Description Logics and OWL

Petr Křemen

petr.kremen@fel.cvut.cz

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Outline

Formal Ontologies

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  - ontologies as “some shared knowledge structures often visualized through UML-like diagrams” ...
- But how to check they are designed correctly? How to reason about the knowledge inside?
- No single language – many graphical/textual languages ranging from informal to formal ones can be used, e.g. *relational algebra*, *Prolog*, *RDFS*, *OWL*, *topic maps*, *thesauri*, *conceptual graphs*

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- propositional logic

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- … what is the meaning of these formulas ?

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- Proof Theory – to enforce the semantics

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proof theory ($\vdash$) – resolution, tableau

complexity – NP-Complete (Cook theorem)

First Order Predicate Logic

Example

What is the meaning of this sentence ?

$$(\forall x_1)((Student(x_1) \wedge (\exists x_2)(GraduateCourse(x_2) \wedge isEnrolledTo(x_1, x_2)))$$
$$\Rightarrow (\forall x_3)(isEnrolledTo(x_1, x_3) \Rightarrow GraduateCourse(x_3)))$$

$$Student \sqcap \exists isEnrolledTo.GraduateCourse \sqsubseteq \forall isEnrolledTo.GraduateCourse$$

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complexity – undecidable (Goedel)

Open World Assumption

OWA

FOPL accepts Open World Assumption, i.e. whatever is not known is not necessarily false.

As a result, FOPL is *monotonic*, i.e.

monotonicity

No conclusion can be invalidated by adding extra knowledge.

This is in contrary to relational databases, or Prolog that accept Closed World Assumption.

Towards Description Logics

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- Well, we have Prolog – wide-spread and optimized implementation of FOPL, right ?
  - ☹ Prolog is not an implementation of FOPL – OWA vs. CWA, negation as failure, problems in expressing disjunctive knowledge, etc.

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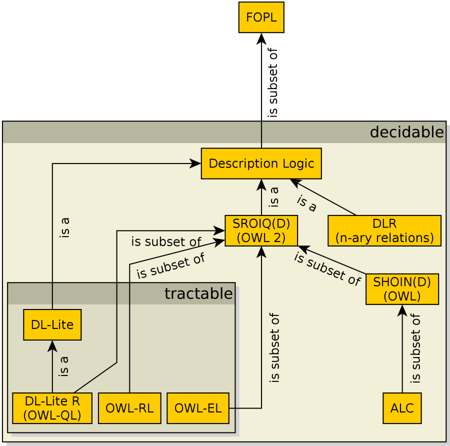

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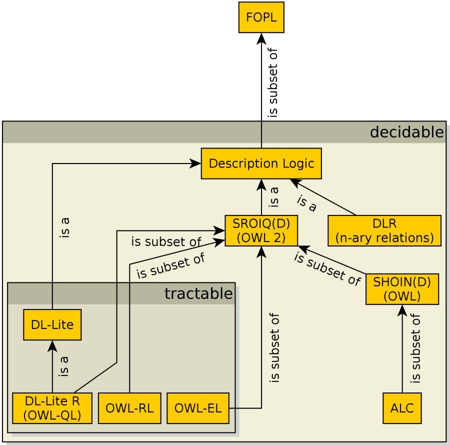
Description logics (DLs) are (almost exclusively) decidable subsets of FOPL aimed at modeling *terminological incomplete knowledge*.



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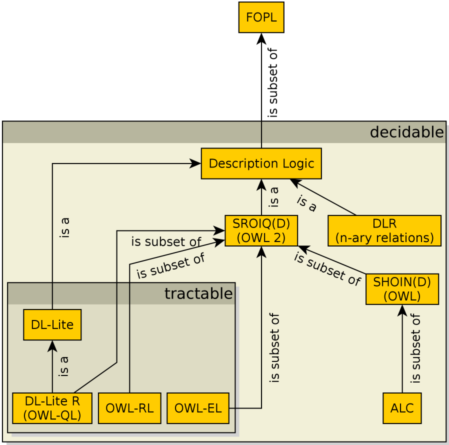

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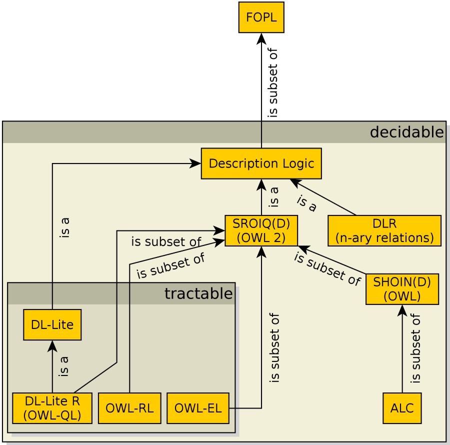

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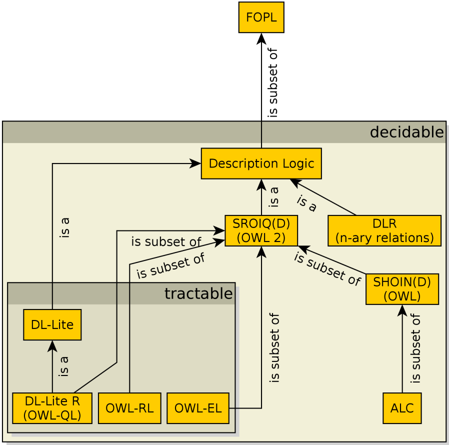

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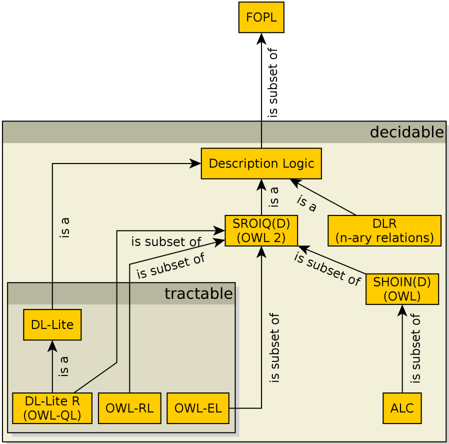

$\mathcal{ALC}$ Language

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- DLs differ in their expressive power (concept/role constructors, axiom types).

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- Having *atomic* concept A, *atomic* role R and individual a, then

$$\begin{aligned} A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \\ R^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \\ a^{\mathcal{I}} &\in \Delta^{\mathcal{I}} \end{aligned}$$

$\mathcal{ALC}$ (= attributive language with complements)

Having concepts C, D, atomic concept A and atomic role R, then for interpretation $\mathcal{I}$:

concept	*concept*$^{\mathcal{I}}$	*description*
$\top$	$\Delta^{\mathcal{I}}$	(universal concept)
$\bot$	$\emptyset$	(unsatisfiable concept)
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	(negation)
$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	(intersection)
$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$	(union)
$\forall R \cdot C$	$\{a \mid \forall b((a, b) \in R^{\mathcal{I}} \implies b \in C^{\mathcal{I}})\}$	(universal restriction)
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TBOX	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$	(inclusion)
	$C_1 \equiv C_2$	$C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$	(equivalence)

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ABOX (UNA = unique name assumption[1])

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$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	(concept assertion)
$R(a_1, a_2)$	$(a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in R^{\mathcal{I}}$	(role assertion)

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$\mathcal{ALC}$ – Example

Example

Consider an information system for genealogical data integrating multiple geneological databases. Let's have atomic concepts *Person*, *Man*, *GrandParent* and atomic role *hasChild*.

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- How does the previous axiom look like in FOPL ?

$$\forall x \, (GrandParent(x) \equiv (Person(x) \wedge \exists y \, (hasChild(x, y) \wedge \exists z \, (hasChild(y, z)))))$$

$\mathcal{ALC}$ Example – $\mathcal{T}$

Example

$$
\begin{aligned}
Woman &\equiv Person \sqcap Female \\
Man &\equiv Person \sqcap \neg Woman \\
Mother &\equiv Woman \sqcap \exists hasChild \cdot Person \\
Father &\equiv Man \sqcap \exists hasChild \cdot Person \\
Parent &\equiv Father \sqcup Mother \\
Grandmother &\equiv Mother \sqcap \exists hasChild \cdot Parent \\
MotherWithoutDaughter &\equiv Mother \sqcap \forall hasChild \cdot \neg Woman \\
Wife &\equiv Woman \sqcap \exists hasHusband \cdot Man
\end{aligned}
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  - $\Delta^{\mathcal{I}_1} = Man^{\mathcal{I}_1} = Person^{\mathcal{I}_1} = \{\text{John}, \text{Phillipe}, \text{Martin}\}$
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Interpretation – Example

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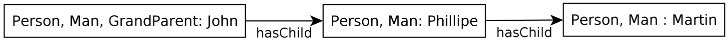
  - $JOHN^{\mathcal{I}_1} = \{\text{John}\}$
- this model is finite and has the form of a tree with the root in the node John :



Shape of DL Models

The last example revealed several important properties of DL models:

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Both properties represent important characteristics of $\mathcal{ALC}$ that significantly speed-up reasoning.

In particular (generalized) TMP is a characteristics that is shared by most DLs and significantly reduces their computational complexity.

Example – CWA × OWA

Example

ABOX

$hasChild(JOCASTA, OEDIPUS)$ $\quad$ $hasChild(JOCASTA, POLYNEIKES)$
$hasChild(OEDIPUS, POLYNEIKES)$ $\quad$ $hasChild(POLYNEIKES, THERSANDROS)$
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Edges represent role assertions of *hasChild*; red/green colors distinguish concepts instances – *Patricide* a $\neg$*Patricide*

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$$JOCASTA \longrightarrow \bullet \longrightarrow \bullet$$

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Q2 Find individuals x such that $\mathcal{K} \models C(x)$, where C is

$$\neg Patricide \sqcap \exists hasChild^{-} \cdot (Patricide \sqcap \exists hasChild^{-} \cdot \{JOCASTA\})$$

What is the difference, when considering CWA ?

$$JOCASTA \longrightarrow \bullet \longrightarrow x$$

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- S is consistent, if S has at least one model

From $\mathcal{ALC}$ to OWL(2)-DL

Extending . . . $\mathcal{ALC}$ ...

- We have introduced $\mathcal{ALC}$. Its expressiveness is higher than the expressiveness of the propositional calculus, still it lacks many constructs needed for practical applications.

Extending . . . $\mathcal{ALC}$ ...

- We have introduced $\mathcal{ALC}$. Its expressiveness is higher than the expressiveness of the propositional calculus, still it lacks many constructs needed for practical applications.
- Let's take a look, how to extend $\mathcal{ALC}$ while preserving decidability.

Extending . . . $\mathcal{ALC}$ ... (2)

$\mathcal{N}$ (Number restructions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq\ n\,R)$	$\left\{ a \middle\vert \quad \left\vert \{ b \mid (a,b) \in R^{\mathcal{I}} \} \right\vert \geq n \right\}$
$(\leq\ n\,R)$	$\left\{ a \middle\vert \quad \left\vert \{ b \mid (a,b) \in R^{\mathcal{I}} \} \right\vert \leq n \right\}$
$(=\ n\,R)$	$\left\{ a \middle\vert \quad \left\vert \{ b \mid (a,b) \in R^{\mathcal{I}} \} \right\vert = n \right\}$

Example

- Concept $Woman \sqcap (\leq\ 3\ hasChild)$ denotes women who have at most 3 children.

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- What denotes the axiom $Car \sqsubseteq (\geq\ 4\ hasWheel)$?
- ... and $Bicycle \equiv (=\ 2\ hasWheel)$?

Extending . . . $\mathcal{ALC}$ ... (3)

$\mathcal{Q}$ (Qualified number restrictions) are used for restricting the number of successors *of the given type* in the given role for the given concept.

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Example

- Concept $Woman \sqcap (\geq\ 3\ hasChild\ Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq\ 4\ hasPart\ Wheel)$?
- Which qualified number restrictions can be expressed in $\mathcal{ALC}$?

Extending ... $\mathcal{ALC}$ ... (4)

$\mathcal{O}$ (Nominals) can be used for naming a concept elements explicitely.

syntax (concept)	semantics
$\{a_1, \ldots, a_n\}$	$\{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$

Example

- Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?

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- $Continent \equiv \{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$?

Extending . . . $\mathcal{ALC}$ ... (5)

$\mathcal{I}$ (Inverse roles) are used for defining role inversion.

syntax (role)	semantics
R^-	$(R^{\mathcal{I}})^{-1}$

Example

- Role *hasChild*$^-$ denotes the relationship *hasParent*.

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- What denotes axiom $Person \sqsubseteq (= 2\ hasChild^-)$?
- What denotes axiom $Person \sqsubseteq \exists hasChild^- \cdot \exists hasChild \cdot \top$?

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$\cdot^{trans}$ (Role transitivity axiom) denotes that a role is transitive. Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

Example

- Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, $hasPart^{-}$, $hasGrandFather^{-}$?

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- What is a transitive closure of a relationship ? What is the difference between a transitive closure of $hasDirectBoss^{\mathcal{I}}$ and $hasBoss^{\mathcal{I}}$.

Extending ... $\mathcal{ALC}$ ...(7)

$\mathcal{H}$ (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

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Example

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- What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

Extending . . . $\mathcal{ALC}$ ... (8)

$\mathcal{R}$ (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R, S)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
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- Whom does the following concept denote $Person \sqcap \exists likes \cdot Self$?

Global restrictions

- *Simple roles* have no (direct or indirect) subroles that are either *transitive* or are defined by means of property chains

$$
\begin{aligned}
hasFather \circ hasBrother &\sqsubseteq hasUncle \\
hasUncle &\sqsubseteq hasRelative \\
hasBiologicalFather &\sqsubseteq hasFather
\end{aligned}
$$

  hasRelative and *hasUncle* are not simple.
- Each concept construct and each axiom from this list contains only *simple roles*:
  - number restrictions – $(\geq\ n\ R)$, $(=\ n\ R)$, $(\leq\ n\ R)$ + their qualified versions
  - $\exists R \cdot Self$
  - functionality/inverse functionality (leads to number restrictions)
  - irreflexivity, asymmetry, and disjoint object properties.

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- From the previously introduced extensions, two prominent decidable supersets of $\mathcal{ALC}$ can be constructed:

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    data types – XSD datatypes are used

Rules and Description Logics

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DL-safe rules

DL-safe rules are decidable conjunctive rules where each variable **only binds individuals** (not domain elements themselves).

Other extensions

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Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

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$$\Box(Man \implies Person \wedge \Box_{hasFather} Man) \tag{2}$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

Data Types ($\mathcal{D}$) allow integrating a data domain (numbers, strings), e.g. $Person \sqcap \exists hasAge \cdot 23$ represents the concept describing "23-years old persons".

References I

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