

Evolutionary Algorithms: Introduction

Jiří Kubalík

The Czech Institute of Informatics, Robotics and Cybernetics
CTU Prague



<http://cw.felk.cvut.cz/doku.php/courses/a0m33eoa/start>

Stochastic Universal Sampling

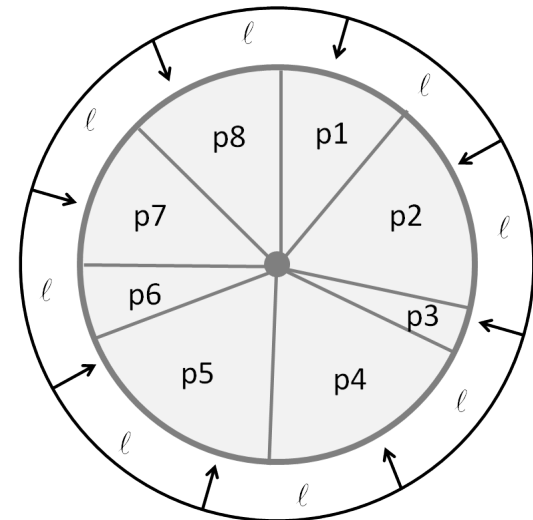
SUS ensures that the observed selection frequencies of each individual are in line with the expected frequencies:

- extra wheel, let's denote it a *pointer wheel*, with equidistantly distributed *PopSize* pointers

Ex.: If we are selecting 8 individuals, the pointer wheel will have 8 pointers distributed with $360/8 = 45$ degrees step size.

- works by making a single spin of the roulette wheel
- an arbitrary rotation of the pointer wheel determines a whole set of *PopSize* selected individuals
- every individual i receives a number of copies in the set of selected individuals from interval $(\lfloor \text{expectedFrequency}(i) \rfloor, \lceil \text{expectedFrequency}(i) \rceil)$

Ex.: If we have an individual that occupies 4.5% of the roulette wheel and we select 100 individuals, we would expect on average 4.5 copies for that individual to be selected. Then, the individual will be selected either four or five times. Neither more, nor less.

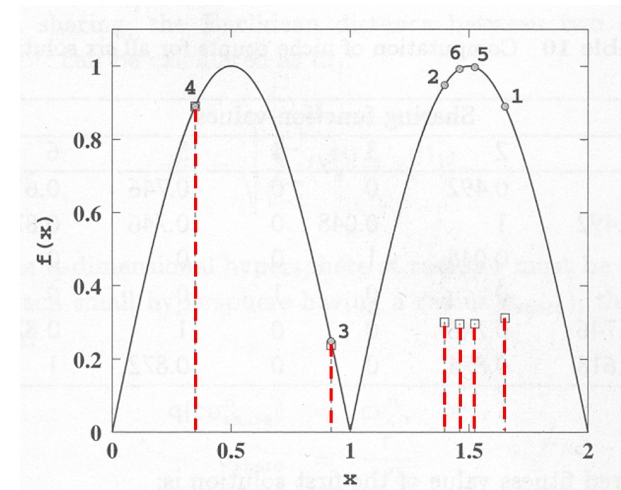


Fitness Sharing: Example

Bimodal function - six solutions and corresponding shared fitness functions

- $\sigma_{share} = 0.5, \alpha = 1.$

Sol. i	String	Decoded value	$x^{(i)}$	f_i	nc_i	f'_i
1	110100	52	1.651	0.890	2.856	0.312
2	101100	44	1.397	0.948	3.160	0.300
3	011101	29	0.921	0.246	1.048	0.235
4	001011	11	0.349	0.890	1.000	0.890
5	110000	48	1.524	0.997	3.364	0.296
6	101110	46	1.460	0.992	3.364	0.295



©Kalyanmoy Deb: Multi-Objective Optimization using Evolutionary Algorithms.

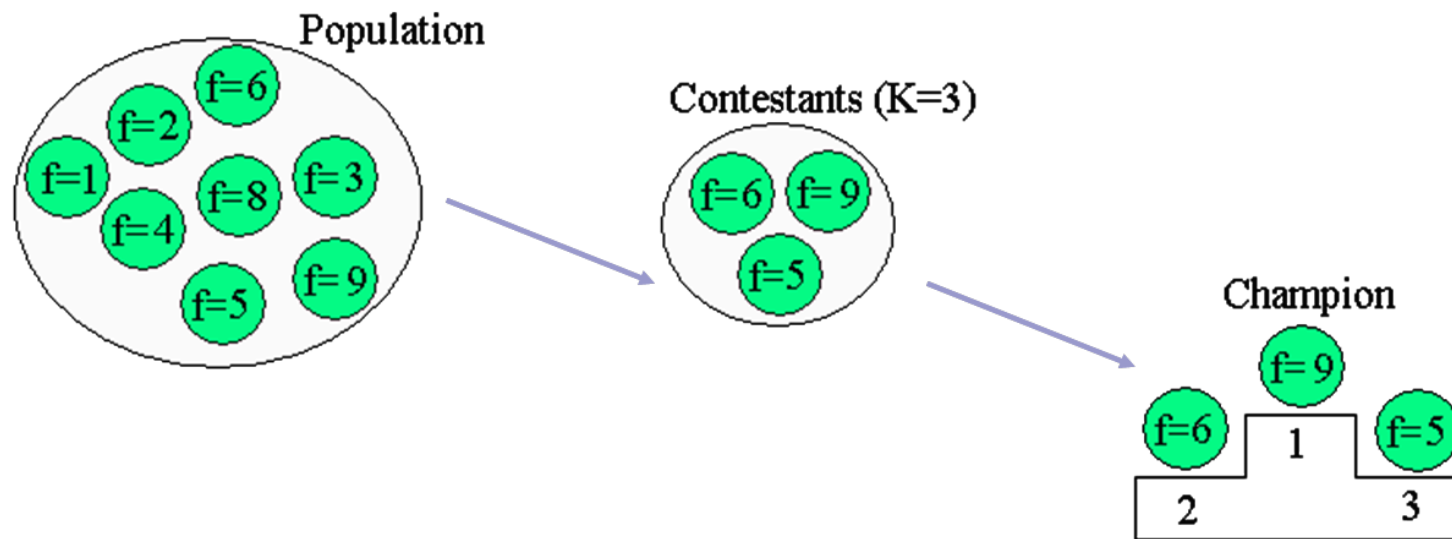
Let's calculate the shared fitness value of the first solution

- $d_{11} = 0.0, d_{12} = 0.254, d_{13} = 0.731, d_{14} = 1.302, d_{15} = 0.127, d_{16} = 0.191$
- $Sh(d_{11}) = 1, Sh(d_{12}) = 0.492, Sh(d_{13}) = 0, Sh(d_{14}) = 0, Sh(d_{15}) = 0.746, Sh(d_{16}) = 0.618.$
- $nc_1 = 1 + 0.492 + 0 + 0 + 0.746 + 0.618 = 2.856$
- $f'(1) = f(1)/nc_1 = 0.890/2.856 = 0.312$

Tournament Selection

Tournament selection – the best out of n randomly chosen individuals is selected.

- n is the size of the tournament,
- rank-based method – absolute differences among individuals do not count.



Job Shop Scheduling Problem

:: **Representation:** Pair-wise relative order of jobs on every machine

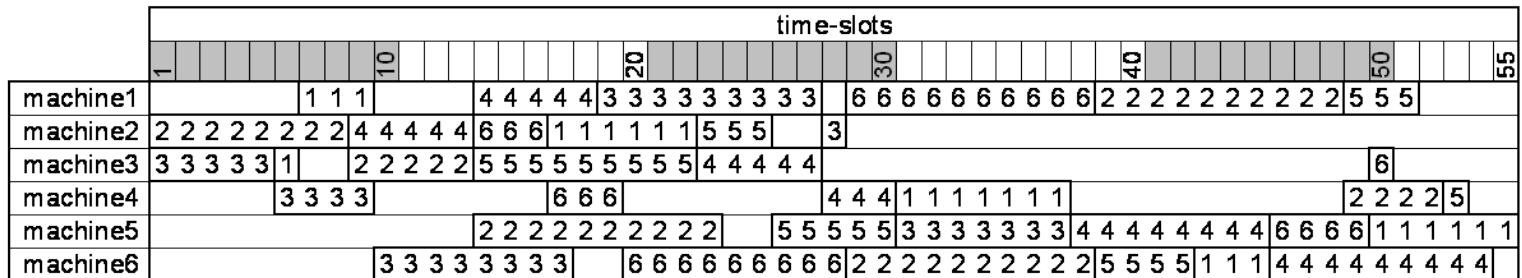
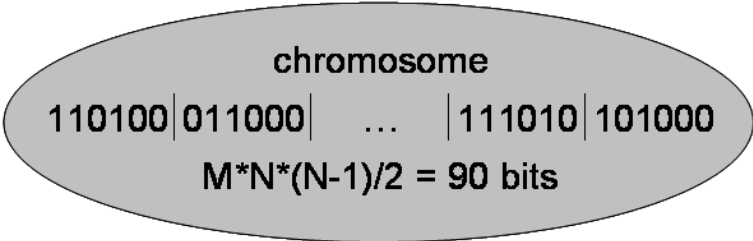
	machine sequence					
job1	3	1	2	4	6	5
job2	2	3	5	6	1	4
job3	3	4	6	1	2	5
job4	2	1	3	4	5	6
job5	3	2	5	6	1	4
job6	2	4	6	1	5	3

job1<job2: 1 1 0 1 0 0
 job1<job3: 0 1 1 0 0 0
 job1<job4: 1 1 0 0 1 0
 job1<job5: 1 1 1 1 0 0
 job1<job6: 1 1 0 0 0 0
 job2<job3: 1 0 1 0 0 0
 job2<job4: 1 1 1 1 0 0
 job2<job5: 1 1 1 1 1 1
 job2<job6: 1 1 1 0 0 0
 job3<job4: 1 1 1 0 0 1
 job3<job5: 1 1 1 1 0 0
 job3<job6: 1 1 1 1 0 1
 job4<job5: 1 1 0 1 0 0
 job4<job6: 1 1 1 0 1 0
 job5<job6: 1 0 1 0 0 0

	processing times					
job1	3	6	1	7	6	3
job2	10	8	5	4	10	10
job3	9	1	5	4	7	8
job4	5	5	5	3	8	9
job5	3	3	9	1	5	4
job6	10	3	1	3	4	9

	job sequence					
machine1	1	4	3	6	2	5
machine2	2	4	6	1	5	3
machine3	3	1	2	5	4	6
machine4	3	6	4	1	2	5
machine5	2	5	3	4	6	1
machine6	3	6	2	5	1	4

Machines: M=6, Jobs: N=6



Artificial Ant Problem

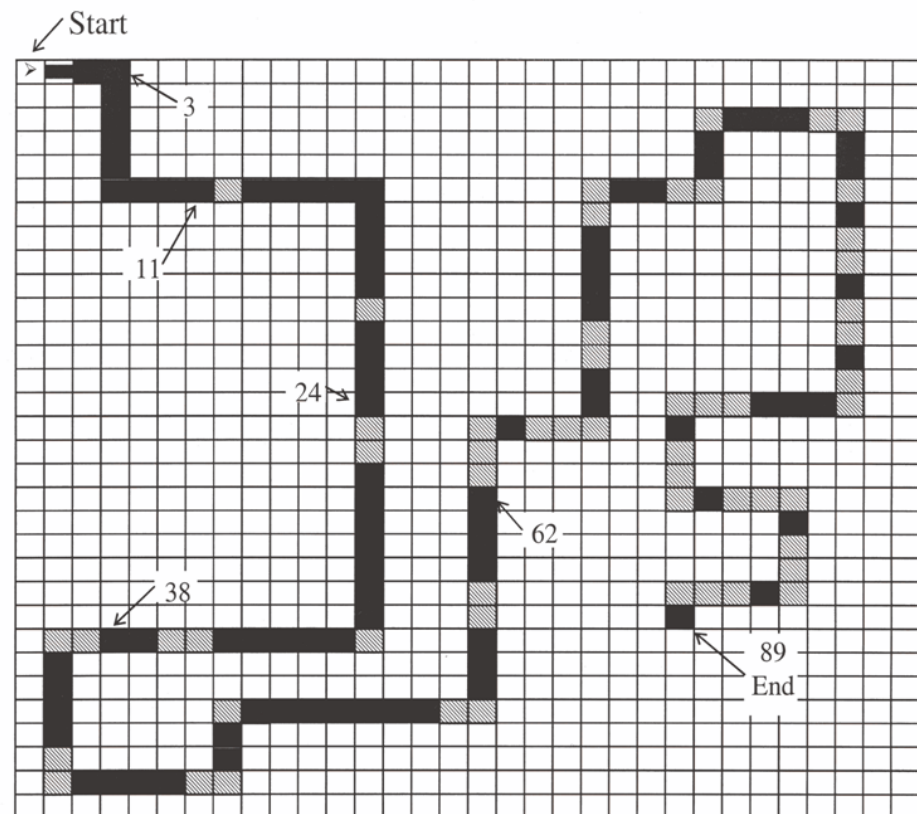
Santa Fe trail

- 32×32 grid with 89 food pieces.
- **Obstacles**
 - $1 \times, 2 \times$ strait,
 - $1 \times, 2 \times, 3 \times$ right/left.

Ant capabilities

- **detects** the food right in front of him in direction he faces.
- **actions** observable from outside
 - MOVE – makes a step and eats a food piece if there is some,
 - LEFT – turns left,
 - RIGHT – turns right,
 - NO-OP – no operation.

Goal is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).



Schema theory

Schema – a template, which defines set of solutions from the search space with certain specific similarities.

- consists of 0s, 1s (fixed values) and wildcard symbols * (any value),
- covers 2^r strings, where r is a number of * used in the schema.

Example: schema $S = \{11*0*\}$ covers strings 11000, 11001, 11100, and 11101

Schema properties

- **Defining length** $\delta(S)$ (compactness) – distance between first and last non-* in a schema (= number of positions where 1-point crossover can disrupt the schema).
- **Order** $o(S)$ (specificity) – a number of non-*'s (= number of positions where simple bit swapping mutation can disrupt the schema).
 - Chromosomes are order l schemata, where l is length of chromosome (in bits or loci).
 - Chromosomes are instances (or members) of lower-order schemata.
- **Fitness** $f(S)$ (quality) – average fitness computed over all covered strings.

Example: $S = \{**1*01*0**\}$: $\delta(S) = 5$, $o(S) = 4$

Schema Properties: Example

8-bit Count Ones problem – maximize a number of ones in 8-bit string.

string	fitness		string	fitness
00000000	0		11011111	7
00000001	1	...	10111111	7
00000010	1		01111111	7
00000100	1		11111111	8

Assume schema $S_a = \{1*1**10*\}$ vs. $S_b = \{*0*0****\}$:

- **defining length:** $\delta(S_a) = 7 - 1 = 6$, $\delta(S_b) = 4 - 2 = 2$
- **order:** $o(S_a) = 4$, $o(S_b) = 2$
- **fitness of S_a :** S_a covers 2^4 strings in total

1 string of fitness 3

4 string of fitness 4 $f(S_a) = (1 \cdot 3 + 4 \cdot 4 + 6 \cdot 5 + 4 \cdot 6 + 1 \cdot 7)/16$

6 string of fitness 5 $f(S_a) = 80/16 = 5$

4 string of fitness 6

1 string of fitness 7

fitness of S_b : $S_b = (1 \cdot 0 + 6 \cdot 1 + 15 \cdot 2 + 20 \cdot 3 + 15 \cdot 4 + 6 \cdot 5 + 1 \cdot 6)/2^6 = 192/64 = 3$

Question: How will be the fitness of $S = \{*0*1****\}$ compared to S_b ?

