ARO **Name:** ____________________

Camera and stereo examples

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1. **3D PCL ⇒ RGB:** Consider a perspective camera with the following intrinsic camera parameters K, camera rotation matrix R, and translation vector t:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

a) Construct camera projection matrix $\mathbf{P} \in \mathcal{R}^{3\times 4}$:

$\mathbf{P} =$

b) Project point

$$\mathbf{q} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

into the camera. What are pixel coordinates $\mathbf{u} \in \mathcal{R}^2$ of the projection?

$u_1 =$

$u_2 =$

2. **RGBD $\Rightarrow$ 3D PCL:** Consider the RGBD perspective camera with the previously defined parameters:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

You are given pixel coordinates $\mathbf{u} = [500; -50]$. Depth estimated at this coordinates by the RGBD sensor is $D(\mathbf{u}) = 3$. What is corresponding 3D point $\mathbf{q} \in \mathcal{R}^3$?
Hint: Start working in camera coordinate frame (i.e. ignore provided $\mathbf{R}$ and $\mathbf{t}$) and find point $\mathbf{p}$ such that $p_z = D(\mathbf{u})$ and its projection is $\mathbf{u}$. Then transform point $\mathbf{p}$ to world c.f. $\mathbf{q}$ such that $\mathbf{p} = \mathbf{R}\mathbf{q} + \mathbf{t}$. You can easily verify that projection of $\mathbf{q}$ to the given yields provided pixel $\mathbf{u}$.

3. **Stereo reconstruction from 2D-2D correspondences:** Consider a stereo pair created from two perspective cameras with the following intrinsic camera parameters $\mathbf{K}$, camera rotation matrices $\mathbf{R}_1, \mathbf{R}_2$ and translation vectors $\mathbf{t_1}, \mathbf{t_2}$:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{t_2} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

You are given one 2D-2D correspondence $\mathbf{u} = [1200, 250]^\top$ (pixel coordinates in the first camera $\mathbf{P}$), $\mathbf{v} = [2200, 250]^\top$ (pixel coordinates in the second camera $\mathbf{S}$). Estimate position of the corresponding 3D point $\mathbf{q} \in \mathcal{R}^3$.

a) Construct camera projection matrix of both matrices $\mathbf{P}, \mathbf{S} \in \mathcal{R}^{3\times 4}$:

$\mathbf{P} =$

$\mathbf{S} =$

b) Formulate the problem of the 3D reconstruction as a solution of the following set of linear equations $\mathbf{A}_{[4\times 3]}\mathbf{q}_{[3\times 1]} = \mathbf{b}_{[4\times 1]}$

$\mathbf{A} =$

$\mathbf{b} =$

c) Use arbitrary language or tool (e.g. python/linalg, matlab, C/C++, mathematica) to solve the problem

$\mathbf{q} =$

d) Check that backprojection of computed point lies in provided pixel coordinates $\mathbf{u}, \mathbf{v}$.

4. **Stereo epipolar line:** Consider the same stereo pair created from two perspective cameras $\mathbf{P}$, $\mathbf{S}$ with the same previously specified parameters.

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{t_2} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

Verify if these two pixel coordinates can create 2D-2D correspondence $\mathbf{u} = [1000, 750]^\top$ (pixel coordinates in camera $\mathbf{P}$), $\mathbf{v} = [1000, 250]^\top$ (pixel coordinates in camera $\mathbf{S}$).

a) Estimate relative pose of camera $\mathbf{S}$ wrt camera $\mathbf{P}$:

$\mathbf{R} =$

$\mathbf{t} =$

b) Compute fundamental matrix.

Hint: $(\mathbf{t} \times \mathbf{R}) = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \cdot \mathbf{R}$

$\mathbf{F} =$

c) Check if point $\mathbf{v}$ lies in the epipolar line generated by point $\mathbf{u}$

YES - these two points can create 2D-2D correspondence (they correspond to a single 3D point).

NO - they cannot create 2D-2D correspondence

5. **Lidar calibration:** Given set of 3D-3D correspondences $\{(\mathbf{p}_i, \mathbf{q}_i), \mid i = 1 \dots N\}$ between two lidar scans. Suggest a suitable optimization problem, which estimates rotation $\mathbf{R}$ and translation $\mathbf{t}$, between these two lidars. Specify also domain of $\mathbf{R}$ and $\mathbf{t}$.
   Hint: Do not copy it from slides, just think about it and you will figure it out.