ARO **Name:** ____________________

SLAM examples

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1. **ICP:** Consider ICP algorithm [Besl and McKey, 1992] presented during the lecture. Derive that the rotation update is:

$$\mathbf{R} := \mathbf{R}'\mathbf{R}$$

and the translation update is:

$$\mathbf{t} := \mathbf{R}'\mathbf{t} + \mathbf{t}'$$

Hint: $\mathbf{R}, \mathbf{t}$ preserves overall transformation of the original pointcloud in the algorithm. Use following induction:

- We will first assume that after the k-th iteration the overall transformation of the original pointcloud is given by rotation $\mathbf{R}$ and translation $\mathbf{t}$.
- Consequently, after $(k+1)$-th iteration the overall transformation of the original pointcloud is the concatenation of (i) the previous transformation $\mathbf{R}, \mathbf{t}$ and (ii) the newly estimated increment $\mathbf{R}', \mathbf{t}'$.

To show, that the suggested update is correct, derive that in the $(k+1)$-th iteration the concatenated transformation is given by rotation $\mathbf{R}'\mathbf{R}$ and translation $\mathbf{R}'\mathbf{t} + \mathbf{t}'$

2. **Harris corner detector:**

(a) Derive following approximation for the Sum-of-squares dissimilarity function:

$$E(\mathbf{t}) = \sum_{\mathbf{x}\in\mathcal{W}(\mathbf{u})} (I(\mathbf{x}+\mathbf{t}) - I(\mathbf{x}))^2 \approx \mathbf{t}^\top \underbrace{\sum_{\mathbf{x}\in\mathcal{W}(\mathbf{u})} \frac{\partial I(\mathbf{x})}{\partial \mathbf{x}} \frac{\partial I(\mathbf{x})}{\partial \mathbf{x}}^\top}_{\mathbf{M}} \mathbf{t}$$

Hint: Approximate $I(\mathbf{x}+\mathbf{t})$ using first order Taylor polynomial

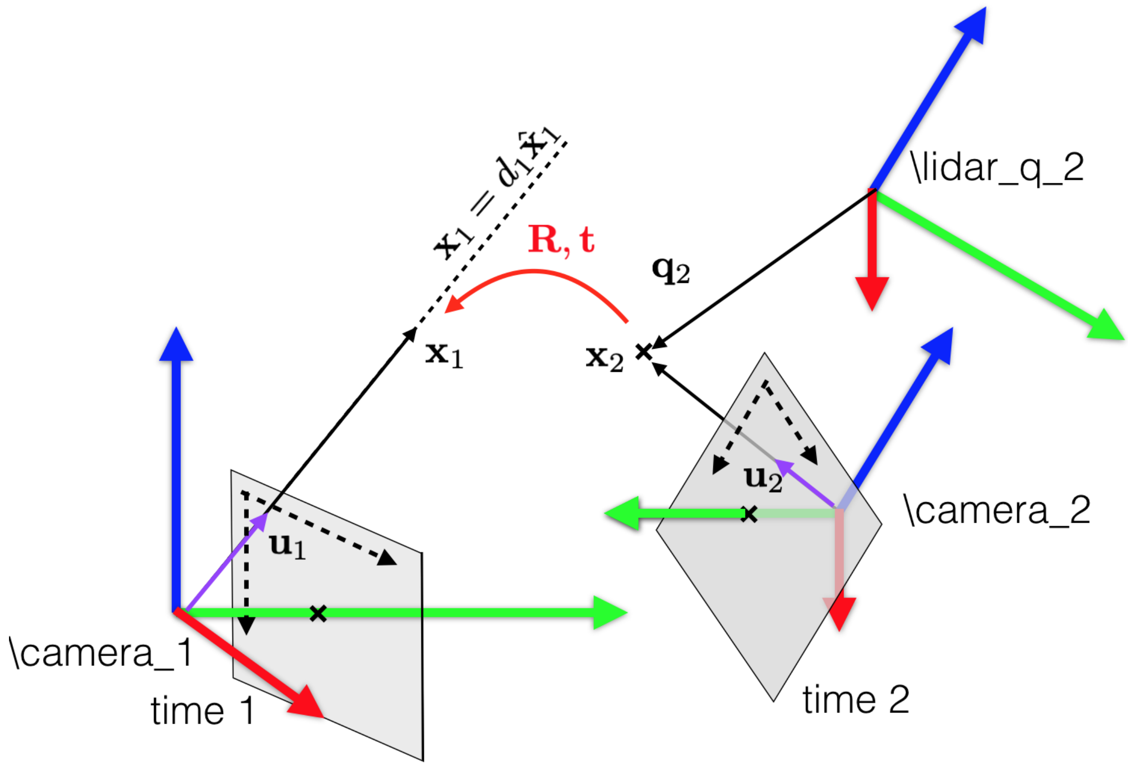
$$I(\mathbf{x}+\mathbf{t}) \approx I(\mathbf{x}) + \frac{\partial I(\mathbf{x})}{\partial \mathbf{x}}^\top \mathbf{t}$$

and substitute it into E(t).

(b) show that $\mathbf{M}$ is positive semi-definite matrix.

Hint: Show that eigen-values of any matrix created as $\mathbf{g}\mathbf{g}^\top$ are non-negative.

3. **SLAM from 2D-3D correspondences:** Consider camera-lidar setup, in which 2D-3D correspondence is provided (lidar measurement for time 1 is mising).



Camera is specified by the following parameters:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R_c} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{t_c} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

You are given one 2D-3D correspondence $\mathbf{u}_1 = [1500, 750]^\top$ (pixel coordinates in camera 1), $\mathbf{x}_2 = [2, 1, 1]^\top$ (3D point measured by lidar in coordinate frame of camera 2). Derive criterion function for the alignment from this correspondence.

a) Construct camera projection matrix and estimate directional vector $\hat{\mathbf{x}}_1 \in \mathcal{R}^3$ of pixel $\mathbf{u}_1$:

$\mathbf{P} =$

$\hat{\mathbf{x}}_1 =$

b) We known that the correct rotation $\mathbf{R}$ and translation $\mathbf{t}$ transform point $\mathbf{x}_2$ on the ray $d_1\hat{\mathbf{x}}_1$ (generated by pixel $\mathbf{u}_1$) as follows:

$$d_1\hat{\mathbf{x}}_1 = \mathbf{R}\mathbf{x}_2 + \mathbf{t}.$$

Split the matrix equation in 3 scalar equations and get rid of the unknown depth d_1, by dividing the first two equations by the third one. Result should be two equations with right-hand-side equal to zero.

Equation1 :

Equation2 :

c) Estimate criterion to be minimized $f_{23}(\mathbf{R}, \mathbf{t})$ as the sum of squares of previously derived left-hand-sides.

$f_{23}(\mathbf{R}, \mathbf{t}) =$

d) What is the value of the criterion for this pose $\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$