Game theory - lab 3

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Overview

1. Area patrol
2. Illuminating Orthogonal Polygons
3. Clearing polygons
4. Double Oracle

Given following patrolling problem, remove unnecessary nodes and edges from the graph.

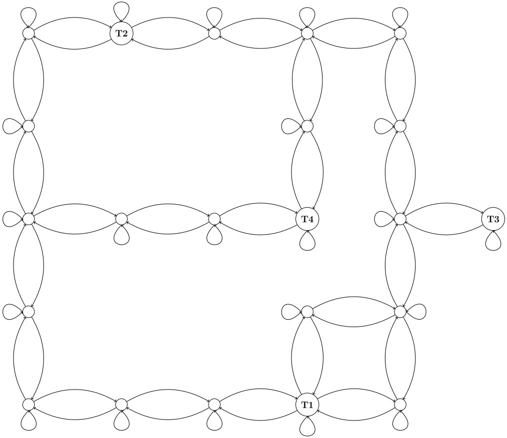
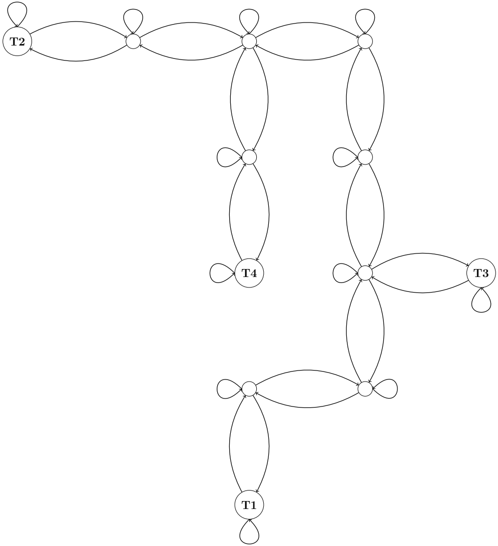


Figure: Given patrolling problem.

Example of reduced graph



Prove the following theorem

Theorem

$\lfloor \frac{n}{4} \rfloor$ *stationary guards are always sufficient and occasionally necessary to illuminate a orthogonal polygon with n vertices.*

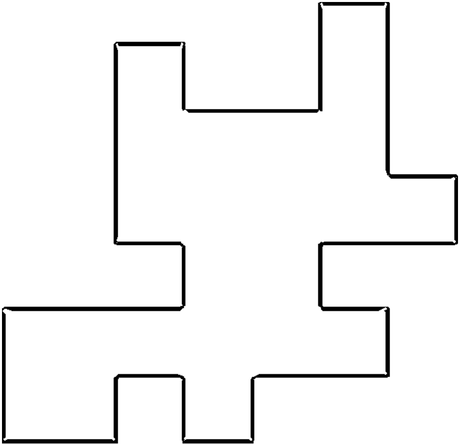


Figure: Orthogonal polygon with 24 vertices

First part of the proof

Theorem

Any orthogonal polygon is convex quadrilaterizable.

Proof.

Showing that every orthogonal polygon can be split to smaller polygons eventually resulting in convex quadrilaterals. Full proof here on page 56. □

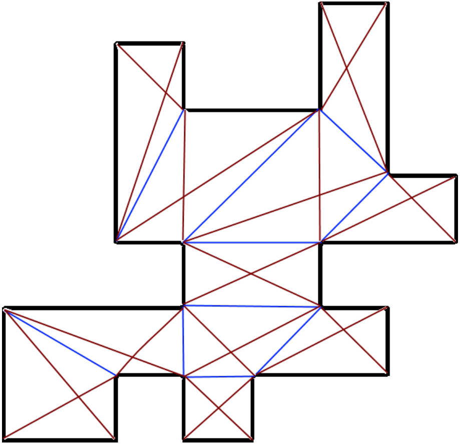


Figure: Quadrilateralization of the polygon (blue lines) and diagonals of resulting quadrilaterals (red)

End of the proof

Theorem

Quadrilaterized polygon with diagonal edges in quadrilaterals forms a 4-colorable graph.

Proof.

Dual graph Q is clearly a tree. When Q is one quadrilateral it is 4-colorable. By induction show that Q with added leaf quadrilateral is still 4-colorable. □

By the construction of the graph each quadrilateral has all four colors. Therefore, placing guards to one color illuminates the polygon. Selecting color with the fewest vertices gives the result.

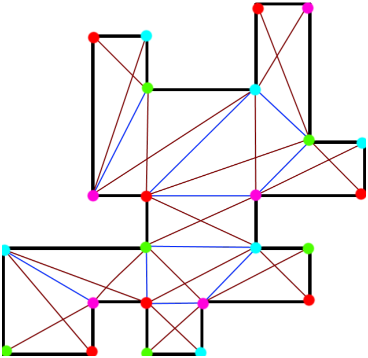


Figure: 4-colored polygon.

Necessary

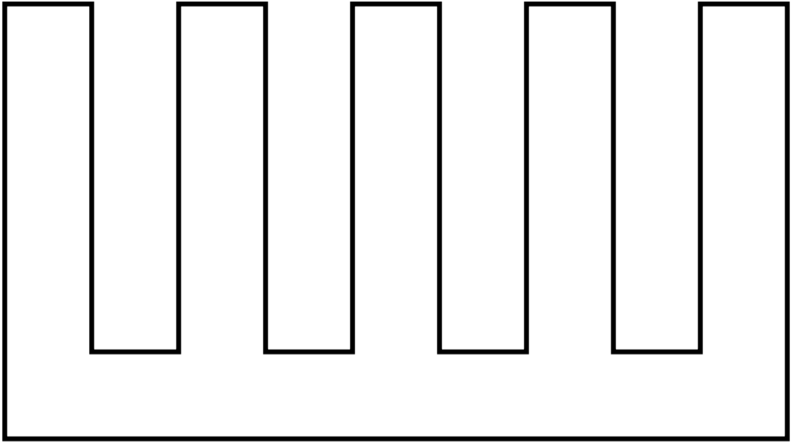


Figure: Class of polygons where $\lfloor \frac{n}{4} \rfloor$ guards is necessary.

Gap edges

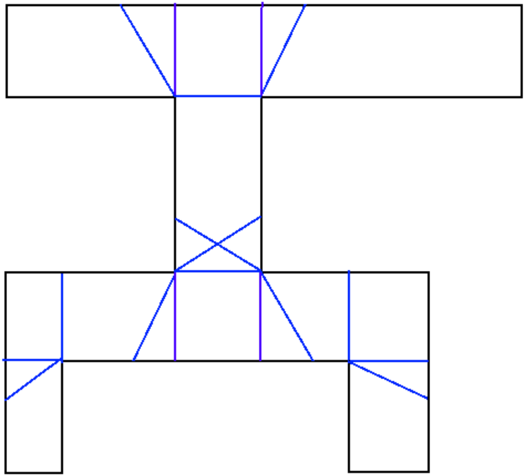


Figure: Gap edges in the example polygon.

Gap edges and dual graph

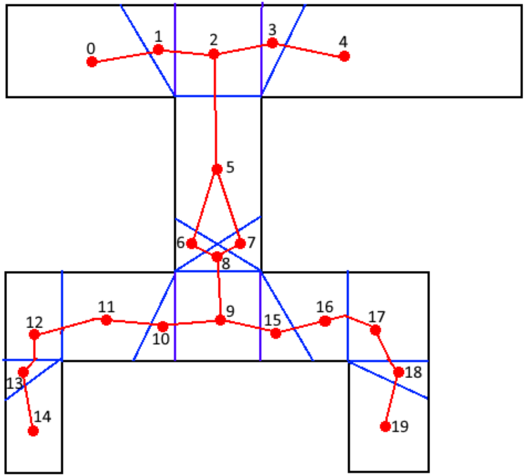


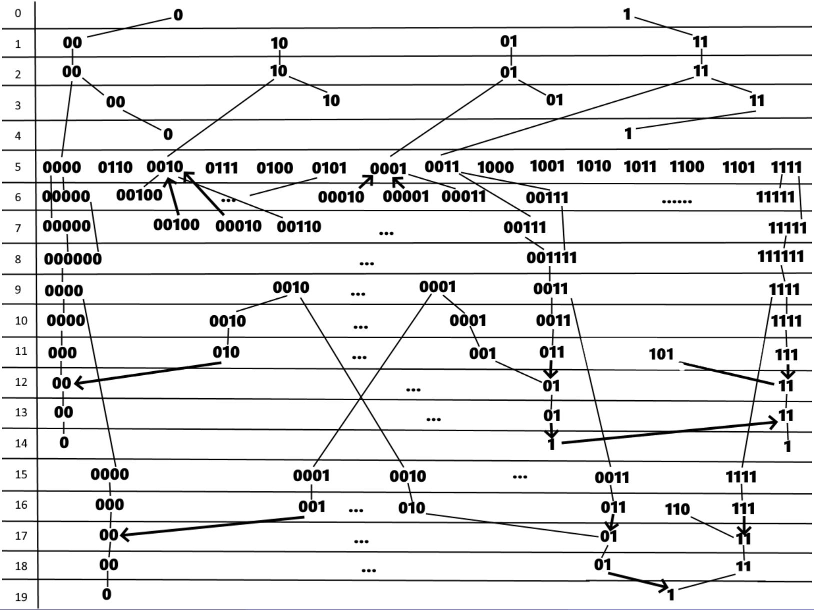
Figure: Gap edges and the dual graph corresponding to them.

Gap edge transitions

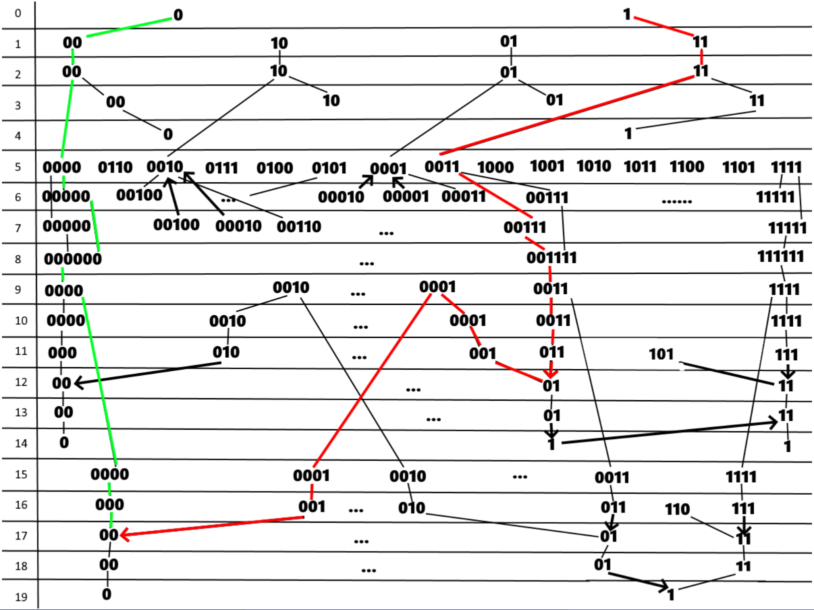
Reminder of transitions in Gap edge algorithm

- When new gap edge appears label it Cleared
- When gap edge splits to two label them according to the original edge label
- When two gap edges join and at least one of them is Contaminated the resulting edge is contaminated, otherwise it is Cleared

Gap edges final graph



Gap edges final solution



Double Oracle reminder

- Used to find Nash Equilibrium of zero-sum two-player game (in our example normal form game)
- Pick randomly one action for each players and form restricted subgame using those actions
- Players are switching during the iterations
- In each iteration find the best response to current strategy for the current player and add it to the restricted subgame, then solve the subgame again
- When in subsequent iterations best responses for both players are already in the restricted subgame, stop

Double Oracle

Try Double Oracle algorithm on the following matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Player 1 selects W $\rightarrow A = 1, V = 0, W = 1$

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Player 2 selects D → solve matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Strategy for Player 2 $A = \frac{1}{8}, D = \frac{7}{8}$

	A	B	C	D	E	
V	-8	9	0	7	-6	5.125
W	6	9	6	5	6	5.125
X	1	-8	3	8	7	7.125
Y	5	2	6	-5	2	-3.75
Z	4	3	3	0	8	0.5

Double Oracle

Player 1 selects X $\rightarrow$ solve matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Strategy for Player 2 $A = \frac{3}{8}, D = \frac{5}{8}$
Strategy for Player 1 $V = 0, W = \frac{7}{8}, X = \frac{1}{8}$

	A	B	C	D	E	
V	-8	9	0	7	-6	1.375
W	6	9	6	5	6	5.375
X	1	-8	3	8	7	5.375
Y	5	2	6	-5	2	-1.25
Z	4	3	3	0	8	1.5
	$5\frac{3}{8}$	$6\frac{7}{8}$	$5\frac{5}{8}$	$5\frac{3}{8}$	$6\frac{1}{8}$	

The End