Studying 4-subsets of {1,2,...,12}

Rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12}.

The number of all 4-subsets of {1,2,...,12} is binCoeff(12, 4) = 495

The rank of {6,8,9,11} is between 0 and 494.

Studying 4-subsets of {1,2,...,12}

All 4-subsets of {1,2,...,12} are (on this and on next 8 slides):

0 {1 2 3 4}
1 {1 2 3 5}
2 {1 2 3 6}
3 {1 2 3 7}
4 {1 2 3 8}
5 {1 2 3 9}
6 {1 2 3 10}
7 {1 2 3 11}
8 {1 2 3 12}

9 {1 2 4 5}
10 {1 2 4 6}
11 {1 2 4 7}
12 {1 2 4 8}
13 {1 2 4 9}
14 {1 2 4 10}
15 {1 2 4 11}
16 {1 2 4 12}

17 {1 2 5 6}
18 {1 2 5 7}
19 {1 2 5 8}
20 {1 2 5 9}
21 {1 2 5 10}
22 {1 2 5 11}
23 {1 2 5 12}

24 {1 2 6 7}
25 {1 2 6 8}
26 {1 2 6 9}
27 {1 2 6 10}
28 {1 2 6 11}
29 {1 2 6 12}

30 {1 2 7 8}
31 {1 2 7 9}
32 {1 2 7 10}
33 {1 2 7 11}
34 {1 2 7 12}

35 {1 2 8 9}
36 {1 2 8 10}
37 {1 2 8 11}
38 {1 2 8 12}

39 {1 2 9 10}
40 {1 2 9 11}
41 {1 2 9 12}

42 {1 2 10 11}
43 {1 2 10 12}

44 {1 2 11 12}

45 {1 3 4 5}
46 {1 3 4 6}
47 {1 3 4 7}
48 {1 3 4 8}
49 {1 3 4 9}
50 {1 3 4 10}
51 {1 3 4 11}
52 {1 3 4 12}

53 {1 3 5 6}
54 {1 3 5 7}
55 {1 3 5 8}
56 {1 3 5 9}
57 {1 3 5 10}
58 {1 3 5 11}
59 {1 3 5 12}

60 {1 3 6 7}
61 {1 3 6 8}
62 {1 3 6 9}
63 {1 3 6 10}
64 {1 3 6 11}
65 {1 3 6 12}

66 {1 3 7 8}
67 {1 3 7 9}
68 {1 3 7 10}
69 {1 3 7 11}
70 {1 3 7 12}

71 {1 3 8 9}
72 {1 3 8 10}
73 {1 3 8 11}
74 {1 3 8 12}

Studying 4-subsets of {1,2,...,12}

The vertical spaces should help to perceive the regularity patterns in the list.

75 {1 3 9 10}
76 {1 3 9 11}
77 {1 3 9 12}

78 {1 3 10 11}
79 {1 3 10 12}

80 {1 3 11 12}

81 {1 4 5 6}
82 {1 4 5 7}
83 {1 4 5 8}
84 {1 4 5 9}
85 {1 4 5 10}
86 {1 4 5 11}
87 {1 4 5 12}

88 {1 4 6 7}
89 {1 4 6 8}
90 {1 4 6 9}
91 {1 4 6 10}
92 {1 4 6 11}
93 {1 4 6 12}

94 {1 4 7 8}
95 {1 4 7 9}
96 {1 4 7 10}
97 {1 4 7 11}
98 {1 4 7 12}

99 {1 4 8 9}
100 {1 4 8 10}
101 {1 4 8 11}
102 {1 4 8 12}

103 {1 4 9 10}
104 {1 4 9 11}
105 {1 4 9 12}
106 {1 4 10 11}
107 {1 4 10 12}
108 {1 4 11 12}

109 {1 5 6 7}
110 {1 5 6 8}
111 {1 5 6 9}
112 {1 5 6 10}
113 {1 5 6 11}
114 {1 5 6 12}

115 {1 5 7 8}
116 {1 5 7 9}
117 {1 5 7 10}
118 {1 5 7 11}
119 {1 5 7 12}

120 {1 5 8 9}
121 {1 5 8 10}
122 {1 5 8 11}
123 {1 5 8 12}

124 {1 5 9 10}
125 {1 5 9 11}
126 {1 5 9 12}

127 {1 5 10 11}
128 {1 5 10 12}

129 {1 5 11 12}

Studying 4-subsets of {1,2,...,12}

130 {1 6 7 8}
131 {1 6 7 9}
132 {1 6 7 10}
133 {1 6 7 11}
134 {1 6 7 12}

135 {1 6 8 9}
136 {1 6 8 10}
137 {1 6 8 11}
138 {1 6 8 12}

139 {1 6 9 10}
140 {1 6 9 11}
141 {1 6 9 12}

142 {1 6 10 11}
143 {1 6 10 12}

144 {1 6 11 12}

145 {1 7 8 9}
146 {1 7 8 10}
147 {1 7 8 11}
148 {1 7 8 12}

149 {1 7 9 10}
150 {1 7 9 11}
151 {1 7 9 12}

152 {1 7 10 11}
153 {1 7 10 12}

154 {1 7 11 12}

155 {1 8 9 10}
156 {1 8 9 11}
157 {1 8 9 12}

158 {1 8 10 11}
159 {1 8 10 12}

160 {1 8 11 12}

161 {1 9 10 11}
162 {1 9 10 12}

163 {1 9 11 12}

164 {1 10 11 12}

165 {2 3 4 5}
166 {2 3 4 6}
167 {2 3 4 7}
168 {2 3 4 8}
169 {2 3 4 9}
170 {2 3 4 10}
171 {2 3 4 11}
172 {2 3 4 12}

173 {2 3 5 6}
174 {2 3 5 7}
175 {2 3 5 8}
176 {2 3 5 9}
177 {2 3 5 10}
178 {2 3 5 11}
179 {2 3 5 12}

Studying 4-subsets of {1,2,...,12}

180 {2 3 6 7}
181 {2 3 6 8}
182 {2 3 6 9}
183 {2 3 6 10}
184 {2 3 6 11}
185 {2 3 6 12}

186 {2 3 7 8}
187 {2 3 7 9}
188 {2 3 7 10}
189 {2 3 7 11}
190 {2 3 7 12}

191 {2 3 8 9}
192 {2 3 8 10}
193 {2 3 8 11}
194 {2 3 8 12}

195 {2 3 9 10}
196 {2 3 9 11}
197 {2 3 9 12}

198 {2 3 10 11}
199 {2 3 10 12}

200 {2 3 11 12}

201 {2 4 5 6}
202 {2 4 5 7}
203 {2 4 5 8}
204 {2 4 5 9}
205 {2 4 5 10}
206 {2 4 5 11}
207 {2 4 5 12}

208 {2 4 6 7}
209 {2 4 6 8}
210 {2 4 6 9}
211 {2 4 6 10}
212 {2 4 6 11}
213 {2 4 6 12}

214 {2 4 7 8}
215 {2 4 7 9}
216 {2 4 7 10}
217 {2 4 7 11}
218 {2 4 7 12}

219 {2 4 8 9}
220 {2 4 8 10}
221 {2 4 8 11}
222 {2 4 8 12}

223 {2 4 9 10}
224 {2 4 9 11}
225 {2 4 9 12}

226 {2 4 10 11}
227 {2 4 10 12}

228 {2 4 11 12}

229 {2 5 6 7}
230 {2 5 6 8}
231 {2 5 6 9}
232 {2 5 6 10}
233 {2 5 6 11}
234 {2 5 6 12}

Studying 4-subsets of {1,2,...,12}

235 {2 5 7 8}
236 {2 5 7 9}
237 {2 5 7 10}
238 {2 5 7 11}
239 {2 5 7 12}

240 {2 5 8 9}
241 {2 5 8 10}
242 {2 5 8 11}
243 {2 5 8 12}

244 {2 5 9 10}
245 {2 5 9 11}
246 {2 5 9 12}

247 {2 5 10 11}
248 {2 5 10 12}

249 {2 5 11 12}

250 {2 6 7 8}
251 {2 6 7 9}
252 {2 6 7 10}
253 {2 6 7 11}
254 {2 6 7 12}

255 {2 6 8 9}
256 {2 6 8 10}
257 {2 6 8 11}
258 {2 6 8 12}

259 {2 6 9 10}
260 {2 6 9 11}
261 {2 6 9 12}

262 {2 6 10 11}
263 {2 6 10 12}

264 {2 6 11 12}

265 {2 7 8 9}
266 {2 7 8 10}
267 {2 7 8 11}
268 {2 7 8 12}

269 {2 7 9 10}
270 {2 7 9 11}
271 {2 7 9 12}

272 {2 7 10 11}
273 {2 7 10 12}

274 {2 7 11 12}

275 {2 8 9 10}
276 {2 8 9 11}
277 {2 8 9 12}

278 {2 8 10 11}
279 {2 8 10 12}

280 {2 8 11 12}

281 {2 9 10 11}
282 {2 9 10 12}

283 {2 9 11 12}

284 {2 10 11 12}

285 {3 4 5 6}
286 {3 4 5 7}
287 {3 4 5 8}
288 {3 4 5 9}
289 {3 4 5 10}
290 {3 4 5 11}
291 {3 4 5 12}

Studying 4-subsets of {1,2,...,12}

292 {3 4 6 7}
293 {3 4 6 8}
294 {3 4 6 9}
295 {3 4 6 10}
296 {3 4 6 11}
297 {3 4 6 12}

298 {3 4 7 8}
299 {3 4 7 9}
300 {3 4 7 10}
301 {3 4 7 11}
302 {3 4 7 12}

303 {3 4 8 9}
304 {3 4 8 10}
305 {3 4 8 11}
306 {3 4 8 12}

307 {3 4 9 10}
308 {3 4 9 11}
309 {3 4 9 12}

310 {3 4 10 11}
311 {3 4 10 12}

312 {3 4 11 12}

313 {3 5 6 7}
314 {3 5 6 8}
315 {3 5 6 9}
316 {3 5 6 10}
317 {3 5 6 11}
318 {3 5 6 12}

319 {3 5 7 8}
320 {3 5 7 9}
321 {3 5 7 10}
322 {3 5 7 11}
323 {3 5 7 12}

324 {3 5 8 9}
325 {3 5 8 10}
326 {3 5 8 11}
327 {3 5 8 12}

328 {3 5 9 10}
329 {3 5 9 11}
330 {3 5 9 12}

331 {3 5 10 11}
332 {3 5 10 12}

333 {3 5 11 12}

334 {3 6 7 8}
335 {3 6 7 9}
336 {3 6 7 10}
337 {3 6 7 11}
338 {3 6 7 12}

339 {3 6 8 9}
340 {3 6 8 10}
341 {3 6 8 11}
342 {3 6 8 12}

343 {3 6 9 10}
344 {3 6 9 11}
345 {3 6 9 12}

346 {3 6 10 11}
347 {3 6 10 12}

348 {3 6 11 12}

Studying 4-subsets of {1,2,...,12}

349 {3 7 8 9}
350 {3 7 8 10}
351 {3 7 8 11}
352 {3 7 8 12}

353 {3 7 9 10}
354 {3 7 9 11}
355 {3 7 9 12}

356 {3 7 10 11}
357 {3 7 10 12}

358 {3 7 11 12}

359 {3 8 9 10}
360 {3 8 9 11}

361 {3 8 9 12}

362 {3 8 10 11}
363 {3 8 10 12}

364 {3 8 11 12}

365 {3 9 10 11}
366 {3 9 10 12}

367 {3 9 11 12}

368 {3 10 11 12}
369 {4 5 6 7}
370 {4 5 6 8}
371 {4 5 6 9}
372 {4 5 6 10}
373 {4 5 6 11}
374 {4 5 6 12}

375 {4 5 7 8}
376 {4 5 7 9}
377 {4 5 7 10}
378 {4 5 7 11}
379 {4 5 7 12}

380 {4 5 8 9}
381 {4 5 8 10}
382 {4 5 8 11}
383 {4 5 8 12}

384 {4 5 9 10}
385 {4 5 9 11}
386 {4 5 9 12}

387 {4 5 10 11}
388 {4 5 10 12}

389 {4 5 11 12}

390 {4 6 7 8}
391 {4 6 7 9}
392 {4 6 7 10}
393 {4 6 7 11}
394 {4 6 7 12}

395 {4 6 8 9}
396 {4 6 8 10}
397 {4 6 8 11}
398 {4 6 8 12}

399 {4 6 9 10}
400 {4 6 9 11}
401 {4 6 9 12}

402 {4 6 10 11}
403 {4 6 10 12}

404 {4 6 11 12}

Studying 4-subsets of {1,2,...,12}

405 {4 7 8 9}
406 {4 7 8 10}
407 {4 7 8 11}
408 {4 7 8 12}

409 {4 7 9 10}
410 {4 7 9 11}
411 {4 7 9 12}

412 {4 7 10 11}
413 {4 7 10 12}

414 {4 7 11 12}

415 {4 8 9 10}
416 {4 8 9 11}
417 {4 8 9 12}

418 {4 8 10 11}
419 {4 8 10 12}

420 {4 8 11 12}

421 {4 9 10 11}
422 {4 9 10 12}

423 {4 9 11 12}

424 {4 10 11 12}

425 {5 6 7 8}
426 {5 6 7 9}
427 {5 6 7 10}
428 {5 6 7 11}
429 {5 6 7 12}

430 {5 6 8 9}
431 {5 6 8 10}
432 {5 6 8 11}
433 {5 6 8 12}

434 {5 6 9 10}
435 {5 6 9 11}
436 {5 6 9 12}

437 {5 6 10 11}
438 {5 6 10 12}

439 {5 6 11 12}

440 {5 7 8 9}
441 {5 7 8 10}
442 {5 7 8 11}
443 {5 7 8 12}

444 {5 7 9 10}
445 {5 7 9 11}
446 {5 7 9 12}

447 {5 7 10 11}
448 {5 7 10 12}

449 {5 7 11 12}

450 {5 8 9 10}
451 {5 8 9 11}
452 {5 8 9 12}

453 {5 8 10 11}
454 {5 8 10 12}

455 {5 8 11 12}

Studying 4-subsets of {1,2,...,12}

456 {5 9 10 11}
457 {5 9 10 12}

458 {5 9 11 12}

459 {5 10 11 12}

460 {6 7 8 9}
461 {6 7 8 10}
462 {6 7 8 11}
463 {6 7 8 12}

464 {6 7 9 10}
465 {6 7 9 11}
466 {6 7 9 12}

467 {6 7 10 11}
468 {6 7 10 12}

469 {6 7 11 12}

470 {6 8 9 10}
471 {6 8 9 11}
472 {6 8 9 12}

473 {6 8 10 11}
474 {6 8 10 12}

475 {6 8 11 12}

476 {6 9 10 11}
477 {6 9 10 12}

478 {6 9 11 12}

479 {6 10 11 12}

480 {7 8 9 10}
481 {7 8 9 11}
482 {7 8 9 12}

483 {7 8 10 11}
484 {7 8 10 12}

485 {7 8 11 12}

486 {7 9 10 11}
487 {7 9 10 12}

488 {7 9 11 12}

489 {7 10 11 12}

490 {8 9 10 11}
491 {8 9 10 12}

492 {8 9 11 12}

493 {8 10 11 12}

494 {9 10 11 12}

Studying 4-subsets of {1,2,...,12}

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471 (see previous slide).

General strategy:

1. Note that the list of all 4-subsets is divided into a number of blocks.

2. Establish the pattern by which the 4-subsets are divided into blocks.

3. Note that this pattern has recursive character.

4. Using the established pattern, count (recursively) the number of blocks which precede the given subset {6,8,9,11} in the list of all subsets. This number is equal to the rank of the subset.

Studying 4-subsets of {1,2,...,12}

Level 0

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471 (see previous slide).

The minimum item in {6,8,9,11} is 6.
Therefore {6,8,9,11} is preceded in the list by all 4- subsets which contain values
-- 1 and bigger
-- 2 and bigger
-- 3 and bigger
-- 4 and bigger
-- 5 and bigger

Specifically, those are:

0 {1 2 3 4}	**165 {2 3 4 5}**	**285 {3 4 5 6}**	**369 {4 5 6 7}**	**425 {5 6 7 8}**
1 {1 2 3 5}	**166 {2 3 4 6}**	**286 {3 4 5 7}**	**370 {4 5 6 8}**	**426 {5 6 7 9}**
...	**...**	**...**	**...**	**...**
...	**...**	**...**	**...**	**...**
164 {1 10 11 12}	**284 {2 10 11 12}**	**368 {3 10 11 12}**	**424 {4 10 11 12}**	**459 {5 10 11 12}**

The size of each of these blocks is computed on next two slides

Studying 4-subsets of {1,2,...,12}

Level 0

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471 (see previous slide).

0 {1 2 3 4}
1 {1 2 3 5}
...
...
164 {1 10 11 12}

Block 1. All 4-subsets which contain 1 and bigger values.
Value 1 is present in all subsets in the block.
When we remove 1 from each item in the block, we find that the size of the block is equal to the number of all 3-subsets of the set {2,3,4,...,12}.
Formally, that number is the same as the number of all 3-subsets of the set {1,2,3,...,11} *).
And that, in turn, is equal to binCoeff(11,3) = 11!/(3!*8!) = 165

165 {2 3 4 5}
166 {2 3 4 6}
...
...
284 {2 10 11 12}

Block 2. All 4-subsets which contain 2 and bigger values.
Value 2 is present in all subsets in the block.
When we remove 2 from each item in the block, we find that the size of the block is equal to the number of all 3-subsets of the set {3,4,5,...,12}.
Formally, that number is the same as the number of all 3-subsets of the set {1,2,3,...,10}.
And that, in turn, is equal to binCoeff(10,3) = 10!/(3!*7!) = 120

*) Should be obvious, as the size of sets {2,3,4,...,12} and {1,2,3,...,11} is clearly the same.

Studying 4-subsets of {1,2,...,12}

Level 0

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471 (see previous slide).

285 {3 4 5 6}
286 {3 4 5 7}
...
...
368 {3 10 11 12}

Block 3. All 4-subsets which contain 3 and bigger values.
The size of the block is equal to the number
of all 3-subsets of the set {4,5,6,...,12}.
Formally, that number is the same as the number
of all 3-subsets of the set {1,2,3,...,9}.
And that, in turn, is equal to binCoeff(9,3) = 9!/(3!*6!) = 84

369 {4 5 6 7}
370 {4 5 6 8}
...
...
424 {4 10 11 12}

Block 4. All 4-subsets which contain 4 and bigger values.
Formally, the number of those subsets is the same as the number
of all 3-subsets of the set {1,2,3,...,8}.
And that is equal to binCoeff(8,3) = 8!/(3!*5!) = 56

425 {5 6 7 8}
426 {5 6 7 9}
...
...
459 {5 10 11 12}

Block 5. All 4-subsets which contain 5 and bigger values.
Formally, the number of those subsets is the same as the number
of all 3-subsets of the set {1,2,3,...,7}.
And that is equal to binCoeff(7,3) = 7!/(3!*4!) = 35

Studying 4-subsets of {1,2,...,12}

Level 0

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471.

The subset {6,8,9,11} is preceded by 5 blocks which total size is
165 + 120 + 84 + 56 + 35 = 460.
Thus, the rank of {6,8,9,11} is 460 or bigger.

The 4-subset {6,8,9,11} is itself in the block 6 which contains values 6 and higher:

460 {6 7 8 9}
461 {6 7 8 10}
...
...
479 {6 10 11 12}

The rank of {6,8,9,11} in all 4-subsets of {1,2,...,12} is equal to
460 + the rank of {6,8,9,11} in all 4-subsets of {6,7,8,...,12}.

Note that the value 6 is common in all subsets in this block.
Remove it from the subsets and from the set {6,7,8,...,12}.

Therefore:

A. The rank of {6,8,9,11} in all 4-subsets of {6,7,8,...,12} is equal to
the rank of {8,9,11} in all 3-subsets of {7,8,...,12}.

B. The rank of {8,9,11} in all 3-subsets of {7,8,...,12} is equal to
the rank of {2,3,5} in all 3-subsets of {1,2,...,6}.
(just formally subtract 6 from all elements in the subset and the set {7,8,...,12})

Studying 4-subsets of {1,2,...,12}

Level 0

The rank of subset {6,8,9,11} in all 4-subsets of {1,2,...,12} is 471.

The subset {6,8,9,11} is preceded by 5 blocks which total size is
165 + 120 + 84 + 56 + 35 = 460.
Thus, the rank of {6,8,9,11} is 460 or bigger.

The 4-subset {6,8,9,11} is itself in the block 6 which contains values 6 and higher:

460 {6 7 8 9}
461 {6 7 8 10}
...
...
479 {6 10 11 12}

Previous slide A+B:
The rank of {6,8,9,11} in all 4-subsets of {6,7,8,...,12} is equal to the rank of {2,3,5} in all 3-subsets of {1,2,...,6}.

Apply recursion -- same problem structure, smaller parameters.

The rank of {2,3,5} in all 3-subsets of {1,2,...,6} is 11.

Studying 3-subsets of {1,2,...,6}

Level 1

All 3-subsets of {1,2,...,6} are

0 {1 2 3}	10 {2 3 4}	16 {3 4 5}	19 {4 5 6}
1 {1 2 4}	**11 {2 3 5}**	17 {3 4 6}	
2 {1 2 5}	12 {2 3 6}		
3 {1 2 6}		18 {3 5 6}	
	13 {2 4 5}		
4 {1 3 4}	14 {2 4 6}		
5 {1 3 5}			
6 {1 3 6}	15 {2 5 6}		
7 {1 4 5}			
8 {1 4 6}			
9 {1 5 6}			

Studying 3-subsets of {1,2,...,12}

Level 1

The rank of subset {2,3,5} in all 3-subsets of {1,2,...,6} is 11 (see previous slide).

The minimum item in {2,3,5} is 1.
Therefore {2,3,5} is preceded in the list by all 3-subsets which contain values
-- 1 and bigger

Specifically, that is:

0 {1 2 3}
1 {1 2 4}
...
9 {1 5 6}

Block 1. All 3-subsets which contain 1 and bigger values.
Value 1 is present in all subsets in the block.
When we remove 1 from each item in the block, we find that the size of the block is equal to the number of all 2-subsets of the set {2,3,4,...,6}.
Formally, that number is the same as the number of all 2-subsets of the set {1,2,3,...,5}.
And that, in turn, is equal to binCoeff(5,2) = 5!/(2!*3!) = 10

Studying 3-subsets of {1,2,...,12}

Level 1

The rank of subset {2,3,5} in all 3-subsets of {1,2,...,6} is 11.

The subset {6,8,9,11} is preceded by 1 blocks which size is10.
Thus, the rank of {2,3,5} is 10 or bigger.

The 3-subset {2,3,5} is itself in the block 2 which contains values 2 and higher:

460 {6 7 8 9}
461 {6 7 8 10}
...
...
479 {6 10 11 12}

The rank of {2,3,5} in all 3-subsets of {1,2,...,6} is equal to
10 + the rank of {2,3,5} in all 4-subsets of {2,3,4,...,6}.

Note that the value 2 is common in all subsets in this block.
Remove it from the subsets and from the set {2,3,4,...,6}.

Therefore:

A. The rank of {2,3,5} in all 3-subsets of {2,3,4,...,6} is equal to
the rank of {3,5} in all 2-subsets of {3,4,...,6}.

B. The rank of {3,5} in all 2-subsets of {3,4,...,6} is equal to
the rank of {1 3} in all 2-subsets of {1,2,...,4}.
(Just formally subtract 2 from all elements in the subset and the set {3,4,...,6}.)

Studying 2-subsets of {1,2,...,4}

All 2-subsets of {1,2,...,4} are

Level 2

0 {1 2}
1 {1 3}
2 {1 4}

3 {2 3}
4 {2 4}

5 {3 4}

The rank of subset {1,3} in all 2-subsets of {1,2,...,4} is 1.

0 {1 2}
1 {1 3}
...
2 {1 4}

The minimum item in {1, 3} is 1.
Therefore {1,3} is in the list, in the first block.
In other words, it is preceded by 0 blocks which contain value 0 and higher.
(Value 0 cannot appear in the subset).

The rank of {1,3} in the first block in the list of all 2-subsets of {1,2,...,4} is equal to the rank of {3} in the list of all 1-subsets of {2,...,4}.
That is the same as the rank of {2} in the list of all 1-subsets of {1,...,3}.
(Just subtract 1 from all elements in the subset and the set {1,...,3}.)

Studying 1-subsets of {1,2,...,4}

Level 3

All 1-subsets of {1,2,...,3} are

0 {1}
1 {2}
2 {3}

The rank of subset {2} in all 1-subsets of {1,2,...,3} is 1.

Finding the rank of 1-element subset {a} of the set {1,2,...,X) is easy, just return a–1.

To conclude:

Finding the rank of a subset required computing recursively the the size of blocks which preceded the given subset in the lexicographically ordered list of all subsets of the given size.

The computations on consecutive levels of recursion yielded total sizes 460 + 10 + 0 + 1 = 471.

```
def rankSubset( subset, n ):
    k = len(subset)
    if k == 1: return subset[0] - 1

    rank = 0
    # total number of all subsets containing
    # values 1, 2, ..., subset[0]-1, which precede the given subset
    # in the list of all subsets lexicographically sorted
    for i in range(1, subset[0]):
        rank += binCoeff( n-i, k-1 )

    # exclude first elem from the subset array
    # and "normalize" the input for recursion
    subset1 = subset[1:] # copy of subset[1..k]
    for j in range( len(subset1) ):
        subset1[j] -= subset[0]
    n1 = n - (subset[0])

    # and recurse
    return rank + rankSubset( subset1, n1)
```

```python
def unrankSubset ( rank, n, k ):
    if k == 1:  return [rank+1]   # list with sigle value

    # jump over appropriate number of blocks
    # which precede the subset with the given rank
    # and simultaneously construct value subset[0]
    blockSizes = 0
    n1 = n-1
    subset0 = 1
    while True:
        bSize = binCoeff( n1, k-1 )
        if bSize <= rank:
            rank -= bSize
            subset0 += 1
            n1 -= 1
        else: break

    subsetRec = unrankSubset ( rank, n1, k-1 )
    for j in range( len(subsetRec) ):
        subsetRec[j] += subset0

    return [subset0] + subsetRec  #list concatenation
```