

Extensive-Form Games

Branislav Bošanský, Dominik Seitz, José Hilario , Michal Šustr

Czech Technical University in Prague

branislav.bosansky@agents.fel.cvut.cz

dominik.seitz@aic.fel.cvut.cz

hilarjos@fel.cvut.cz

michal.sustr@aic.fel.cvut.cz

November 5, 2019

Previously ... on multi-agent systems (tutorials and lectures).

- 1 Sequence-form LP

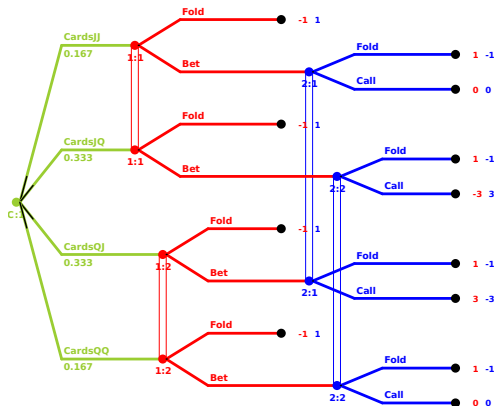
Semestral project: Agent and Bandits

You can use Gambit for visualization
(small poker, game value: -0.167):

5
7

#E---E#
#S#-#D#
#E--EG#

1
0.7



Normal-form LP

Reminder from lectures:

$$\max_{s,U} U \quad (1)$$

$$\text{s.t.} \quad \sum_{a_1 \in \mathcal{A}_1} s(a_1) u_1(a_1, a_2) \geq U \quad \forall a_2 \in \mathcal{A}_2 \quad (2)$$

$$\sum_{a_1 \in \mathcal{A}_1} s(a_1) = 1 \quad (3)$$

$$s(a_1) \geq 0 \quad \forall a_1 \in \mathcal{A}_1 \quad (4)$$

Sequence-form LP

Reminder from lectures:

$$\max_{r_1, v} v(\text{root}) \quad (5)$$

$$\text{s.t.} \quad r_1(\emptyset) = 1 \quad (6)$$

$$0 \leq r_1(\sigma_1) \leq 1 \quad \forall \sigma_1 \in \Sigma_1 \quad (7)$$

$$\sum_{a \in \mathcal{A}(I_1)} r_1(\sigma_1 a) = r_1(\sigma_1) \quad \forall I_1 \in \mathcal{I}_1, \sigma_1 = \text{seq}_1(I_1) \quad (8)$$

$$\sum_{I' \in \mathcal{I}_2: \sigma_2 a = \text{seq}_2(I')} v(I') + \sum_{\sigma_1 \in \Sigma_1} g(\sigma_1, \sigma_2 a) r_1(\sigma_1) \geq v(I) \quad \forall I \in \mathcal{I}_2, \sigma_2 = \text{seq}_2(I), \forall a \in \mathcal{A}(I) \quad (9)$$

- $\text{seq}_i(I)$ is a sequence of player i to information set,
- $I \in \mathcal{I}_i$, v_I is an expected utility in an information set,
- $\text{inf}_i(\sigma_i)$ is an information set, where the last action of σ_i has been executed,
- $\sigma_i a$ denotes an extension of a sequence σ_i with action a

Task 1: Write down a sequence-form linear program for both players for the following game of a small “poker”:

- there is an ante of 1\$
- there is a limited deck of cards $\{J, J, Q, Q\}$
- each player receives a card
- player 1 either folds or bets 2\$
- player 2 either calls or folds
- player with the higher card wins

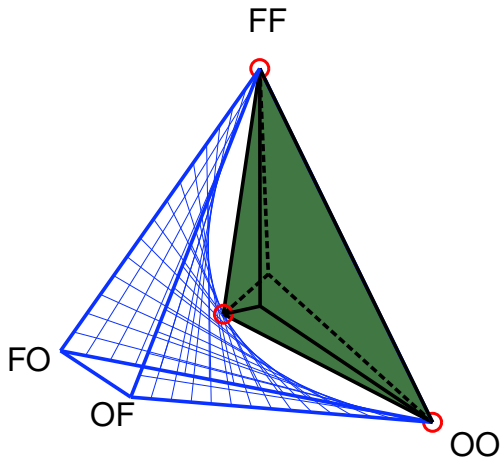
Task 2: Consider the game of battle of sexes. Find a CE that is not a NE. Show that it is indeed not a NE.

	Dare	Chicken
Dare	0,0	7,2
Chicken	2,7	6,6

Task 3: Find another game which has CE that is not a convex combination of NE.

Task 4: Prove or disprove that all convex combinations of (two) NE are CE.

Geometry of NE and CE ¹ for the game battle of sexes:



¹From <https://www.cs.cmu.edu/~ggordon/CE/>

Task 5: Consider following game. Write down a normal-form representation it.

