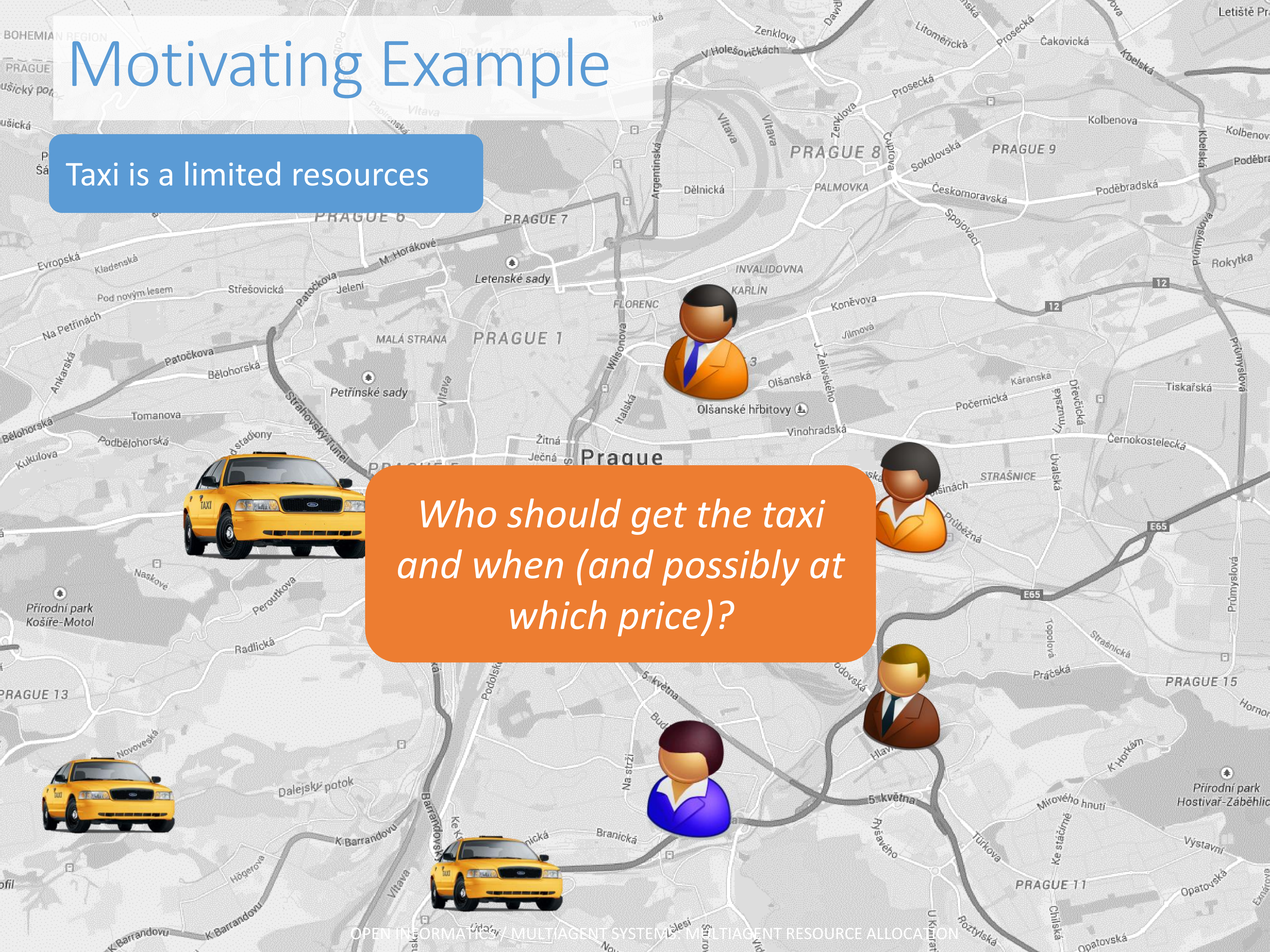
Multiagent Resource Allocation

Michal Jakob
Artificial Intelligence Center,
Dept. of Computer Science and Engineering,
FEE, Czech Technical University

AE4M36MAS Autumn 2020 - Lecture 8

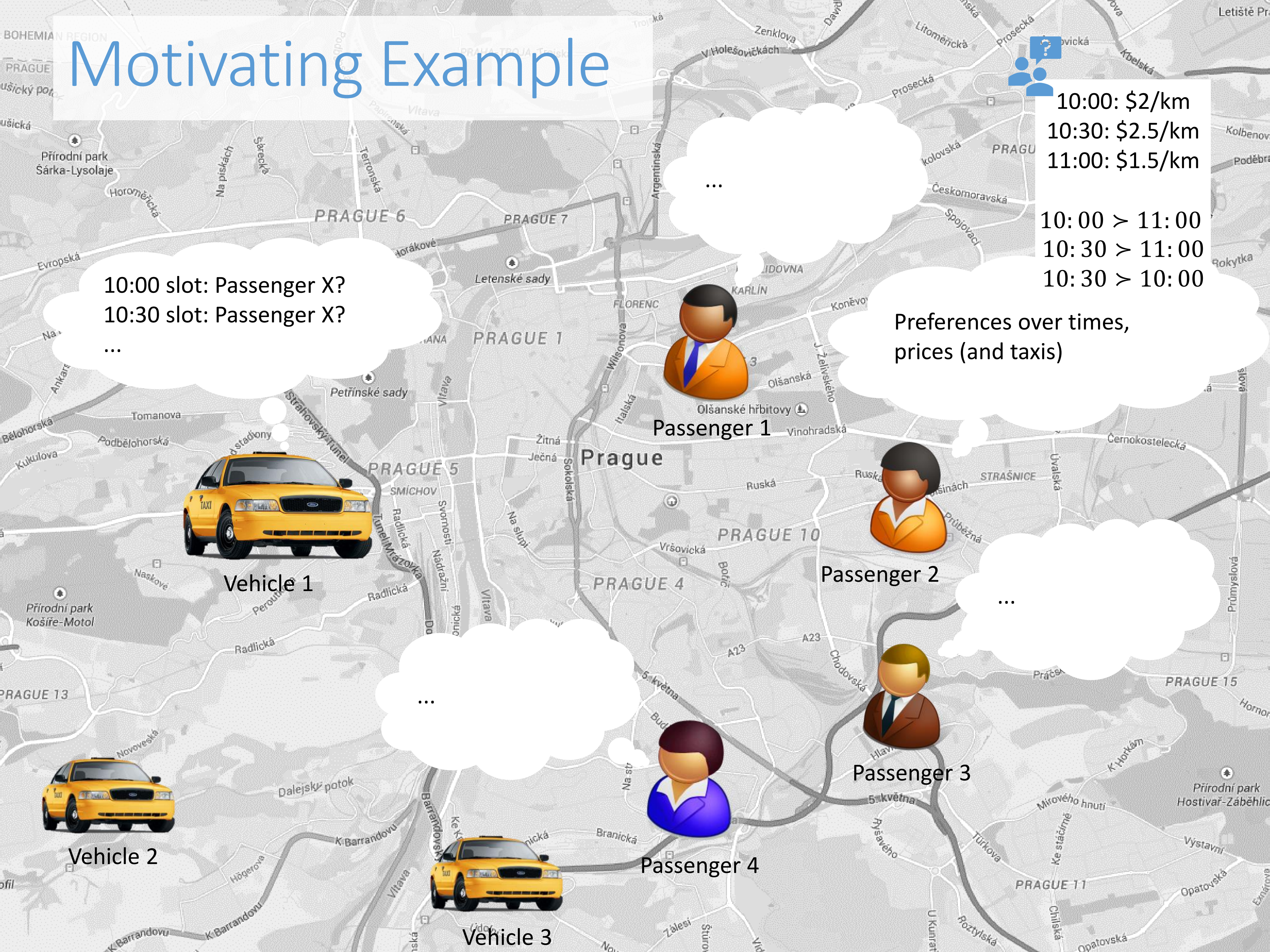
Motivating Example

Taxi is a limited resources



Who should get the taxi and when (and possibly at which price)?

Motivating Example



Multiagent Resource Allocation (MARA)

What is Multiagent Resource Allocation?

Multiagent Resource Allocation (MARA) is the process of distributing a number of items amongst a number of agents.

- **What** kind of items (resources) are being distributed?
- **How** are they being distributed?
- **Why** are they being distributed?

Classification of MARA

1. **Resources** (What)
2. Agent (i.e. individual) **preferences** (Why)
3. Social (i.e. collective) **welfare** (Why)
4. Allocation **mechanism** (How)

Link to **social choice**: allocations are the alternatives over which agents express their preferences.

Link to **game theory**: allocation mechanisms are games (that needs to be designed and for which strategies can be studied).

Types of Resources

Different **types** of resources may require different resource allocation **techniques**.

- **Continuous** vs. **Discrete**
  Continuous resource can be arbitrarily divided.
- **Divisible** vs. **Indivisible**
  Discrete resources indivisible; continuous can be treated either way.
- **Sharable** vs. **Non-Sharable**
  Sharable can be assigned multiple times: e.g. a path in a network.
- **Static** vs. **Non-Static**
  static = properties do not change; non-static = properties do change e.g. perishable goods.
- **Single-Unit** vs. **Multi-Unit**
  One copy vs. multiple copies (ten trucks of the same type).

Resources vs. Tasks

Tasks may be considered resources with **negative** utility (cost).

Task allocation may be regarded a multiagent resource allocation problem.

- However, tasks are often coupled with **constraints** regarding their **coherent combination** (timing and ordering).

Preference Representation

How can we represent agent's preferences over allocations?

Preference Representation

Agents may have **preferences** over

- the bundle of resources they receive
- the bundles of resources received by others (**externalities**)

What are suitable **languages** for representing agent **preferences**?

Notation

Set of **agents** $\mathcal{A} = \{1, \ldots, n\}$

Set of **resources** $\mathcal{R}$

Agents have **preferences over allocations** $X \in \mathcal{X}$

Allocation X is a *partial* mapping of $\mathcal{R}$ to $\mathcal{A}$
(not all resources need to be allocated)

Cardinal vs. Ordinal Preferences

A **preference structure** represents an agent's preferences over allocations $X \in \mathcal{X}$.

Cardinal preferences	Ordinal preferences
Cardinal preference structure is a **function** $u\colon \mathcal{X} \mapsto Val$, where Val is usually a set of numerical values such as $\mathbb{N}$ or $\mathbb{R}$ (and typically non-negative)	Ordinal preference structure is a *reflexive, transitive and connected* **binary *relation*** $\preccurlyeq$ on the set $\mathcal{X} \times \mathcal{X}$ which can used to *compare* allocations.

If the allocations over which agents must express preferences are *bundles of indivisible resources* from the set $\mathcal{R}$, then we have $\mathcal{X} = 2^{\mathcal{R}}$.

Example

Hanging a picture with a **frame** (f), a **hammer** (h) and a **nail** (n)

Cardinal

X	$u(X)$
$\{\,\}$	0
$\{f\}$	0
$\{h\}$	0
$\{n\}$	10
$\{f,h\}$	0
$\{f,n\}$	20
$\{h,n\}$	15
$\{f,h,n\}$	50

Ordinal

$\succcurlyeq$	$\{\,\}$	$\{f\}$	$\{h\}$	$\{n\}$	$\{f,h\}$	$\{f,n\}$	$\{h,n\}$	$\{f,h,n\}$
$\{\,\}$	1	0	0	0	0	0	0	0
$\{f\}$	1	1	1	0	1	0	0	0
$\{h\}$	1	1	1	0	1	0	0	0
$\{n\}$	1	1	1	1	1	0	0	0
$\{f,h\}$	1	1	1	0	1	0	0	0
$\{f,n\}$	1	1	1	1	1	1	1	0
$\{h,n\}$	1	1	1	1	1	0	1	0
$\{f,h,n\}$	1	1	1	1	1	1	1	1

Cardinal **can be** always translated to ordinal.
Ordinal **cannot be** always translated to cardinal
(→ ordinal preferences more expressive).

Preference Representation Languages

Expressive power
Can the chosen language encode all the preference structures we are interested in?

Succinctness
Is the representation of (typical) preference structures succinct? Is one language more succinct than the other?

Complexity
What is the computational complexity of related decision problems, such as comparing two alternatives?

Cognitive relevance
How close is a given language to the way in which humans would express their preferences?

Elicitation
How dicult is it to elicit the preferences of an agent so as to represent them in the chosen language?

Preferences Properties

	Cardinal	Ordinal
Intrapersonal comparison	yes	Yes
Interpersonal comparison ("Ann likes x more than Bob likes y")	yes	No
Preference intensity	yes	No
Cognitive relevance	lower	higher
Explicit representation	$\mathcal{O}(\lvert\mathcal{X}\rvert)$	$\mathcal{O}(\lvert\mathcal{X}\rvert^2)$

Representation can be an issue ➔ **compact** representations

Social Welfare

How can we express preferences from the collective perspective?

Social Welfare

A third parameter in the specification of a MARA problem concerns our goals: **What kind of allocation do we want to achieve?**

We use the term **social welfare** in a very broad sense to describe **metrics** for assessing the **quality** of an **allocation** of resources.

Efficiency and Fairness

Two key indicators of social welfare.

Aspects of **efficiency*** include:

- The chosen agreement should be such that there is no alternative agreement that would be better for some and not worse for any of the other agents (**Pareto optimality**).
- If preferences are quantitative, the sum of all payoffs should be as high as possible (**utilitarianism**).

Aspects of **fairness** include:

- No agent should prefer to take the bundle allocated to one of its peers rather than keeping their own (**envy-freeness**).
- The agent that is going to be worst off should be as well off as possible (**egalitarianism**).

*not in the computational sense

Efficiency and Fairness

Two key indicators of social welfare.

Efficiency*

The aspect of efficiency include:

- **Pareto optimality:** There should be no alternative allocation that would be better for some and not worse for any of the other agents that the chosen allocation.
- **Utilitarism** (for cardinal preferences)**:** the sum of all utilities should be as high as possible.

Fairness

The aspect of fairness include:

- **Envy-freeness:** No agent should prefer to take the bundle allocated to one of its peers rather than keeping their own.
- **Egalitarianism** (for cardinal preferences)**:** The agent that is going to be worst off should be as well off as possible.

*not in the computational sense

Utilitarian Social Welfare

Utilitarian Social Welfare

The **utilitarian** social welfare function (also called collective utility function) sw_u is defined as the sum of individual utilities:

$$sw_u(X) = \sum_{i \in \mathcal{A}} u_i(X)$$

Maximizing utilitarian CUF improves **efficiency**.

The utilitarian CUF is **zero-independent**: adding a constant value to your utility function will not affect social welfare judgements.

Egalitarian Social Welfare

Egalitarian Social Welfare

The **egalitarian** social welfare function sw_e is defined as the sum of individual utilities:

$$sw_e(X) = \min_{i \in \mathcal{A}} u_i(X)$$

Maximising this function amounts to improving the situation of the weakest members of society (→ **fairness**).

Nash Product Social Welfare

Nash Social Welfare

The **Nash** social welfare function sw_e is defined as the sum of individual utilities:

$$sw_n(X) = \prod_{i \in \mathcal{A}} u_i(X)$$

This is a useful measure of social welfare as long as all utility functions can be assumed to be **positive**.

Nash CUF favours increases in overall utility, but also inequality-reducing redistributions ($2 \cdot 6 < 4 \cdot 4$) ➔ **proportional fairness**.

The Nash CUF is **scale independent**: whether a particular agent measures their own utility in euros or dollars does not affect social welfare judgements.

Efficiency vs. Fairness Example

Allocation problem

- **Agents** $\mathcal{A} = \{Alice, Bob\}$
- **Items** $\mathcal{R} = \{phone, bike, shoes\}$
- **Utility functions:**

Resources	**Alice $u_a(r)$**	**Bob $u_b(r)$**
phone	20	6
bike	10	8
shoes	10	4

Additive utilities:

$$u_a(X) = \sum_{r, X(R)=Alice} u_a(r)$$

$$u_b(X) = \sum_{r, X(R)=Bob} u_b(r)$$

Allocations

Res.	**Allocations**		
	efficient X_e	**fair** X_f	**Propor.** X_n
phone	Alice	Bob	Alice
bike	Alice	Bob	Bob
shoes	Alice	Alice	Alice
sw_u	**40**	24	38
sw_f	0	**14**	8
sw_n	0	140	**240**

Efficiency vs. Fairness Trade-off

Efficient Allocation

We assume **cardinal preferences**.

Utilitarian welfare function is considered to **measure the efficiency** of an allocation.

An allocation is called **efficient** (also **utilitarian**) if it **maximizes** the sum of utilities of all agents (=**social value**).

We denote the social value of an efficient allocation as $\mathrm{EFFICIENT}(\mathcal{X})$, i.e.,

$$\mathrm{EFFICIENT}(\mathcal{X}) = \sup\{\sum_{i \in \mathcal{A}} u_i(X) \,|X \in \mathcal{X}\}$$

α-Fair Allocation

Constant Elasticity Social Welfare Function

Constant Elasticity Social Welfare Function sw_α with **inequality aversion parameter** α is defined as

$$sw(X,\alpha) = \begin{cases} \sum_{i \in \mathcal{A}} \frac{u_i(X)^{1-\alpha}}{1-\alpha} & \text{for } \alpha \geq 0, \alpha \neq 1 \\ \sum_{i \in \mathcal{A}} \log u_i(X) & \text{for } \alpha = 1 \end{cases}$$

$\alpha = 0$: Utilitarian SWF
$\alpha = 1$: Proportional fairness (~Nash SWF)
$\alpha \to \infty$: Egalitarian (Max-min) SWF

α-Fair Allocation

$\boldsymbol{\alpha}$-fair allocation $X^*(\alpha)$ is an allocation that maximizes the constant elasticity social welfare function for the corresponding value of α, i.e,

$$X^*(\alpha) = \operatorname*{argmax}_{X \in \mathcal{X}} sw(X, \alpha)$$

We denote the social value of the α-fair allocation as $\mathbf{FAIR}(\boldsymbol{\mathcal{X}}, \boldsymbol{\alpha})$, i.e.,

$$\mathrm{FAIR}(\mathcal{X}, \alpha) = sw_u(X^*(\alpha))$$

Price of Fairness

Quantifies the **loss of efficiency** due to the requirement for fairness.

Price of Fairness

$$\mathrm{POF}(\mathcal{X}, \alpha) = \frac{\mathrm{EFFICIENT}(\mathcal{X}) - \mathrm{FAIR}(\mathcal{X}, \alpha)}{\mathrm{EFFICIENT}(\mathcal{X})}$$

Price is fairness is always **between zero and one**, and it corresponds to the **percentage efficiency loss** compared to the maximum system efficiency.

Note: $\mathrm{POF}(\mathcal{X}, 0) = 0$

Price of Fairness

Theorem

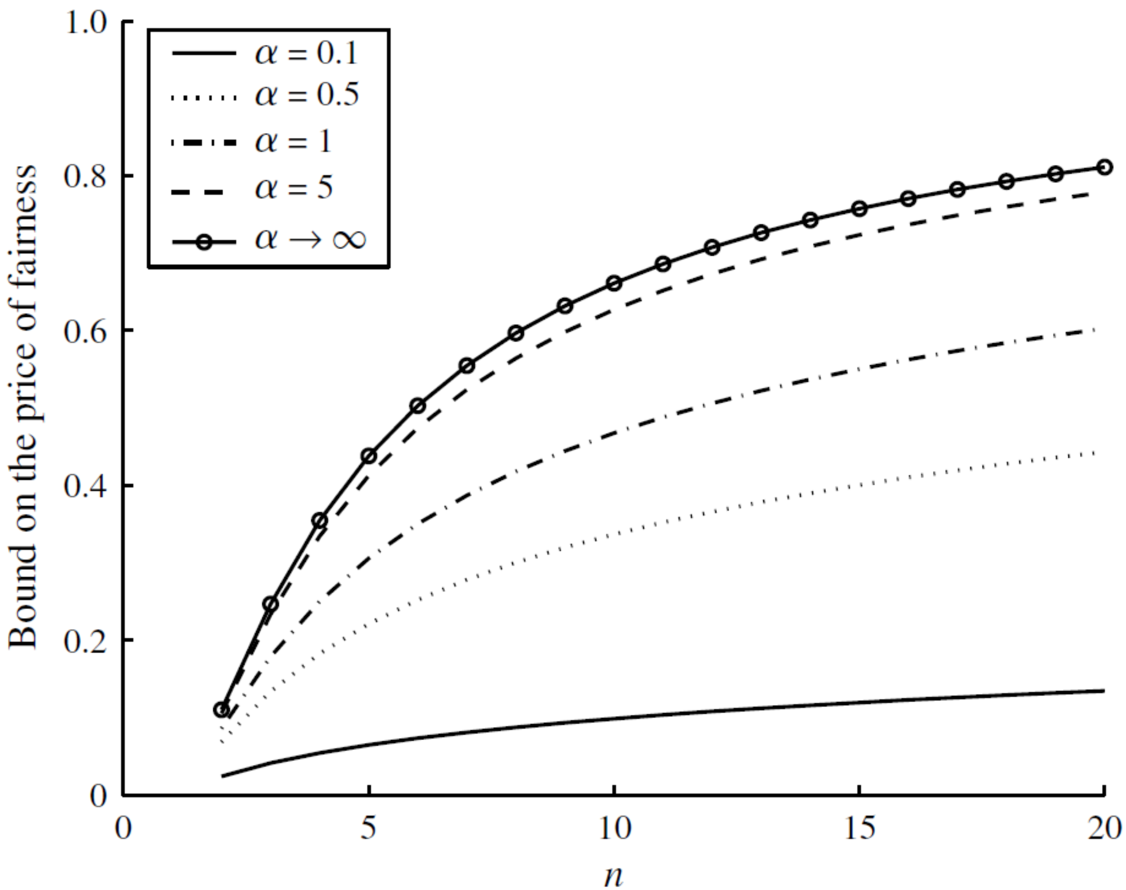
Consider a resource allocation problem with $n \geq 2$ agents where all agents have non-negative utilities with the same maximum achievable utility and the set of all feasible utility allocation is convex.
Then for the α-fair allocations, $\alpha \geq 0$, **the price of fairness** is bounded by

$$\mathrm{POF}(\mathcal{X}, \alpha) \leq 1 - \Theta\left(n^{-\frac{\alpha}{1+\alpha}}\right)$$

Generalization to heterogeneous utilities possible

- the price then increases with the ratio between the highest and lowest achievable utility

Price of Fairness



The worst-case price is **increasing** with the **number of agents** and the **value of α**.

Bounds are very strong, **near-tight**.

Price of Efficiency

Quantifies the **loss of fairness** due to the requirement for efficiency.

We adopt the **minimum utility** egalitarian social welfare function as the fairness metric.

Price of Efficiency

$$\mathrm{POE}(\mathcal{X}, \alpha) = \frac{\max_{X \in \mathcal{X}} \min_{i \in \mathcal{A}} u_i(X) - \min_{i \in \mathcal{A}} u_i(X_i^*(\alpha))}{\max_{X \in \mathcal{X}} \min_{i \in \mathcal{A}} u_i(X)}$$

(where $X^*(\alpha)$ is the α-fair allocation)

Price of Efficiency

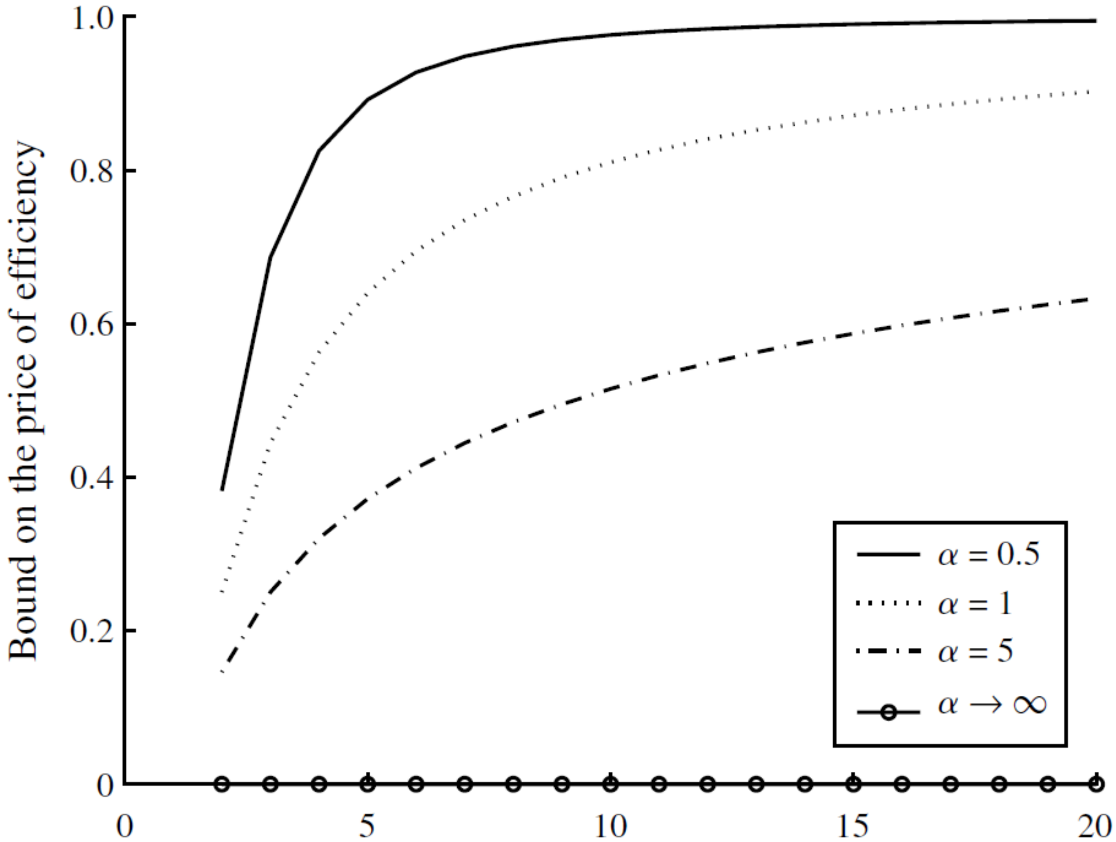
Theorem

Consider a resource allocation with $n \geq 2$ agents where all agents have non-negative utilities and the same maximum achievable utility and the set of all feasible utility allocations is convex.

Then for the α-fair allocations, $\alpha \geq 0$, the **price of efficiency** is bounded by

$$\mathrm{POE}(\mathcal{X}, \alpha) \leq 1 - \Theta(n^{-\frac{1}{\alpha}})$$

Price of Efficiency

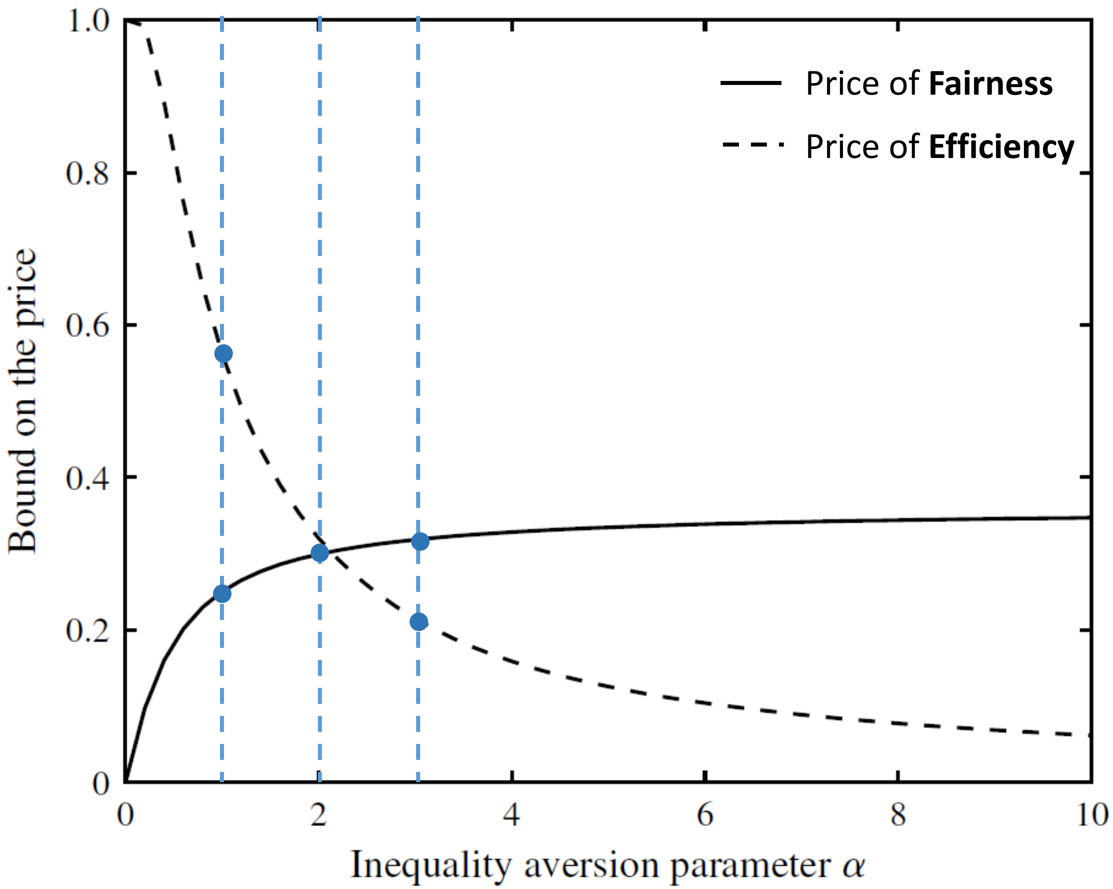


The worst-case price of efficiency is increasing with the number of agents and the value of α.

Bounds are very strong, near-tight.

Example for four agents

Bounds Bounds on the Price of Fairness and the Price of Efficiency of α-Fair Allocations for $\boldsymbol{n = 4}$ **agents**.



$\alpha = 1$: **Maximizes fairness** while guaranteeing max ~25% loss of system efficiency.

$\alpha = 3$: **Maximizes efficiency** while guaranteeing maximum ~20% drop in the utility for the worst-off agent (compared to egalitarian allocation).

$\alpha = 2$: **Balances efficiency and fairness**

Allocation Procedures

Allocation Procedures

Protocols: What messages do agents have to exchange and in which order?

Strategies: What strategies may an agent use for a given protocol? How can we give incentives to agents to behave in a certain way?

Algorithms: How do we solve the computational problems faced by agents when engaged in negotiation?

Centralised vs. Distributed Allocation

Centralised case	Distributed case
• A **single entity decides** on the final allocation, possibly after having elicited the preferences of the other agents. • Example: auctions	• **Allocations emerge** as the result of a sequence of local negotiation steps. • Such local steps may or may not be subject to structural restrictions (say, bilateral deals).

Which approach is appropriate under what circumstances?

Centralised vs. Distributed Comparison

Centralised	Distributed
• The **communication** protocols required are relatively **simple**. • Many **results** from **economics** and **game theory** , in particular on mechanism design, can be exploited. • **Powerful algorithms** for winner determination in combinatorial auctions. • Possible **trust** issues. • Difficult to deal with **unbounded problems**.	• Avoids **trust** issues. • Inherently **scalable**. • Can take an **initial allocation** into account. • More natural to model **step-wise improvements** over the status quo. • Can deal with **unbounded domains**. • More **complex** protocols significantly more **difficult** to analyse (convergence etc.)

➔ Auctions

Conclusions

Solving allocation problems requires defining 1) resources, 2) agents and their preferences, 3) system/social preferences and 4) mechanism.

There is an inherent trade-off between efficiency and fairness in allocation.

Auctions are a widely adopted centralized allocation mechanism which (typically) aims to optimize efficiency and is neutral toward fairness.

Reading:

- Chevaleyre, Y., Dunne, P.E., Endriss, U., Lang, J., Lemaitre, M., Maudet, N., Padget, J., Phelps, S., Rodríguez-Aguilar, J.A. and Sousa, P., 2006. Issues in multiagent resource allocation.
- Bertsimas, D., Farias, V.F. and Trichakis, N., 2012. On the efficiency-fairness trade-off. *Management Science*, *58*(12), pp.2234-2250.