

Logical reasoning and programming, lab session 1

(September 21, 2020)

I.1 Are the following two C programs equivalent?

```
if (!a && !b) h();           if (a) f();
else if (!a) g();          else if (b) g();
else f();                  else h();
```

Prove their equivalence formally, or provide a counterexample. Please, check your solution against these slides.

I.2 Decide which of the following formulae are tautologies:

- (a) $((p \rightarrow q) \rightarrow q) \rightarrow q$,
- (b) $((p \rightarrow q) \rightarrow p) \rightarrow p$,
- (c) $(p \rightarrow q) \vee (q \rightarrow p)$,
- (d) $((p \rightarrow q) \wedge q) \rightarrow p$,
- (e) $\neg p \rightarrow \neg(p \vee (p \wedge q))$.

Although formulae contain only two variables, try to find a better method than truth tables if possible.

I.3 Decide which of the following claims is true:

- (a) $\neg(\varphi \leftrightarrow \psi) \models \varphi \wedge \psi$,
- (b) $\neg(\varphi \leftrightarrow \psi) \models \varphi \vee \psi$,
- (c) $\neg(\varphi \leftrightarrow \psi) \models \varphi \rightarrow \psi$.

I.4 If $\varphi \rightarrow \psi \in \text{TAUT}$ and $\psi \rightarrow \chi \in \text{TAUT}$, then $\varphi \rightarrow \chi \in \text{TAUT}$. Why? Does the claim still hold if we replace TAUT with SAT and why?

I.5 Let $\text{Cl}(\Gamma) = \{\varphi : \Gamma \models \varphi\}$. What is $\text{Cl}(\emptyset)$ equivalent to? Let Γ and Δ be sets of formulae. Check whether:

- (a) $\Gamma \subseteq \text{Cl}(\Gamma)$,
- (b) $\text{Cl}(\text{Cl}(\Gamma)) = \text{Cl}(\Gamma)$,
- (c) $\text{Cl}(\Gamma \cup \Delta) = \text{Cl}(\Gamma) \cup \text{Cl}(\Delta)$.

If the equality does not hold in (b) or (c), does at least one of the inclusions hold?

I.6 Recall that a Boolean function of n -variables is a function $f: \{0, 1\}^n \rightarrow \{0, 1\}$. Describe all functions of one variable. How many distinct Boolean functions of n variables exist? Try $n = 2$ first. Do you know, why are functions NAND (\uparrow) and NOR (\downarrow) interesting?¹

¹These connectives are also called Sheffer stroke and Peirce arrow, respectively. They are defined as $x \uparrow y := \neg(x \wedge y)$ and $x \downarrow y := \neg(x \vee y)$.