

Logical reasoning and programming, lab session 3

(October 12, 2020)

- 3.1** Let φ be a formula in CNF such that it contains only Horn clauses; they contain at most one positive literal. Show that SAT for φ is decidable in polynomial time.

Hint: Perform all the unit propagations first.

- 3.2** In fact, it is possible to improve the previous algorithm in such a way that it works in linear time.

- 3.3** Decide the satisfiability of

$$\{\{\bar{p}, r, s, t\}, \{\bar{r}, s, t\}, \{\bar{p}, r, \bar{s}\}, \{p, q\}, \{p, \bar{q}\}, \{\bar{p}, \bar{t}\}, \{\bar{r}, \bar{s}, t\}\}$$

by DPLL. Use the order of branching: p, q, r, \dots .

- 3.4** Decide the satisfiability of

$$\begin{aligned} &\{\{p_1, p_{13}\}, \{\bar{p}_1, \bar{p}_2, p_{14}\}, \{p_3, p_{15}\}, \{p_4, p_{16}\}, \\ &\quad \{\bar{p}_5, \bar{p}_3, p_6\}, \{\bar{p}_5, \bar{p}_7\}, \{\bar{p}_6, p_7, p_8\}, \{\bar{p}_4, \bar{p}_8, \bar{p}_9\}, \{\bar{p}_1, p_9, \bar{p}_{10}\}, \\ &\quad \{p_9, p_{11}, \bar{p}_{14}\}, \{p_{10}, \bar{p}_{11}, p_{12}\}, \{\bar{p}_2, \bar{p}_{11}, \bar{p}_{12}\}\} \end{aligned}$$

by CDCL using the first UIP. Use the order of branching: p_1, p_2, p_3, \dots .

- 3.5** Given the formula $\{\{p_1, p_2\}, \{\bar{p}_1, p_3\}, \{p_2, \bar{p}_3\}, \{\bar{p}_2, \bar{p}_4\}, \{\bar{p}_3, p_4\}\}$, what clause will a CDCL solver learn first if it begins by deciding that p_1 is true?

- 3.6** Assume that our version of CDCL fails to produce a satisfiable valuation for φ and hence φ is unsatisfiable. How would you produce a resolution proof of this fact from the run of CDCL?

- 3.7** Check this video, where zChaff colors the McGregor graph.

- 3.8** How many symmetries does your formulation of PHP_n^{n+1} have?

- 3.9** We can define the lexicographic order on two bit vectors x_1, \dots, x_n and y_1, \dots, y_n , denoted $x_1 \dots x_n \leq_{lex} y_1 \dots y_n$, as follows

$$\bigwedge_{i=1}^n ((\bar{x}_i \vee y_i \vee \bar{a}_{i-1}) \wedge (\bar{x}_i \vee a_i \vee \bar{a}_{i-1}) \wedge (y_i \vee a_i \vee \bar{a}_{i-1}))$$

using new auxiliary variables $a_0, a_1, \dots, a_{n-1}, a_n$.

- (a) What is the purpose of auxiliary variables?

Hint: When is it necessary to satisfy $x_i \leq y_i$?

- (b) Why is \bar{a}_0 always false and hence useless?

- (c) Why can we replace $(\bar{x}_n \vee y_n \vee \bar{a}_{n-1}) \wedge (\bar{x}_n \vee a_n \vee \bar{a}_{n-1}) \wedge (y_n \vee a_n \vee \bar{a}_{n-1})$ just by $(\bar{x}_n \vee y_n \vee \bar{a}_{n-1})$? Hence we need only $3n - 2$ clauses and $n - 1$ auxiliary variables (a_n is also useless).

(d) How does the meaning of the formula change if you replace $(\overline{x_n} \vee y_n \vee \overline{a_{n-1}})$ by $(\overline{x_n} \vee \overline{a_{n-1}}) \wedge (y_n \vee \overline{a_{n-1}})$?

3.10 How can we exploit the lexicographic order to decrease the number of symmetries in PHP_n^{n+1} ?

Hint: Order hole-occupancy or pigeon-occupancy vectors.

3.11 A very nice symmetry breaker for PHP_n^{n+1} is based on columnwise symmetry, namely we can add the following clauses

$$p_{i(i+1)} \vee \overline{p_{ij}}$$

for $1 \leq i < j \leq n$, where p_{kl} means that pigeon k is in hole l , for $1 \leq k \leq n+1$ and $1 \leq l \leq n$. Why?

3.12 Try PicoSAT/pycosat and PySAT on PHP_n^{n+1} with various symmetry breakers.

3.13 Define constraints *at least one* and *at most one* in CNF and discuss various variants of them.