

Multi-goal Planning

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Lecture 05

B4M36UIR – Artificial Intelligence in Robotics



Overview of the Lecture

- Part 1 – Multi-goal Planning
 - Inspection Planning
 - Multi-goal Planning
- Part 2 – Unsupervised Learning for Multi-goal Planning
 - Unsupervised Learning for Multi-goal Planning
 - TSPN in Multi-goal Planning with Localization Uncertainty



Part I Part 1 – Multi-goal Planning



Robotic Information Gathering in Inspection of Vessel's Propeller

- The planning problem is to determine a shortest inspection path for an Autonomous Underwater Vehicle (AUV) to inspect the vessel's propeller.



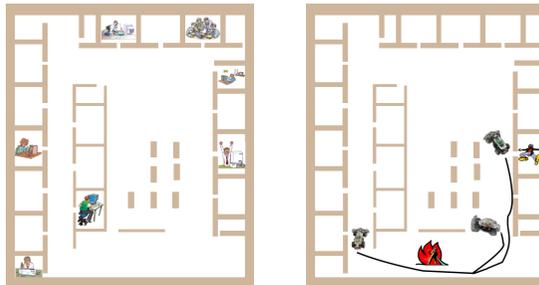
https://www.youtube.com/watch?v=8azP_9VnMtM

Englot, B., Hover, F.S.: *Three-dimensional coverage planning for an underwater inspection robot*, International Journal of Robotics Research, 32(9-10):1048-1073, 2013.



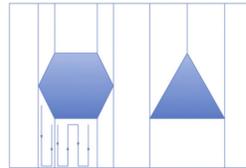
Example of Inspection Planning in Search Scenario

- Periodically visit particular locations of the environment and return to the starting locations.
- Use available floor plans to guide the search, e.g., finding victims in search-and-rescue scenario.

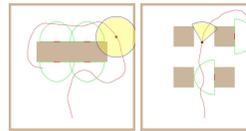
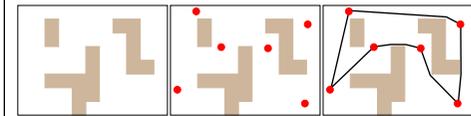


Inspection Planning

- **Inspection/coverage planning** stands to determine a plan (path) to inspect/cover the given areas or point of interest.
- We can directly find inspection/coverage plan using
 - predefined covering patterns such as *ox-plow* motion;
 - a "general" path satisfying coverage constraints.
- **Decoupled** approach where locations to be visited are determined before path planning as the **sensor placement** problem.



Trapezoidal decomposition method

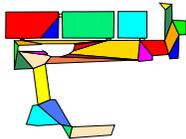


Kafka, Faigl, Vána: ICRA 2016

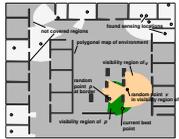
Inspection Planning – Decoupled Approach

1. Determine sensing locations such that the whole environment would be inspected (seen) by visiting them (**Sampling design problem**).

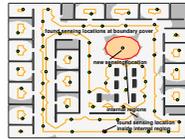
In the geometrical-based approach, a solution of the *Art Gallery Problem*.



Convex Partitioning (Kazazakis and Argyros, 2002)



Randomized Dual Sampling (González-Baños et al., 1998)



Boundary Placement (Faigl et al., 2006)

The problem is related to the **sensor placement** and **sampling design**.

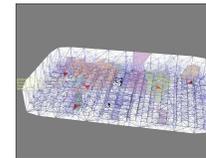
E.g., using **visibility graph** or **randomized sampling based approaches**.

2. Create a roadmap connecting the sensing location.
3. Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning (a solution of the robotic TSP).



Planning to Capture Areas of Interest using UAV

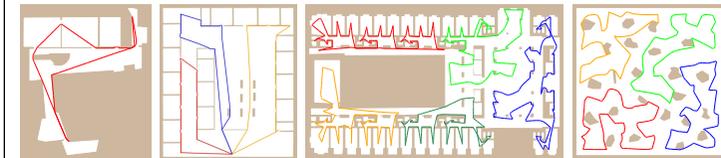
- Determine a cost-efficient path from which a given set of target regions is covered.
- For each target region a subspace $S \subset \mathbb{R}^3$ from which the target can be covered is determined. S represents the neighborhood.
- We search for the best sequence of visits to the regions. *Combinatorial optimization*
- The PRM is utilized to construct the planning roadmap (a graph).
PRM – Probabilistic Roadmap Method – sampling-based motion planner, see lecture 8.
- The problem can be formulated as the **Traveling Salesman Problem with Neighborhoods**, as it is not necessary to visit exactly a single location to capture the area of interest.



Inspection Planning – “Continuous Sensing”

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**.

Given a map of the environment \mathcal{W} determine the shortest, closed, and collision-free path, from which the whole environment is covered by an omnidirectional sensor with the radius ρ .

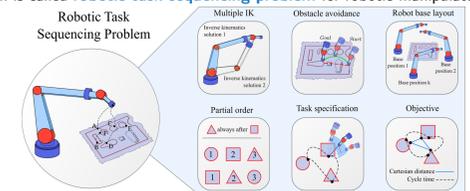


Faigl, J.: *Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.

Inspection Planning Multi-goal Planning

Multi-Goal Planning

- Having a **set of locations** to be visited, determine the cost-efficient path to visit them.
 - Locations where a robotic arm or mobile robot performs some task. *The operation can be repeated-closed path.*
- The problem is called **robotic task sequencing problem** for robotic manipulators.



Alatartsev, S., Stellmacher, S., Ortmeier, F. (2015): *Robotic Task Sequencing Problem: A Survey*. Journal of Intelligent & Robotic Systems.

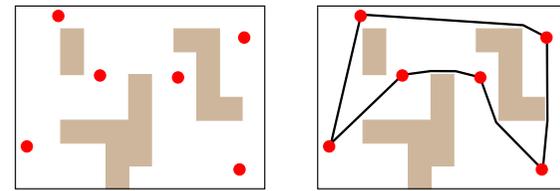
- The problem is also called **Multi-goal Path Planning (MTP)** problem or **Multi-goal Planning (MGP)**.

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Inspection Planning Multi-goal Planning

Multi-Goal Path Planning (MTP)

- Multi-goal planning problem is a problem how to visit the given set of locations.
- It consists of **point-to-point path planning** on how to reach one location from another.
- The **challenge** is to determine the optimal sequence of the visits to the locations w.r.t. cost-efficient path to visit all the given locations.



- Determination the sequence of visits is a **combinatorial optimization problem** that can be formulated as the **Traveling Salesman Problem (TSP)**.

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Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph $G(V, E)$, where V denotes a set of locations (cities) and E represents edges connecting two cities with the associated travel cost c (distance), i.e., for each $v_i, v_j \in V$ there is an edge $e_{ij} \in E$, $e_{ij} = (v_i, v_j)$ with the cost c_{ij} .
- If the associated cost of the edge (v_i, v_j) is the Euclidean distance $c_{ij} = |(v_i, v_j)|$, the problem is called the **Euclidean TSP (ETSP)**.
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

William J. Cook (2012) – *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation.*

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Traveling Salesman Problem (TSP)

- Let S be a set of n sensor locations $S = \{s_1, \dots, s_n\}$, $s_i \in \mathbb{R}^2$ and $c(s_i, s_j)$ is a cost of travel from s_i to s_j
- Traveling Salesman Problem (TSP)** is a problem to determine a closed tour visiting each $s \in S$ such that the total tour length is minimal.
 - We are searching for the optimal **sequence of visits** $\Sigma = (\sigma_1, \dots, \sigma_n)$ such that

$$\text{minimize } L = \left(\sum_{i=1}^{n-1} c(s_{\sigma_i}, s_{\sigma_{i+1}}) \right) + c(s_{\sigma_n}, s_{\sigma_1}) \quad (1)$$

subject to $\Sigma = (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n, \sigma_i \neq \sigma_j \text{ for } i \neq j.$

- The TSP can be considered on a graph $G(V, E)$ where the set of vertices V represents sensor locations S and E are edges connecting the nodes with the cost $c(s_i, s_j)$.
- For simplicity we can consider $c(s_i, s_j)$ to be Euclidean distance; otherwise, we also need to address the path/motion planning problem. **Euclidean TSP**
- If $c(s_i, s_j) \neq c(s_j, s_i)$ it is the **Asymmetric TSP**.
- The TSP is known to be NP-hard unless P=NP.

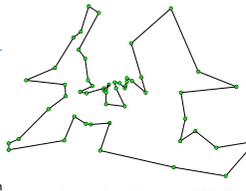
Traveling vs Travelling – <http://www.math.uwaterloo.ca/tsp/history/travelling.html>

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Existing Approaches to the TSP

- Exact solutions
 - Branch&Bound, Branch&Cut, and Integer Linear Programming (ILP).
 - Concorde-<http://www.math.uwaterloo.ca/tsp/concorde.html>
- Approximation algorithms
 - Minimum Spanning Tree (MST) heuristic with $L \leq 2L_{opt}$.
 - Christofides's algorithm with $L \leq \frac{3}{2}L_{opt}$.
- Heuristic algorithms
 - Constructive heuristic – Nearest Neighborhood (NN) algorithm
 - 2-Opt – local search algorithm proposed by Croes 1958
 - LKH – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998). <http://www.akira.ruc.dk/~keld/research/LKH/>
- Combinatorial meta-heuristics
 - Variable Neighborhood Search (VNS)
 - Greedy Randomized Adaptive Search Procedures (GRASP)
- Soft-computing techniques, evolutionary methods, and **unsupervised learning**



Problem Berlin52 from the TSPLIB

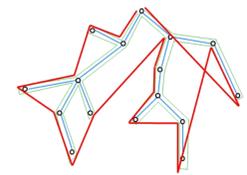
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MST-based Approximation Algorithm to the TSP

- Minimum Spanning Tree heuristic

- Compute the MST (denoted T) of the input graph G .
- Construct a graph H by doubling every edge of T .
- Shortcut repeated occurrences of a vertex in the tour.



- For the triangle inequality, the length of such a tour L is

$$L \leq 2L_{optimal}$$

where $L_{optimal}$ is the cost of the optimal solution of the TSP.

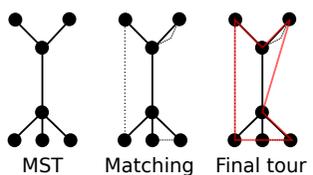
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Christofides's Algorithm to the TSP

- Christofides's algorithm

- Compute the MST of the input graph G .
- Compute the minimal matching on the odd-degree vertices.
- Shortcut a traversal of the resulting Eulerian graph.



MST Matching Final tour

- For the triangle inequality, the length of such a tour L is

$$L \leq \frac{3}{2}L_{optimal}$$

where $L_{optimal}$ is the cost of the optimal solution of the TSP.

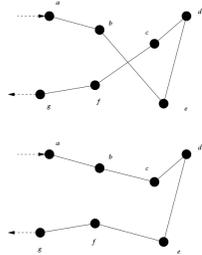
Length of the MST is $\leq L_{optimal}$
Sum of lengths of the edges in the matching $\leq \frac{1}{2}L_{optimal}$

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2-Opt Heuristic

- Use a construction heuristic to create an initial route
 - NN algorithm, cheapest insertion, farther insertion
- Repeat until no improvement is made
 - Determine swapping that can shorten the tour (i, j) for $1 \leq i \leq n$ and $i+1 \leq j \leq n$
 - route[0] to route[i-1];
 - route[i] to route[j] in reverse order;
 - route[j] to route[end];
 - Determine length of the route;
 - Update the current route if the length is shorter than the existing solution.



Croes, G.A.: *A method for solving traveling salesman problems*, Operations Research 6:791–812. 1958.

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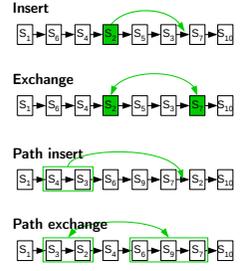
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Overview of the Variable Neighborhood Search (VNS) for the TSP

- The **Variable Neighborhood Search (VNS)** is a metaheuristic for solving combinatorial optimization and global optimization problems by searching distant neighborhoods of the current **incumbent solution** using **shake** and **local search** procedures.

Mladenović and Hansen, 1997

- Shake** explores the neighborhood of the current solution to escape from a local minima using operators
 - Insert – moves one element;
 - Exchange – exchanges two elements.
- Local search** improves the solution by
 - Path insert – moves a subsequence;
 - Path exchange – exchanges two subsequences.



Insert
Exchange
Path insert
Path exchange

```

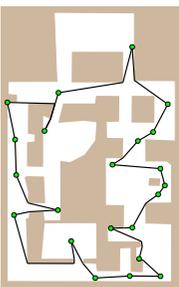
Algorithm 1: VNS-based Solver to the TSP
Input: S – Set of the target locations to be visited.
Output: Σ – Found sequence of visits to locations S.
Σ* ← Initial sequence found by cheapest insertion
while terminal condition is not met do
    Σ' ← shake(Σ*)
    for n=1 to times do
        Σ'' ← localSearch(Σ')
        if Σ'' is 'better' than Σ' then
            Σ' ← Σ'' // Select Σ'' instead of Σ'
    if Σ' is 'better' than Σ* then
        Σ* ← Σ' // Replace the incumbent sequence.
return Σ*
    
```

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Multi-Goal Path Planning (MTP) Problem

- MTP problem is a robotic variant of the TSP with the edge costs as the length of the shortest path connecting the locations.
- Variants of the robotic TSP includes additional constraints arising from limitations of real robotic systems such as
 - obstacles, curvature-constraints, sensing range, location precision.
- For n locations, we need to compute up to n^2 shortest paths.
- Having a **roadmap** (graph) representing C_{free} , the paths can be found in the graph (roadmap), from which the $G(V, E)$ for the TSP can be constructed. *Visibility graph as a roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots.*
- We can determine the roadmap using randomized sampling-based motion planning techniques. See lecture 8.

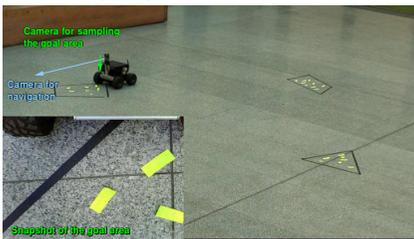


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Multi-goal Path Planning with Goal Regions

- It may be sufficient to visit a goal region instead of the particular point location.



Not only a **sequence** of goals visit has to be determined, but also an **appropriate location** at each region has to be found.

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods (TSPN)**.

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Traveling Salesman Problem with Neighborhoods

Given a set of n regions (neighbourhoods), what is the shortest closed path that visits each region.

- The problem is NP-hard and APX-hard, it cannot be approximated to within factor $2 - \epsilon$, where $\epsilon > 0$. *Safra and Schwartz (2006) – Computational Complexity*
- Approximate algorithms exist for particular problem variants such as disjoint unit disk neighborhoods.
- TSPN provides a suitable problem formulation for planning various inspection and data collection missions.
- It enables to exploit non-zero sensing range, and thus find shortest (more cost-efficient) **data collection plans**.

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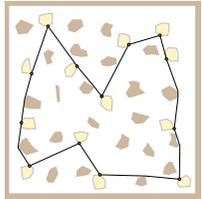
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Traveling Salesman Problem with Neighborhoods (TSPN)

- Instead visiting a particular location $s \in S$, $s \in \mathbb{R}^2$ as in the TSP, we request to visit a set of regions $R = \{r_1, \dots, r_n\}$, $r_i \subset \mathbb{R}^2$ to save travel cost.
- The TSP becomes the **TSP with Neighborhoods (TSPN)** where, in addition to the determination of the **sequence** Σ , we determine a suitable locations of visits $P = \{p_1, \dots, p_n\}$, $p_i \in r_i$.
- The problem is a combination of combinatorial optimization to determine Σ with **continuous optimization** to determine P .

$$\text{minimize}_{\Sigma, P} L = \left(\sum_{i=1}^{n-1} c(p_{\sigma_i}, p_{\sigma_{i+1}}) \right) + c(p_{\sigma_n}, p_{\sigma_1})$$

subject to $R = \{r_1, \dots, r_n\}, r_i \subset \mathbb{R}^2$
 $P = \{p_1, \dots, p_n\}, p_i \in r_i$
 $\Sigma = (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n,$
 $\sigma_i \neq \sigma_j \text{ for } i \neq j$
 Foreach $r_i \in R$ there is $p_i \in r_i$.



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Approaches to the TSPN

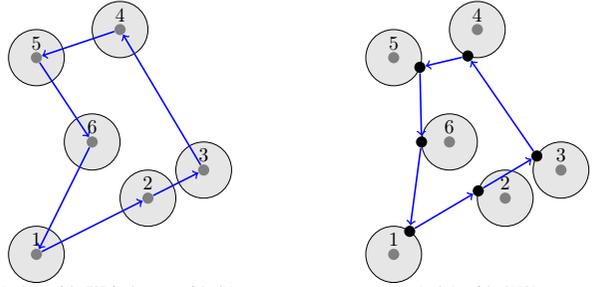
- A direct solution of the TSPN – approximation algorithms and heuristics E.g., using evolutionary techniques or unsupervised learning
- Euclidean TSPN with disk-shaped δ neighborhoods is called **Closed Enough TSP (CETSP)**.
 - Simplified variant with regions as disks with radius δ – remote sensing with the δ communication range.
- Decoupled approach**
 - Determine sequence of visits Σ independently on the locations P , e.g., as a solution of the TSP using centroids of the (convex) regions R .
 - For the sequence Σ determine the locations P to minimize the total tour length using
 - Touring polygon problem (TPP);
 - Sampling possible locations and use a forward search for finding the best locations;
 - Continuous optimization such as hill-climbing. E.g., Local Iterative Optimization (LIO), Vána & Faigl (IROS 2015)
- Sampling-based approaches**
 - For each region, sample possible locations of visits into a discrete set of locations for each region.
 - The problem can be then formulated as the **Generalized Traveling Salesman Problem (GTSP)**.

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Close Enough Traveling Salesman Problem (CETSP)

- Close Enough TSP (CETSP)** is a variant of the TSPN with disk shaped δ -neighborhoods.



A solution of the TSP for the centers of the disks

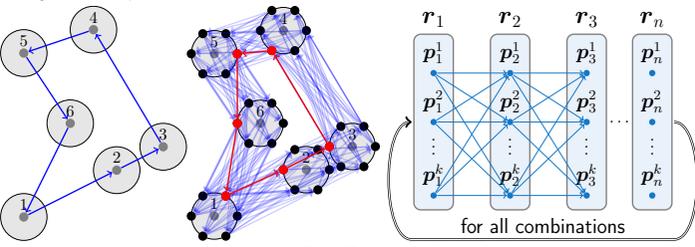
A solution of the CETSP

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Decoupled Sampling-based Solution of the TSPN / CETSP

- Decoupled** – Determine sequence of visits as a solution of the Euclidean TSP for the representatives of the regions R_i , e.g., using centroids.
- Sample each region (neighborhood) with k samples, e.g., $k = 6$.
- Construct graph and find the shortest tour in $\mathcal{O}(nk^3)$ for n regions and nk^2 edges in the sequence. For the closed path, we need to examine all k possible starting locations.

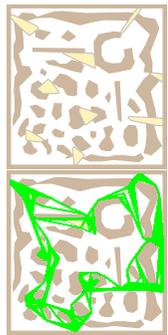


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Iterative Refinement in the Multi-goal Planning Problem with Regions

- Let the sequence of n polygon regions be $R = (r_1, \dots, r_n)$. Li, F., Klette, R.: Approximate algorithms for touring a sequence of polygons. 2008
- Sampling regions into a discrete set of points and determine all shortest paths between each sampled points in the sequence of visits to the regions. E.g., using visibility graph
- Initialization:** Construct an initial touring polygons path using a sampled point of each region. Let the path be defined by $P = (p_1, p_2, \dots, p_n)$, where $p_i \in r_i$ and $L(P)$ be the length of the shortest path induced by P .
- Refinement:** For $i = 1, 2, \dots, n$:
 - Find p_i^* $\in r_i$ minimizing the length of the path $d(p_{i-1}, p_i^*) + d(p_i^*, p_{i+1})$, where $d(p_k, p_l)$ is the path length from p_k to p_l , $p_0 = p_n$, and $p_{n+1} = p_1$.
 - If the total length of the current path over point p_i^* is shorter than over p_i , replace the point p_i by p_i^* .
- Compute the path length L_{new} using the refined points.
- Termination condition:** If $L_{new} - L < \epsilon$ Stop the refinement. Otherwise $L \leftarrow L_{new}$ and go to Step 3.
- Final path construction:** Use the last points and construct the path using the shortest paths among obstacles between two consecutive points. On-line sampling during the iterations – Local Iterative Optimization (LIO), Vána & Faigl (IROS 2015).



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Unsupervised Learning for Multi-goal Planning

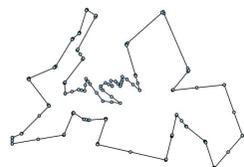
Part II

Part 2 – Unsupervised Learning for Multi-goal Planning

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Unsupervised Learning based Solution of the TSP

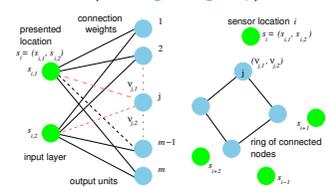
- Iterative learning procedure where neurons (nodes) adapt to the target locations.
- Based on self-organizing map by T. Kohonen.
- Deployed in robotic problems such as inspection and search-and-rescue planning.
- Evolved to **Growing Self-Organizing Array (GSOA)**.



Unsupervised Learning based Solution of the TSP

Kohonen's type of **unsupervised** two-layered neural network (**Self-Organizing Map**)

- Neurons' **weights** represent **nodes** $\mathcal{N} = \{\nu_1, \dots, \nu_m\}$ in a plane (input space \mathbb{R}^2).
- Nodes are organized into a **ring** that evolved in the output space \mathbb{R}^2 .
- Target locations $S = \{s_1, \dots, s_n\}$ are presented to the network in a **random** order.
- Nodes **compete** to be winner according to their distance to the presented goal s



- The **winner** and its **neighbouring** nodes are adapted (**moved**) towards the target according to the neighbouring function $\nu^* \leftarrow \mu f(\sigma, d)(\nu - s)$
- Best matching unit ν to the presented prototype s is determined according to the distance function $|D(\nu, s)|$.

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < m/nr, \\ 0 & \text{otherwise,} \end{cases}$$

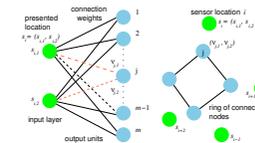
- For the Euclidean TSP, D is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning, D should correspond to the length of the shortest, collision-free path.

Unsupervised Learning based Solution of the TSP - Detail

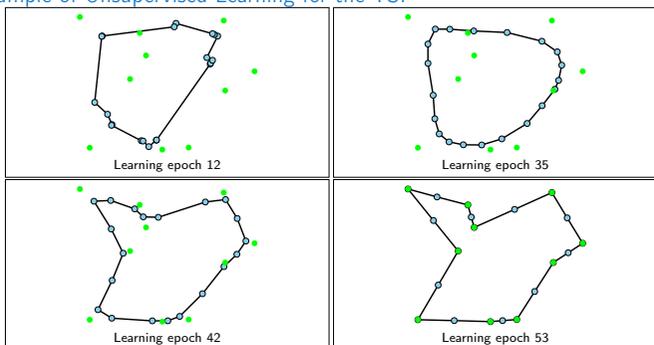
- Target (sensor) locations $S = \{s_1, \dots, s_n\}$, $s_i \in \mathbb{R}^2$; Neurons $\mathcal{N} = \{\nu_1, \dots, \nu_m\}$, $\nu_i \in \mathbb{R}^2$, $m = 2.5n$.
 - Learning gain σ ; epoch counter i ; gain decreasing rate $\alpha = 0.1$; learning rate $\mu = 0.6$.
- $\mathcal{N} \leftarrow$ init ring of neurons as a small ring around some $s_i \in S$, e.g., a circle with radius 0.5.
 - $i \leftarrow 0$; $\sigma \leftarrow 12.41n + 0.06$;
 - $i \leftarrow 0$ // clear inhibited neurons
 - foreach** $s \in \Pi(S)$ (a permutation of S)
 - $\nu^* \leftarrow \text{argmin}_{\nu \in \mathcal{N} \setminus I} \|\nu - s\|$
 - foreach** ν in d neighborhood of ν^*

$$\nu \leftarrow \nu + \mu f(\sigma, d)(s - \nu)$$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < 0.2m, \\ 0 & \text{otherwise,} \end{cases}$$
 - $I \leftarrow I \cup \{\nu^*\}$ // inhibit the winner
 - $\sigma \leftarrow (1 - \alpha)\sigma$; $i \leftarrow i + 1$;
 - If (termination condition is not satisfied) Goto Step 3;**
 - Maximal number of learning epochs $i \leq i_{max}$, e.g., $i_{max} = 120$.
 - Winner neurons are negligibly close to sensor locations, e.g., less than 0.001.



Example of Unsupervised Learning for the TSP



Unsupervised Learning for the Multi-Goal Path Planning

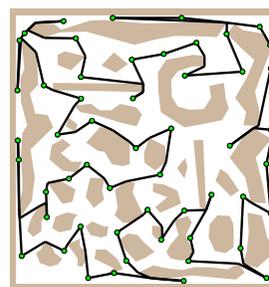
- Unsupervised learning procedure for the **Multi-goal Path Planning (MTP)** problem a robotic variant of the Traveling Salesman Problem (TSP).

Algorithm 2: SOM-based MTP solver

```

 $\mathcal{N} \leftarrow$  initialization( $\nu_1, \dots, \nu_m$ );
repeat
  error  $\leftarrow$  0;
  foreach  $g \in \Pi(S)$  do
     $\nu^* \leftarrow$ 
    selectWinner argmin $_{\nu \in \mathcal{N}} |S(g, \nu)|$ ;
    adapt( $S(g, \nu), \mu f(\sigma, d)|S(g, \nu)|$ );
    error  $\leftarrow$  max{error,  $|S(g, \nu^*)|$ };
   $\sigma \leftarrow (1 - \alpha)\sigma$ ;
until error  $\leq$   $\delta$ ;
    
```

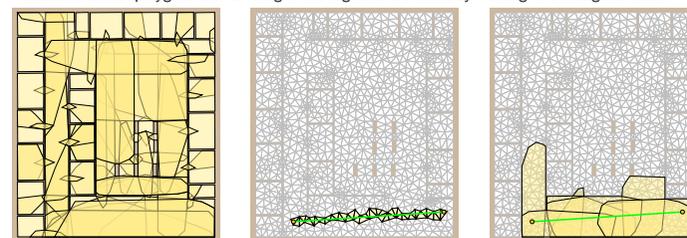
- For multi-goal path planning – the **selectWinner** and **adapt** procedures are based on the solution of the **path planning problem**.



SOM for the TSP in the Watchman Route Problem – Inspection Planning

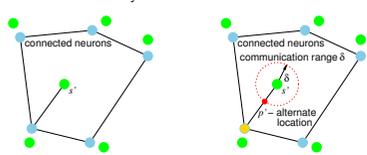
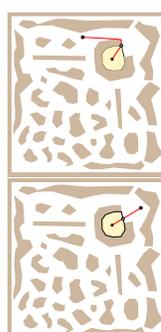
During the unsupervised learning, we can compute **coverage** of \mathcal{W} from the current **ring** (solution represented by the neurons) and **adapt** the network **towards uncovered parts** of \mathcal{W} .

- Convex cover set of \mathcal{W} created on top of a triangular mesh.
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh.



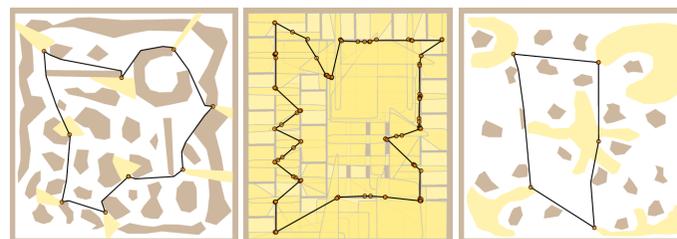
Unsupervised Learning for the TSPN

- A suitable location of the region can be sampled during the winner selection.
- We can use the centroid of the region for the shortest path computation from ν to the region r presented to the network.
- Then, an intersection point of the path with the region can be used as an alternate location.
- For the Euclidean TSPN with disk-shaped δ neighborhoods, we can compute the alternate location directly from the Euclidean distance.



SOM for the Traveling Salesman Problem with Neighborhoods (TSPN)

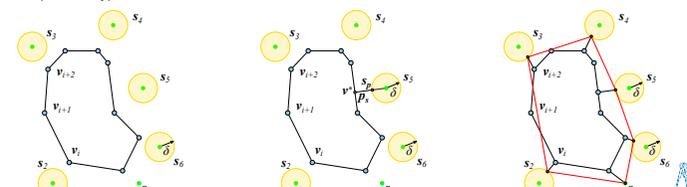
- Unsupervised learning of the SOM for the TSP allows to generalize the adaptation procedure to the TSPN.
- It also provides solutions for non-convex regions, overlapping regions, and coverage problems.



Polygonal Goals $n=9, T=0.32$ s
Convex Cover Set $n=106, T=5.1$ s
Non-Convex Goals $n=5, T=0.1$ s

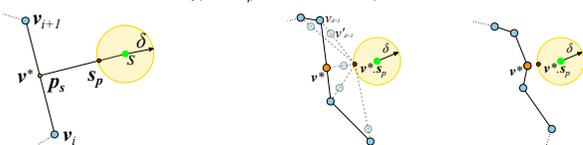
Growing Self-Organizing Array (GSOA)

- Growing Self-Organizing Array (GSOA)** is generalization of the unsupervised learning to routing problems motivated by data collection planning, i.e., routing with neighborhoods such as the **Close Enough TSP**.
- The GSOA is an array of nodes $\mathcal{N} = \{\nu_1, \dots, \nu_m\}$ that evolves in the problem space using unsupervised learning.
- The array adapts to each $s \in S$ (in a random order) and for each s a **new winner node** ν^* is determined together with the corresponding s_p , such that $\|\nu^* - s\| \leq \delta(s)$. It adaptively adjusts the number of nodes.
- The winner and its neighborhoods are adapted (moved) towards s_p .
- After the adaptation to all $s \in S$, each s has its ν and s_p , and the array defines the sequence Σ and the requested waypoints P .



GSOA – Winner Selection and Its Adaptation

- Selecting winner node v^* for s and its waypoint s_p
- Winner adaptation

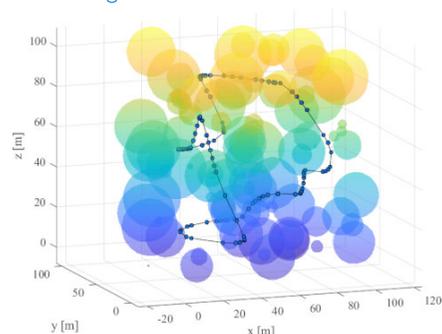


- For each $s \in S$, we create new node v^* , and therefore, all not winning nodes are removed after processing all locations in S (one learning epoch) to balance the number of nodes in the GSOA.
- After **each learning epoch**, the **GSOA encodes a feasible solution of the CETSP**.
- The power of adaptation is decreasing using a cooling schedule after each learning epoch.
- The GSOA converges to a stable solution in tens of epochs. Number of epochs can be set.

Faigl, J. (2018): GSOA: Growing Self-Organizing Array - Unsupervised learning for the Close-Enough Traveling Salesman Problem and other routing problems. Neurocomputing 312: 120-134 (2018).

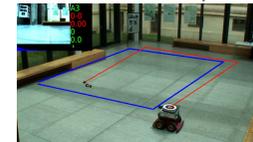


GSOA Evolution in solving the 3D CETSP



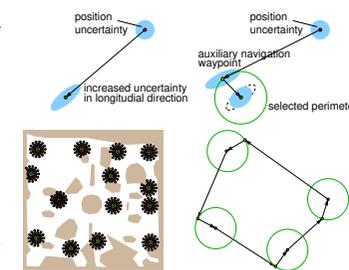
Example – TSPN for Planning with Localization Uncertainty

- Teach-and-repeat autonomous navigation using vision-based bearing corrections that are more precise than estimation of the traveled distance based on odometry measurements.



Krajník, T., Faigl, J., Vonásek, V., Košnar, K., Kulich, M., and Přečil, L.: Simple yet stable bearing-only navigation, Journal of Field Robotics, 27(5):511-533, 2010.

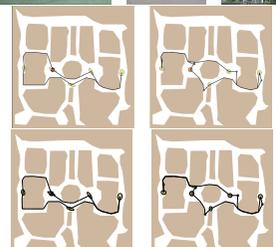
- The localization uncertainty can be decreased by visiting auxiliary navigation waypoints prior the target locations.
- It can be formulated as a variant of the TSPN with auxiliary navigation waypoints.
- The adaptation procedure is modified to select the auxiliary navigation waypoint to decrease the expected localization error at the target locations.



Faigl, J., Krajník, T., Vonásek, V., and Přečil, L.: On localization uncertainty in an autonomous inspection, IEEE International Conference on Robotics and Automation (ICRA), 2012, pp. 1119-1124.

Example – Results on the TSPN for Planning with Localization Uncertainty

- Deployment of the method in indoor and outdoor environment with ground mobile robots and aerial vehicle in indoor environment.
- In the indoor with the small MMP5 robot, the error decreased from 16.6 cm → 12.8 cm.
- In the outdoor with the P3AT robot, the real overall error at the goals decreased from 0.89 m → 0.58 m (about 35%).
- Deployment with a small aerial vehicle the Parrot AR.Drone, the success of the locations' visits improved from 83% to 95%.



TSP: L=184 m, E_{avg}=0.57 m TSPN: L=202 m, E_{avg}=0.35 m

Summary of the Lecture



Topics Discussed

- Robotic information gathering in inspection missions
- Inspection planning and multi-goal path planning - coverage planning
- Multi-goal path planning (MTP)
 - Robotic Traveling Salesman Problem (TSP)
 - Traveling Salesman Problem with Neighborhoods (TSPN) and Close Enough Traveling Salesman Problem (CETSP)
 - Decoupled and Sampling-based approaches
 - TSP can be solved by efficient heuristics such as LKH
 - Optimal, approximation, and heuristics solutions
 - Generalized TSP (GTSP)
- Next: Data collection planning

