1$

2D classification

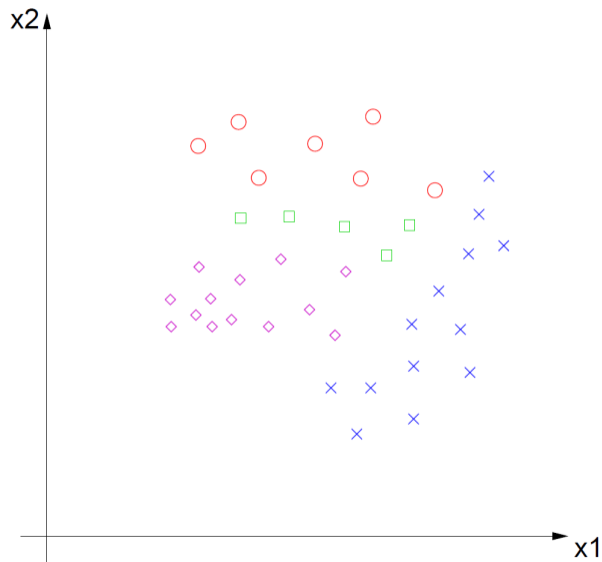


Is it possible to get zero classification errors on the given training multiset using only a linear classifier?

Yes

No

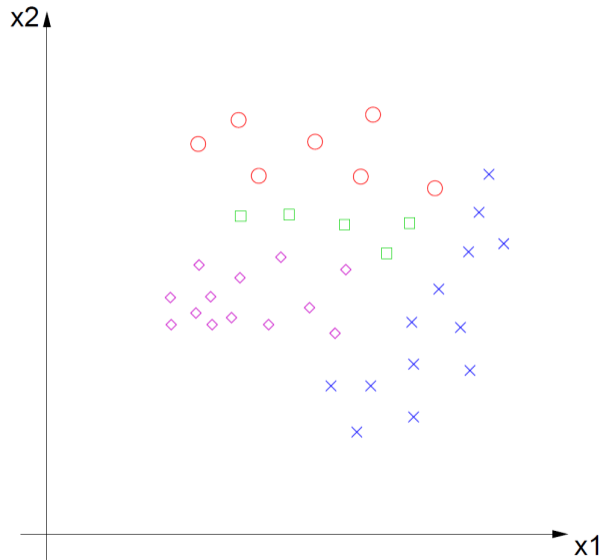
2D classification



Is it possible to get zero classification errors on the given training multiset using only a linear classifier?

► **Yes.**

2D classification



► Draw the solution.