Auctions

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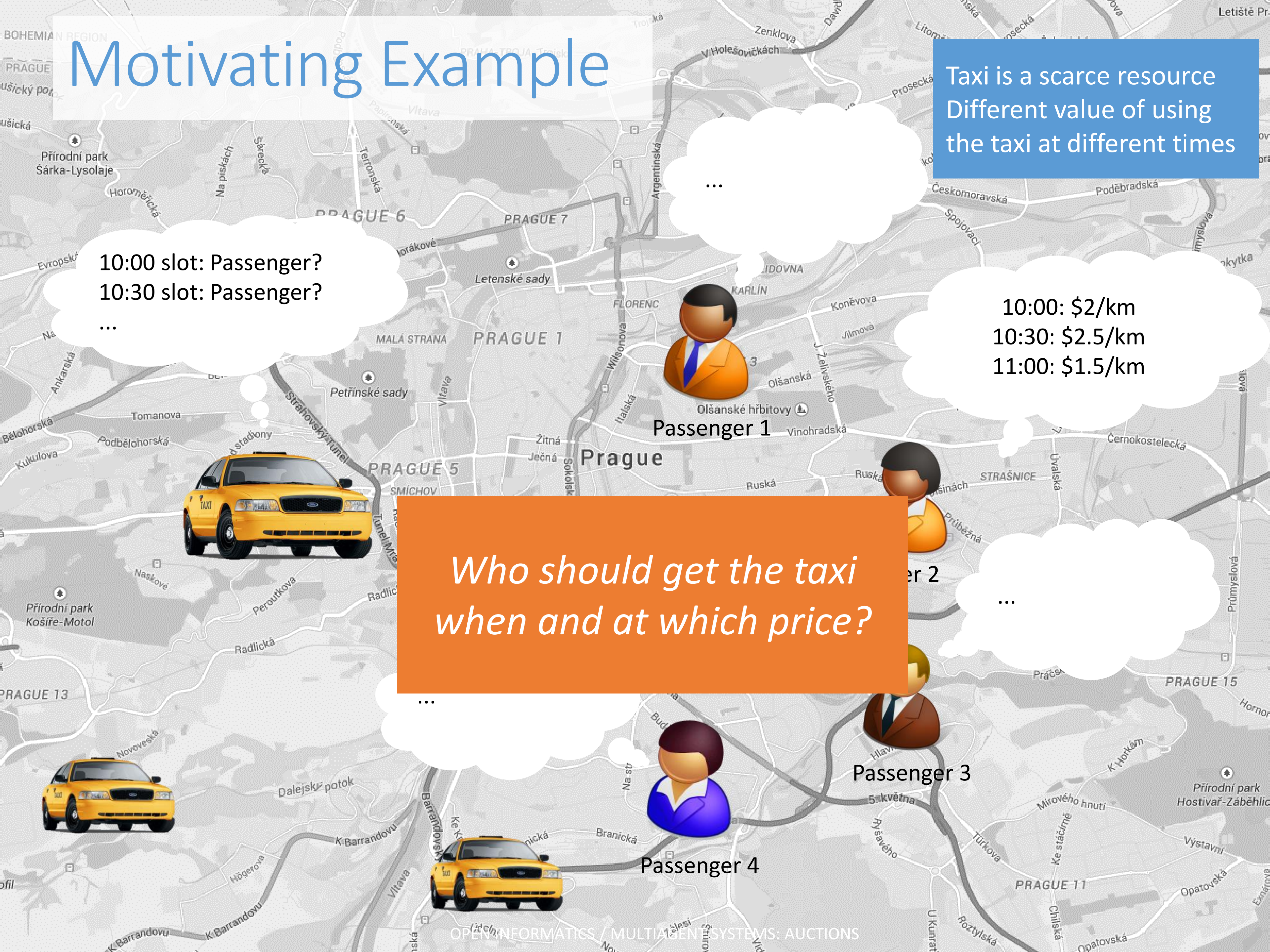
Motivating Example

Taxi is a scarce resource
Different value of using the taxi at different times



10:00 slot: Passenger?
10:30 slot: Passenger?
…

…

10:00: $2/km
10:30: $2.5/km
11:00: $1.5/km

…

…

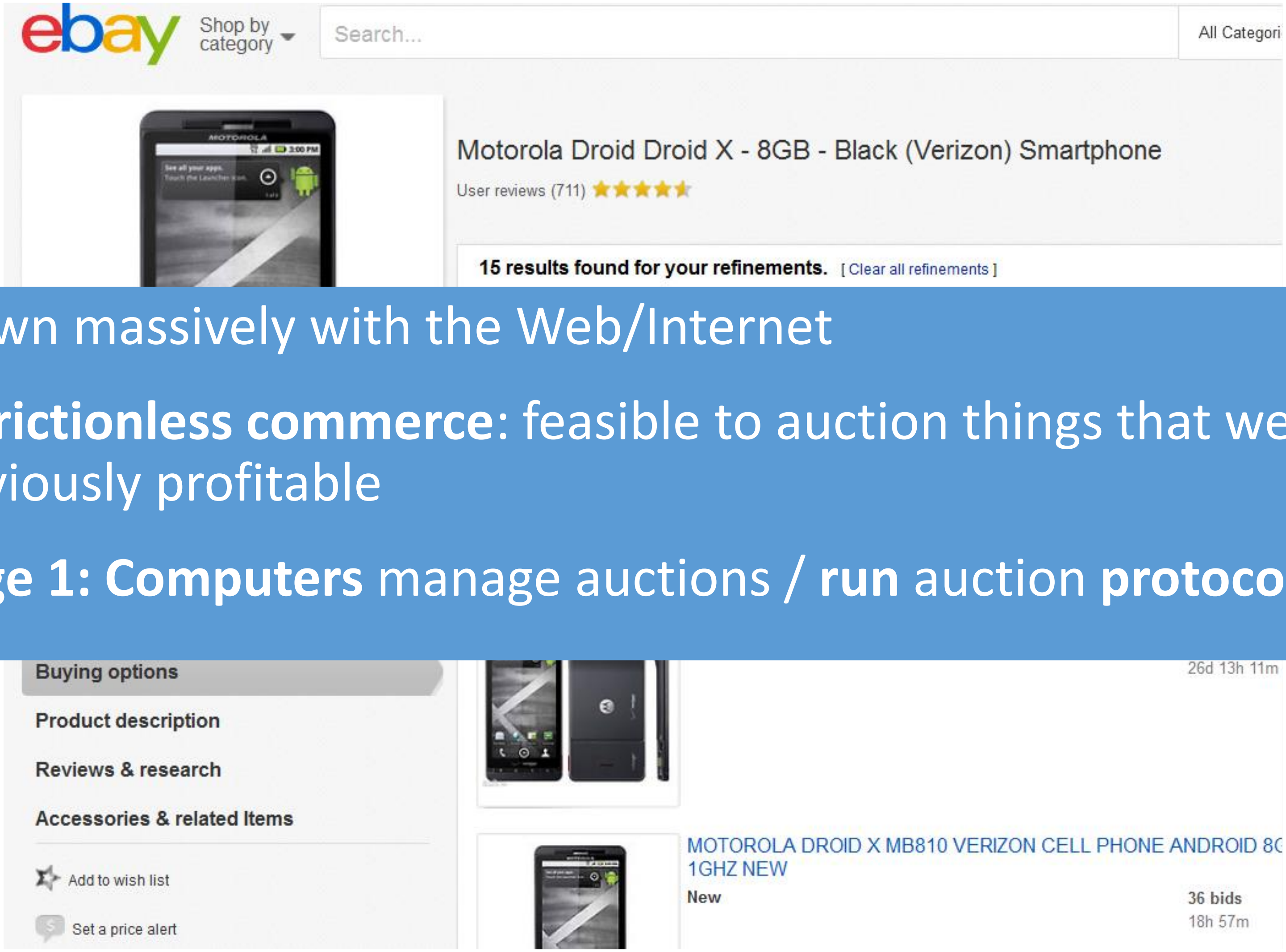
Who should get the taxi when and at which price?

Auctions: Traditional

Auctions used in Babylon as early as 500 B.C. but until relatively recently used only for high-value items for which it was difficult to asses the market price

Stage 0: No automation

Auctions: Partial Automation



Motorola Droid Droid X - 8GB - Black (Verizon) Smartphone

User reviews (711)

15 results found for your refinements. [Clear all refinements]

26d 13h 11m

MOTOROLA DROID X MB810 VERIZON CELL PHONE ANDROID 8
1GHZ NEW

New

36 bids

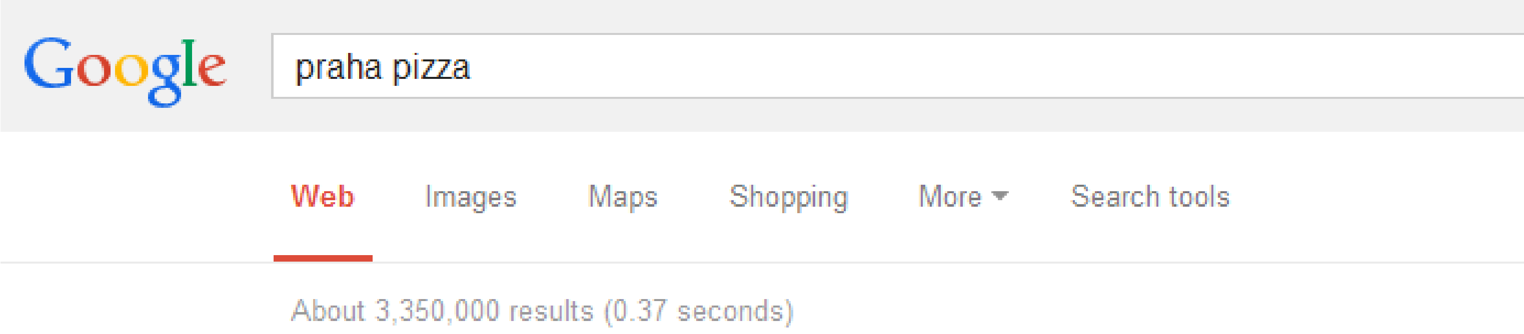
18h 57m

Grown massively with the Web/Internet

→ **Frictionless commerce**: feasible to auction things that weren't previously profitable

Stage 1: Computers manage auctions / **run** auction **protocols**

Auctions: (Almost) Full automation



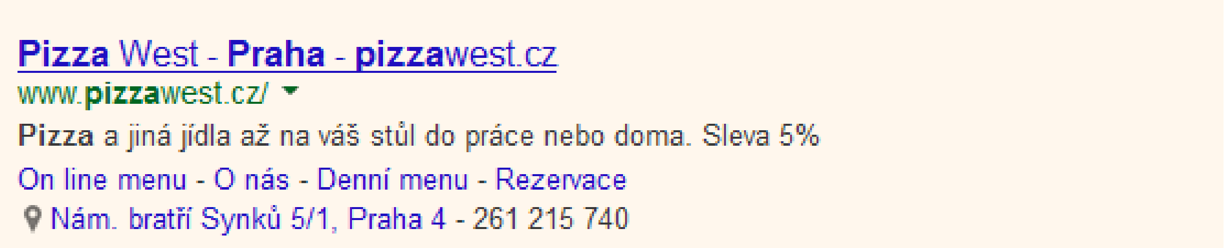
praha pizza

About 3,350,000 results (0.37 seconds)

Stage 2: Computers also automate the **decision making of bidders**

Concerns:

1) the most **relevant adds** are shown (→ user's are reasonably happy)
2) auctioner's **profit is maximized** (over longer time)



Pizza West - **Praha** - **pizza**west.cz
www.**pizza**west.cz/
Pizza a jiná jídla až na váš stůl do práce nebo doma. Sleva 5%
On line menu - O nás - Denní menu - Rezervace
Nám. bratří Synků 5/1, Praha 4 - 261 215 740

Lots of Applications

Industrial procurement

Transport and logistics

Energy markets

Cloud and grid computing

Internet auctions

(Electromagnetic spectrum allocation)

... and counting!

Introduction to Auctions

English Auction

1. Auctioneer starts the bidding at some **reservation price**
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

What is an Auction?

*An **auction** is a protocol that allows **agents** (=bidders) to indicate their **interests** in one or more **resources** and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest .

Auctions are mechanisms **with money**.

Auctions can be viewed as **games** of a specific structure.

Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be **automated**

- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and **efficient allocations** are achievable

Auctions Rules

Auctions are **structured negotiations** governed by **auction rules** **(→** *rules of the game)*

Bidding rules

How **offers (bids)** are made:

- by whom
- when
- what their content is

Clearing rules

Who gets which goods (**allocation**) and what money changes hands (**payment**).

Information rules

What information about the state of the negotiation is *revealed* **to whom** and **when**.

Payoff

Agent's payoff
from participating in an auction

If winner

payoff* =
valuation of the item −
price paid for the item

* Also termed **surplus**

If *not* winner

payoff=**zero**

Individual rationality: the agent never bids higher than its valuation

Risk Attitudes

Risk neutrality: the payoff is (as above) a *linear function* of the difference between the item's valuation and the price paid

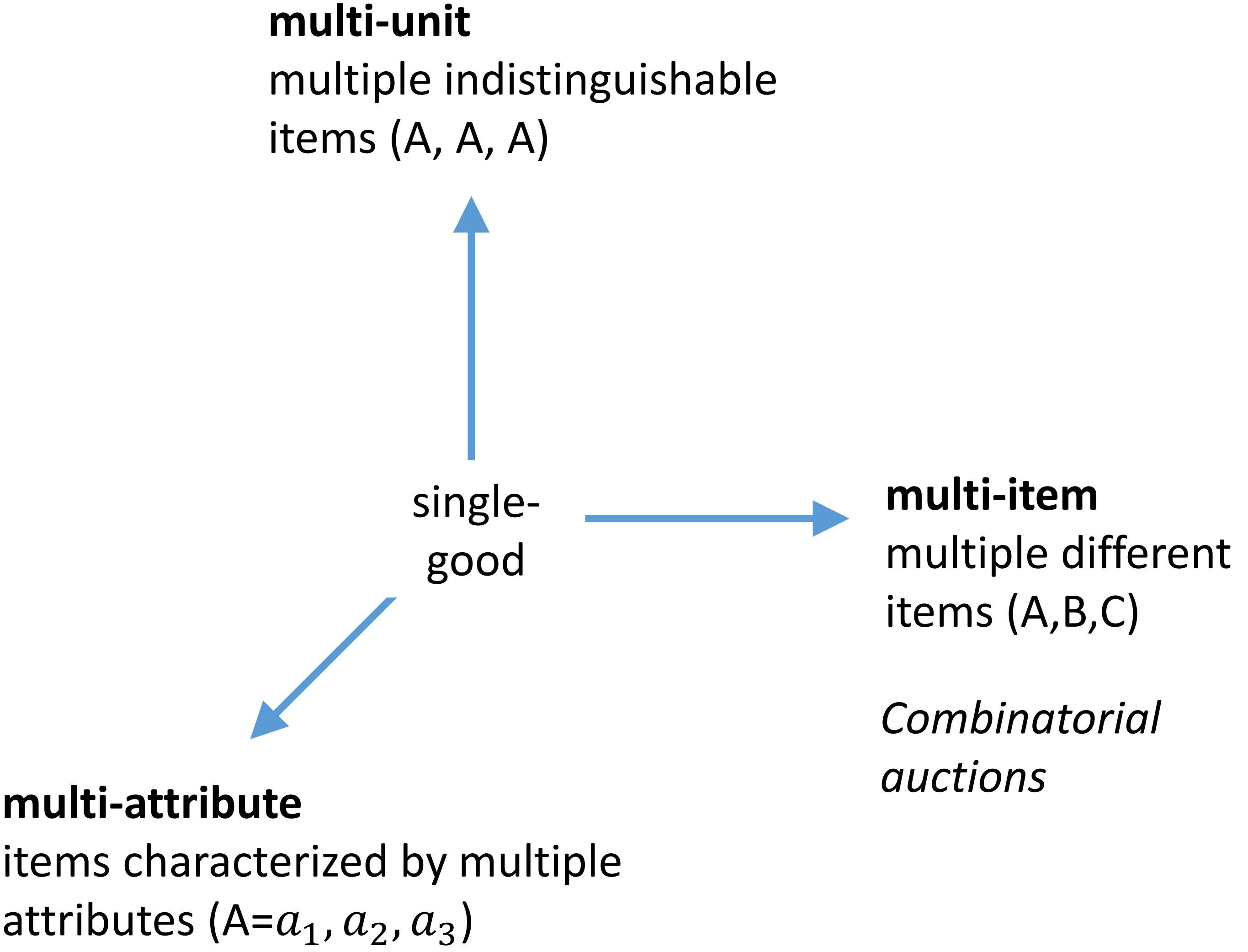
Risk seeking: the payoff is a convex function of the difference (aggressively seeking high gains is prioritized)

Risk aversion: the payoff is a concave function of the difference (conservatively ensuring at least some gains is prioritized)

Valuation Models

Independent private value (IPV)	Correlated value
An agent A's valuation of the good is **independent from other agent's** valuation of the good (e.g. a taxi ride to the airport).	Valuations of the good are **related between agents** (typically the more other agents are prepared to pay, the more the agent A prepared to pay – e.g. purchase of items for later resale).

Types of Auctions



multi-unit
multiple indistinguishable items (A, A, A)

single-good

multi-item
multiple different items (A,B,C)

Combinatorial auctions

multi-attribute
items characterized by multiple attributes (A=a_1, a_2, a_3)

Types of Auctions

Forward (sell-side) auction: selling

Reverse (buy-side) auction: buying

Single-sided: either selling or buying

Double-sided: both selling and buying (→ exchange)

There are other allocation mechanisms: facility location, allocation of divisible goods (cake cutting), allocation of indivisible goods (CPU, memory), …

Single-Item Auctions

Basic Auction Mechanisms

English

Japanese

Dutch

First-Price

Second-Price

English Auction

1. Auctioneer starts the bidding at some **reservation price**
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

1. All bidders start out **standing**
2. When the price reaches a level that a bidder is not willing to pay, that bidder **sits down;** once a bidder sits down, they **can't get back up.**
3. The **last** person **standing** gets the good

Dutch Auction

1. The auctioneer starts a clock at some high value; it descends
2. At some point, a bidder shouts “mine!" and gets the good at the price shown on the clock

Good when items need to be sold **quickly** (similar to Japanese)

No information is revealed during auction

First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount of **his bid**

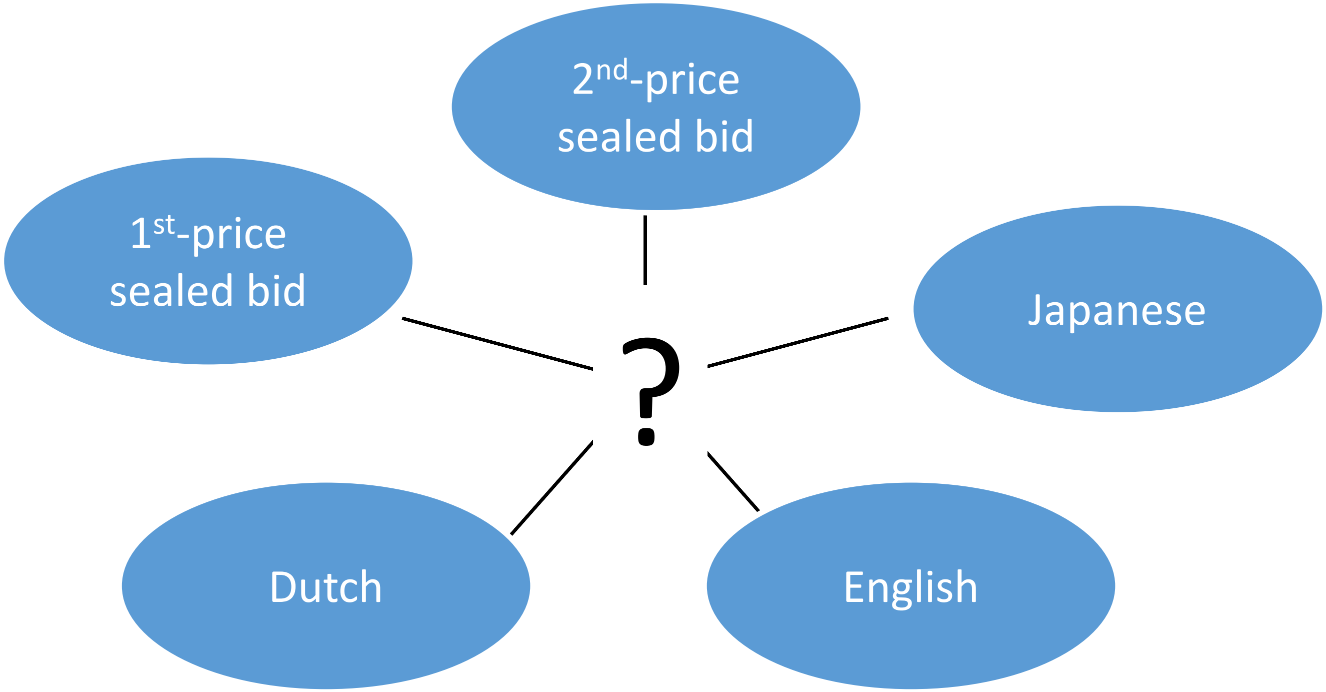
Second-price sealed bid auction (**Vickerey** auction)

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount bid by **the second-highest** bidder

Intuitive Comparison

	English	Dutch	Japanese	1st-Price	2nd-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2nd-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

Analysing Auctions



2nd-price sealed bid

1st-price sealed bid

Japanese

?

Dutch

English

Are there fundamental similarities / differences between mechanisms described?

Two Problems

Analysis of auction mechanisms

- determine the properties of a given auction mechanism
- methodology: treat auctions as (extended-form) *Bayesian games* and analyse players’ (i.e. bidders’) strategies

Design of auction mechanisms

- design the auction mechanism (i.e. the game for the bidders) with the desirable properties
- methodology: apply mechanism design techniques

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \dots, u_n)$, where $u_i: \mathrm{A} \times \Theta \mapsto \mathbb{R}$ is the **utility function** of player i

We assume that all of the above is **common knowledge** among the players, and that each **agent knows** his **own type**.

Bayes-Nash equilibrium: rational, risk-neutral players are seeking to maximize their **expected payoff**, given their **beliefs** about the other players' types.

Relation to (sealed bid) Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's **actions** correspond to his **bids** $\widehat{v_i}$
- player types Θ_i correspond to players' **private valuations** v_i over the auctioned item(s)
- the **payoff** of player i corresponds to: ***i*'s valuation of the item $\boldsymbol{v_i}$ – price paid** (if winner); zero otherwise.

Similar analogies for more complicated auction mechanisms

(Desirable) Properties (semi-formally)

Truthfulness: bidders are incentivized to bid their *true* valuations, i.e.

$$v_i = \widehat{v_i} \; \forall i \forall v_i$$

Efficiency: the aggregated value of bidders is maximized, i.e.

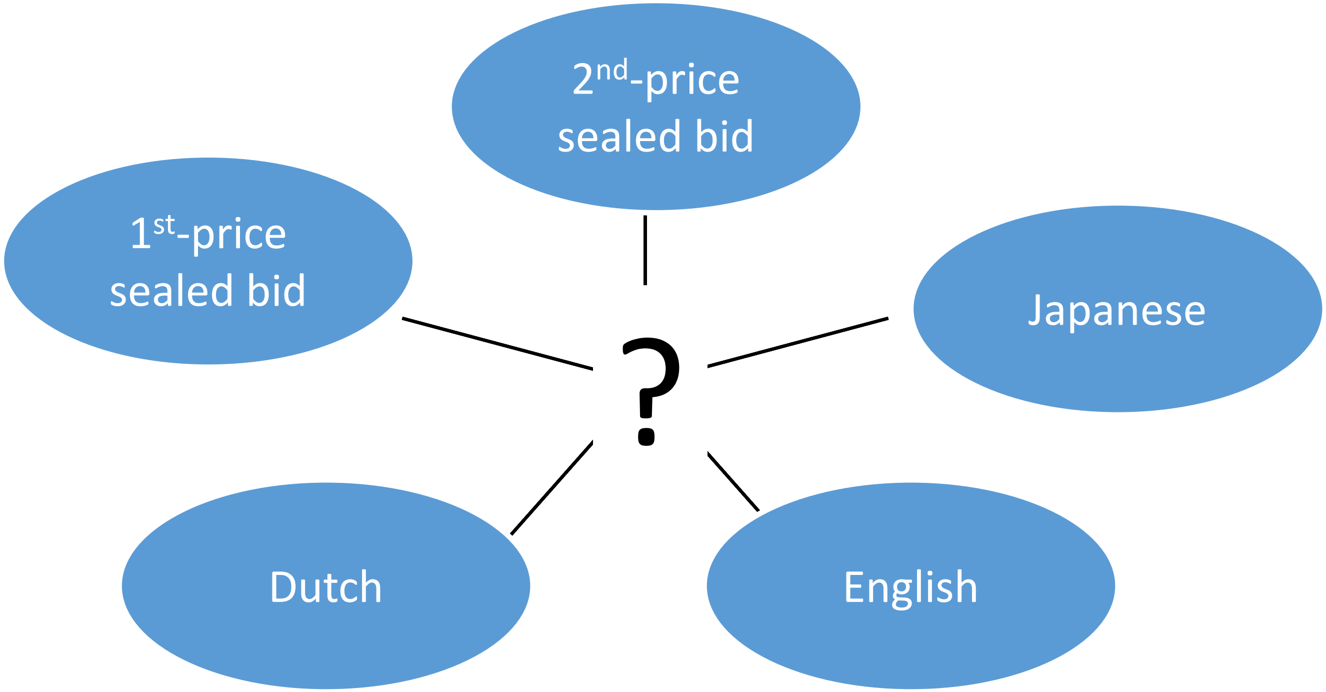
$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x')$$

Optimality: maximization of seller's revenue

Strategy: existence of dominant strategy

Manipulation vulnerability: lying auctioner, shills, bidder collusion

Other consideration: communication complexity, private information revelation, ...



2nd-price sealed bid

1st-price sealed bid

Japanese

?

Dutch

English

Are there fundamental similarities / differences between mechanisms described?

Bidding in Second-Price Sealed Bid Auction

Second-Price Sealed Bid

How should agents bid in the second-price sealed bid auctions?

Theorem

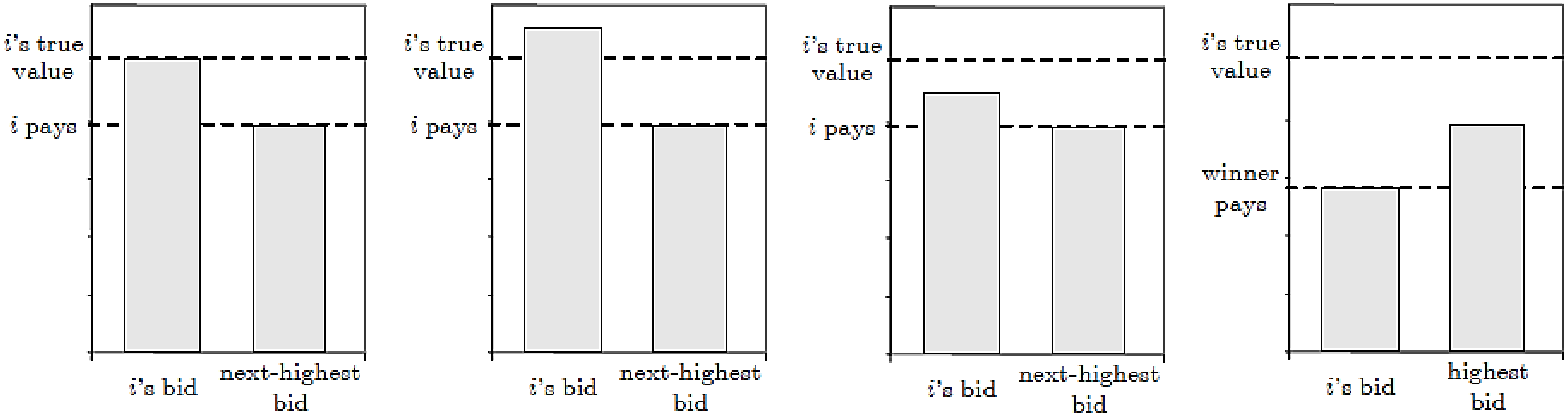
Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values – IPV).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof

Bidding honestly, i is the winner



If i bids higher, he will still win and still pay the same amount

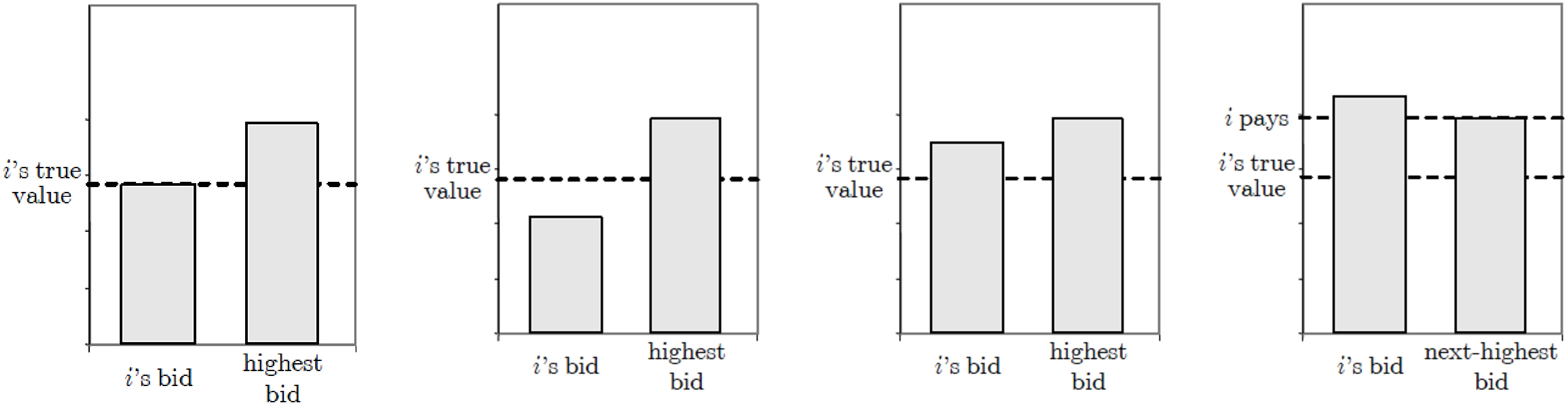
If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

➔ There is a disadvantage bidding lower and no advantage bidding higher

Second-Price Sealed Bid Proof

Bidding honestly, i is not the winner



If i bids lower, he will still lose and still pay nothing

If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation (⇒ negative payoff).

➔ There is a disadvantage bidding higher and no advantage bidding lower

Second-Price Sealed Bid

Advantages:

- **Truthful** bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing
- Reveals true valuations
- Not revenue maximizing

Bidding in First-Price Sealed Bid Auctions

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents’ bids (i.e. difference are technical implementation)

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held **asynchronously**
- Dutch auctions are **fast**, and require **minimal communication**
  - only **one bit** needs to be transferred from the bidders to the auctioner

Bidding in Dutch / First Price Sealed Bid

How should bidders bid in these auctions?

There's a **trade-off** between **probability of winning** vs. **amount paid** upon winning (and thus the winner’s surplus)

➔ Bidders don't have a **dominant strategy** any more:

Individually optimal strategy depends on the **assumptions** about **others’ valuations.**

Equilibrium Strategy

Assume a **first-price auction** with **two risk-neutral bidders** whose valuations are drawn independently and **uniformly** at random from the interval [0, 1] - what is the equilibrium strategy?

→ $\left(\frac{1}{2}v_1, \frac{1}{2}v_2\right)$ is the Bayes-Nash equilibrium strategy profile

Proof

Assume that bidder 2 bids $\frac{1}{2}v_2$, and that bidder 1 bids s_1.

Bidder 1 wins when $\frac{1}{2}v_2 < s_1$, gaining payoff $v_1 - s_1$, but loses when $v_2 > 2s_1$ and then gets utility 0 (we can ignore the case where the agents have the same valuation, because this occurs with probability zero).

$$\mathbb{E}[u_1] = \int_0^{2s_1}(v_1 - s_1)dv_2 + \int_{2s_1}^{1}(0)dv_2 =$$
$$= (v_1 - s_1)v_2\Big|_0^{2s_1} =$$
$$= 2v_1s_1 - 2s_1^2$$

Proof Continued

We can determine bidder 1’s best response to bidder 2’ strategy by taking the derivative and setting it to zero:

$$\frac{\partial}{\partial s_1}(2v_1 s_1 - 2s_1^2) = 0$$

$$2v_1 - 4s_1 = 0$$

$$s_1 = \frac{1}{2}v_1$$

Thus when player 2 is bidding half her valuation, player 1’s best reply is to **bid half his valuation** (and analogously for playr 2, given the symmetry of the game)

Bidding in Dutch / First Price Sealed Bid

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \ldots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $(\frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n)$.

For non-uniform valuation distributions: Each bidder should bid **the expectation of the second-highest valuation**, conditioned on the assumption that his own valuation is the highest.

⇒ Dutch / FPSB auctions **not incentive compatible,** i.e., there are incentives to **counter-speculate**.

Equilibrium in more general cases?

Note we only **verified** the equilibrium.

What about more general assumptions?

→ We need to guess the equilibrium and it gets more complicated as we relax the assumptions about the distributions of valuations (non-uniformity, no symmetry etc.).

Even determining a Nash equilibrium exists gets difficult

This because auctions are **non-continuous games**: even a small variation in the bid amount can lead to not-winning and thus large changes in the payoff.

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** does not make any **difference** in the **independent-private value** (IPV) setting.

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate

Revenue Equivalence

Which auction should an auctioneer choose?

To some extent, it doesn't matter...

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the **same expected payment**.

Revenue Equivalence

Informally: As long as two mechanism **allocate in the same way** and they **do not charge anything** to the agent with the lowest valuation, the **rest of payment functions** is the same.

You cannot get **extra money** from bidder without changing the allocation function or the payment to the lowest-valued bidder.

In fact, the revenue equivalence holds beyond IPV and single good.

Assuming bidders are risk neutral and have independent private valuations, **all the auctions** we have spoken about so far—English, Japanese, Dutch, and all sealed bid auction protocols—are **revenue equivalent**.

What about Efficiency?

Efficiency in single-item auctions: the item allocated to the agent who values it the most.

With independent private values (IPV):

Auction	Efficient
English (without reserve price)	yes
Japanese	yes
Dutch	no
Sealed bid second price	yes
Sealed bid first price	no

Efficiency (often) lost in the correlated value setting.

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Many auction mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

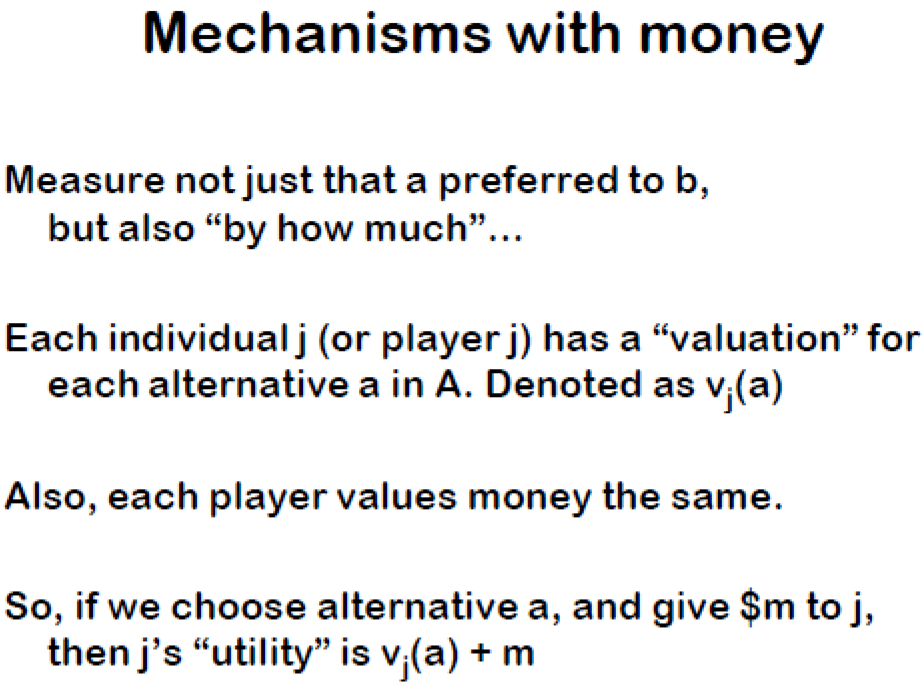
Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:

- [Shoham] – Chapter 11

How to get around impossibility results



Mechanisms with money

Measure not just that a preferred to b,
but also "by how much"…

Each individual j (or player j) has a "valuation" for each alternative a in A. Denoted as $v_j(a)$

Also, each player values money the same.

So, if we choose alternative a, and give \$m to j, then j's "utility" is $v_j(a) + m$

Auction Protocols

Auctions are centralised mechanisms for the allocation of goods amongst several agents. Agents report their preferences (bidding) and the auctioneer decides on the final allocation (and on prices).

- Distinguish *direct* and *reverse* auctions (auctioneer buying).
- Bidding may be *open-cry* (English) or by *sealed bids*.
- Open-cry: *ascending* (English) or *descending* bids (Dutch).
- Pricing rule: *first-price* or *second-price* (Vickrey).
- *Combinatorial auctions:* several goods, sold/bought in bundles.

R.P. McAfee and J. McMillan. Auctions and Bidding. *Journal of Economic Literature*, 25:699–738, 1987.

P. Cramton, Y. Shoham, and R. Steinberg (eds.). *Combinatorial Auctions*. MIT Press, 2006.