## **Grammatical Evolution**

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http://cw.felk.cvut.cz/doku.php/courses/a0m33eoa/start

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- Grammatical Evolution
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  - Crossover operators
  - Automatically defined functions
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## **GP** and **Closure Problem:** Motivation Example

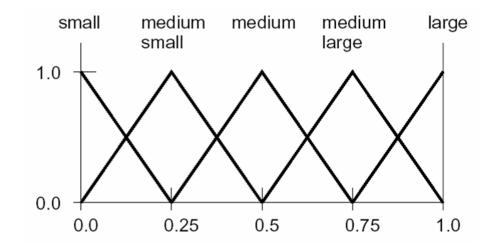
**Closure** - any non-terminal should be able to handle as an argument any data type and value returned from a terminal or non-terminal.

Fuzzy-rule based classifier consists of fuzzy if-then rules of type

```
IF(x1 \text{ is } medium) \text{ and } (x3 \text{ is } large) \text{ THEN } class = 1 \text{ with } cf = 0.73
```

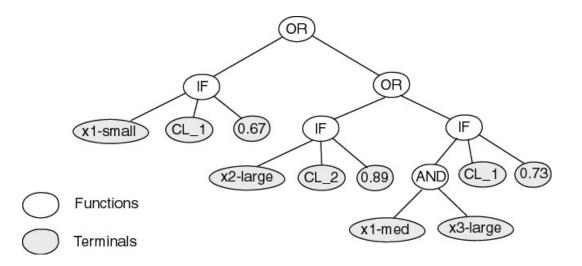
where

- Linguistic terms small, medium small, medium, medium large, large,
- Fuzzy membership functions approximate the confidence in that the crisp value is represented by the linguistic term.



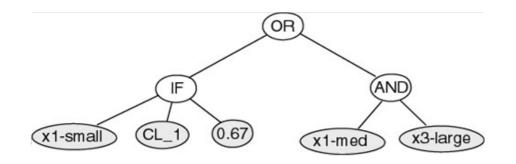
A syntactically correct tree representing a classifier as a disjunction of the three rules:

- IF(x1 is small) THEN class = 1 with cf = 0.67,
- IF(x2 is large) THEN class = 2 with cf = 0.89,
- IF(x1 is medium) and (x3 is large) THEN class = 1 with cf = 0.73.



### **GP** and **Closure Problem:** Motivation Example

Using simple subtree-swapping crossover or subtree-regeneration mutation an invalid tree that does not represent a syntactically correct rule base can be generated.

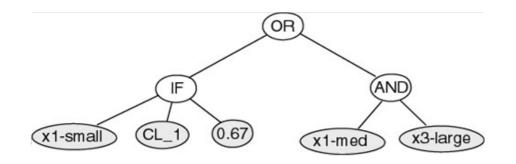


Obviously, this happens due to the fact that the **closure constraint does not hold here**.

Standard GP is not designed to handle a mixture of data types.

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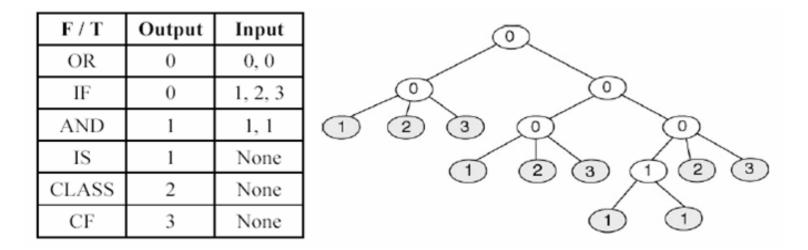
Obviously, this happens due to the fact that the **closure constraint does not hold here**.

#### Standard GP is not designed to handle a mixture of data types.

What can we do to get around the problem with the closure constraint?

**STGP** - defines syntax of evolved tree structures by specifying the data types of each argument of each non-terminal and the data types returned by each terminal and non-terminal.

- prevents generating illegal individuals,
- quite a big overhead  $\implies$  inefficient for manipulating large trees.



Any other elegant way to get around the closure constraint?

**Grammatical Evolution** (GE) – a grammatical-based GP system that can evolve complete programs in an arbitrary language using a variable-length binary/integer strings.

- The evolutionary process is performed on variable-length binary strings.
- A genotype-phenotype mapping process is used to generate programs (*expression trees*) in any language by using the binary strings to select production rules in a Backus-Naur form (BNF) grammar definition.
- By the use of the mapping between linear chromosomes and programs the "closure" problem is overcome, only valid programs are generated.

The syntactically correct programs are evaluated by a fitness function.

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A user can specify and modify the grammar, whilst ignoring the task of designing specific genetic search operators.

**Backus-Naur form** (BNF) is a notation for expressing the grammar of a language in the form of production rules.

BNF is represented by the tuple  $\{N, T, P, S\}$ , where

- T is a set of terminals items that can appear in the language (+, -, X, ...),
- N is a set of nonterminals items that can be further expanded into one or more terminals or nonterminals,
- P is a set of production rules that map the elements of N to T,
- S is a start symbol that is a member of N.

Remark: Do not mistake terminals/nonterminals used in GP for terminals/nonterminals used in GP!

$$N = \{expr, op, pre_{op}\} P: \langle expr \rangle ::= \langle expr \rangle \langle op \rangle \langle expr \rangle (0)$$

$$T = \{sin, +, -, /, *, X, 1.0, (,)\} P: \langle expr \rangle ::= \langle expr \rangle \langle op \rangle \langle expr \rangle (1)$$

$$S = \langle expr \rangle (2)$$

$$\langle op \rangle ::= + (0)$$

$$| - (1)$$

$$| / (2)$$

$$| * (3)$$

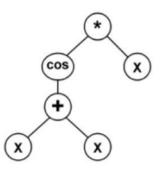
$$\langle pre-op \rangle ::= \cos (0)$$

$$\langle var \rangle ::= X (0)$$

$$| 1.0 (1)$$

Each nonterminal has one or more possible ways of expansion.

Example of a tree compliant with the BNF.



Variable-length binary chromosomes

#### 11011100|11110000|11011100|...|11100110

are transcribed to **codons** (a consecutive group of 8 bits encoding an integer number)

220 | 240 | 220 | ... | 102.

The **codons are used** in a mapping function **to select an appropriate production rule** from the BNF definition to expand a given nonterminal using the following mapping function

rule=(codon integer value) modulo (number of rules for the current nonterminal)

This implies that only syntactically correct programs can be generated!!!

## **Genotype-Phenotype Mapping Process: Modulo Rule**

Example: Assume that a nonterminal <op> is to be expanded, and the codon being read produces the integer 6.

There are four production rules to choose from

 ::= + (0)
$$| - (1)$$
 $| / (2)$ 
 $| * (3)$ 

Then

 $6 \mod 4 = 2$ 

would select rule (2).

Mapping finishes when all of the nonterminals have been expanded.

During the genotype-phenotype mapping process it is possible for individuals to run out of codons and in this case so called **wrapping** is used – the chromosome is being traversed multiple times, so the codons are reused several times.

Each time the same codon is expressed

- it will always generate the same integer value,
- but depending on the current nonterminal to which it is being applied, it may result in the selection of a different production rule.

The codon 240 can be read one time to expand the nonterminal <expr>, another time to expand the nonterminal <pre-op>, etc.

```
chromosome: 220|240|220|...|102.
```

A maximum number of wrapping events is specified – if an incomplete mapping occurs after the specified number of wrapping events, the individual is assigned the lowest possible fitness value.

 To reduce the number of invalid individuals in the population, a steady-state replacement is used.

```
:: Grammar in Backus-Naur Form :: Chromosome: 649358862
  N = \{expr, op, pre-op\}
  T = \{+, -, *, l, sin, cos, exp, log, X\}
  expr = starting symbol
  P:
   <expr> ::= <expr> <op> <expr> [0]
               <pre-op> <expr>
                                    [1]
                                    [2]
   <op>
                                    [0]
                                    [1]
                                    [2]
                                    [3]
   <pre-op>::= sin
                                    [0]
                                    [1]
                COS
```

exp log

[2]

[3]

## **Grammatical Evolution: Genetic Code Degeneracy**

**Neutral mutations** – mutations that have no effect on the phenotypic fitness of an individual.

• cause **diversity** within equally fit individuals in the search space.

Adaptive mutations – mutations that change the phenotype.

explore the solution space.

**Neutral theory of evolution** – most mutations driving the evolutionary process are neutral with respect to the phenotype.

**Degenerate genetic code** – a genetic code with redundancy.

 Each codon can represent a number of distinct integer values – many of these values can represent the same production rule.

Various genotypes can represent the same phenotype, thus facilitating the maintenance of genetic diversity within a population.

Example: Subtle changes in the genotype may have no effect on the phenotype.

A rule for operator selection:

<op> ::= + (0)

 $| - (1) \\ | / (2) \\ | * (3)$ 

If the current codon value were 8, then

8 modulo 4=2

would select rule (0); the same rule that would be selected by codon value 4, 12, 16, etc.

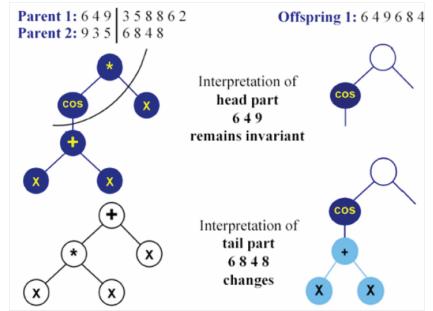
## **Grammatical Evolution: 1-point Crossover**

**Ripple effect** – a single crossover event can remove any number of subtrees to the right of the crossover point.

- It is more exploratory than the subtree crossover used in GP.
- It transmits on average half of the genetic material for each parent,
- It is equally recombinative regardless of the size of the individuals involved,

The subtree crossover exchanges less and less genetic material as the trees are growing.

 It is less likely to get trapped in a local optimum than the subtree crossover.



The head sequence of codons does not change its meaning, while the tale sequence may or may not change its interpretation (**the function of a gene depends on the genes that precede it**) implying a limited exploitation capability of the recombination operation.

Typically, the search is carried out by an EA. However, any search method with the ability to operate over variable-length binary strings could be employed.

- Grammatical Differential Evolution,
- Grammatical Swarm.

## **Grammatical Evolution for Symbolic Regression**

#### The grammar used

 $N = \{expr, op, pre_{op}\}$  P: <expr> ::= <expr><op><expr> (0) $T = \{sin, cos, exp, log,$ (<expr><op><expr>) (1)+, -, /, \*, X, 1.0, (,)<pre-op>(<expr>) (2) $S = \langle expr \rangle$ (3)<var> (0)<op> ::= + (1)(2)(3)\* (0)<pre-op> ::= sin (1)COS (2)exp (3)log (0)::= Х <var> 1.0 (1)

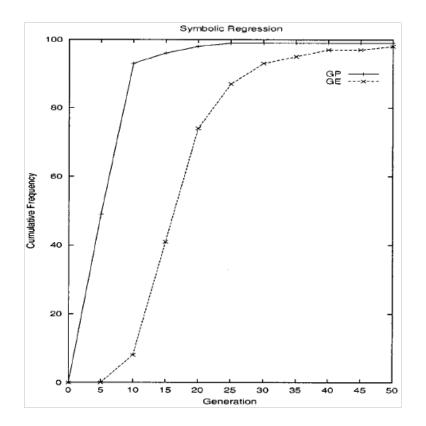
# **Grammatical Evolution for Symbolic Regression**

#### Experimental setup:

Objective :	Find a function of one independent verichle and
Objective :	Find a function of one independent variable and
	one dependent variable, in symbolic form
	that fits a given sample of 20 $(x_i, y_i)$
	data points, where the target function is the
	quartic polynomial $X^4 + X^3 + X^2 + X$
Terminal Operands:	X (the independent variable), 1.0
Terminal Operators	The binary operators $+, *, /, \text{ and } -$
	The unary operators Sin, Cos, Exp and Log
Fitness cases	The given sample of 20 data points in the
	interval $[-1, +1]$ i.e. $\{-1,9,8,76,72,68,64,$
	4,2, 0, .2, .4, .63, .72, .81, .90, .93, .96, .99, 1 }
Raw Fitness	The sum, taken over the 20 fitness cases,
	of the error
Standardised Fitness	Same as raw fitness
Wrapper	Standard productions to generate
	C functions
Parameters	Population Size $= 500$ , Termination when Generations $= 51$
	Prob. Mutation = $0.01$ , Prob. Crossover = $0.9$
	Prob. Duplication $= 0.01$

## **Grammatical Evolution for Symbolic Regression**

Results: GE compared to standard GP.



- GE successfully found the target function.
- GP slightly outperforms GE this might be attributed to more "careful" initialization of the initial population in the GP.

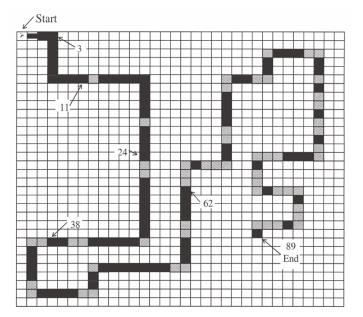
#### Ant capabilities

- detection of the food right in front of him in direction he faces.
- actions observable from outside
  - MOVE makes a step and eats a food piece if there is some,
  - LEFT turns left,
  - RIGHT turns right,
  - NOP no operation.

**Goal** is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

#### Santa Fe trail

- $32 \times 32$  toroidal grid with 89 food pieces.
- Obstacles:  $1 \times$ ,  $2 \times$  strait;  $1 \times$ ,  $2 \times$ ,  $3 \times$  right/left.



#### The grammar used

move()

#### Experimental setup:

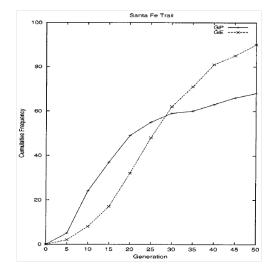
Objective :	Find a computer program to control an artificial ant so that it can
	find all 89 pieces of food located on the Santa Fe Trail.
Terminal Operators:	$left(), right(), move(), food_ahead()$
Terminal Operands:	None
Fitness cases	One fitness case
Raw Fitness	Number of pieces of food before the ant times out
	with 600 operations.
Standardised Fitness	Total number of pieces of food less the raw fitness.
Wrapper	None
Parameters	Population Size $= 500$ , Termination when Generations $= 51$
	Prob. Mutation = $0.01$ , Prob. Crossover = $0.9$
	Prob. Duplication $= 0.01$

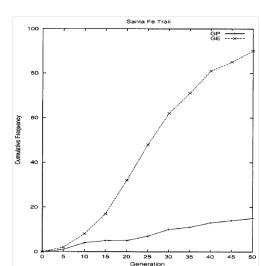
GE was successful at finding a solution to the Santa Fe trail.

Solutions have a form of a multiline code.

GE outperforms GP:

- The left side figure shows the performance of the GP using solution length and the number of food pieces eaten in the fitness.
- The right side figure shows the performance of the GP using just the number of food pieces eaten in the fitness measure.





```
move();
left();
if(food_ahead())
left();
else
right();
right();
if(food_ahead())
move();
else
left();
```

Each solution is executed in a loop until the number of time steps allowed is reached.

## **Grammatical Evolution and Automatically Defined Functions**

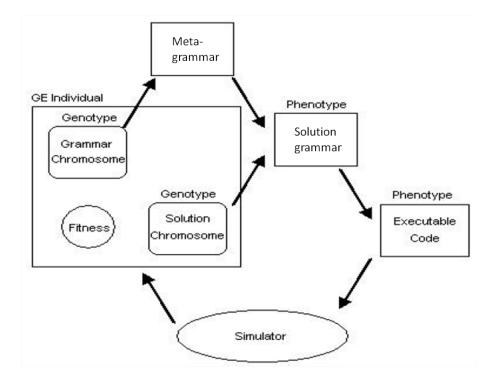
Three approaches illustrating the possibilities to adopt the principles of automatically capturing modularity from GP

- 1. Grammatical Evolution by Grammatical Evolution or meta-Grammar GE  $(GE)^2$ 
  - the input grammar is used to specify the construction of another syntactically correct grammar, which is then used in a mapping process to construct a solution.
- 2. GE grammar with the ability to define one ADF.
- 3. GE grammar with the ability to define any number of ADFs.

## **Grammatical Evolution by Grammatical Evolution**

 $(GE)^2$  is a variant of GE which evolves the input grammar itself, thus it has an ability to automatically define and evolve a number of ADFs.

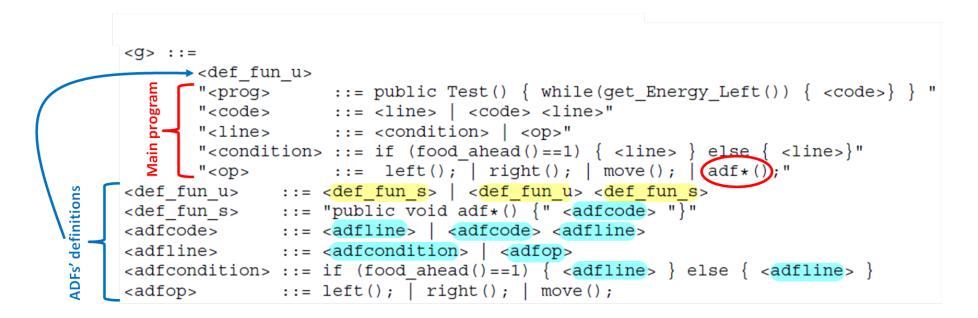
- The meta grammar dictates the construction of the solution grammar.
- Two separate, variable-length binary chromosomes are used
  - Solution grammar chrom. to generate the solution grammar from the meta grammar.
  - Solution structure chromosome to generate the solution itself.
- Crossover operates between homologous chromosomes.
- The solution grammar and the structure of the solution are evolved simultaneously.



## **Grammatical Evolution by Grammatical Evolution**

Example: Meta grammar for the artificial ant problem.

 Quotes are used to escape symbols, e.g. not expand non-terminals in the meta-grammar, instead expand them in the solution grammar.

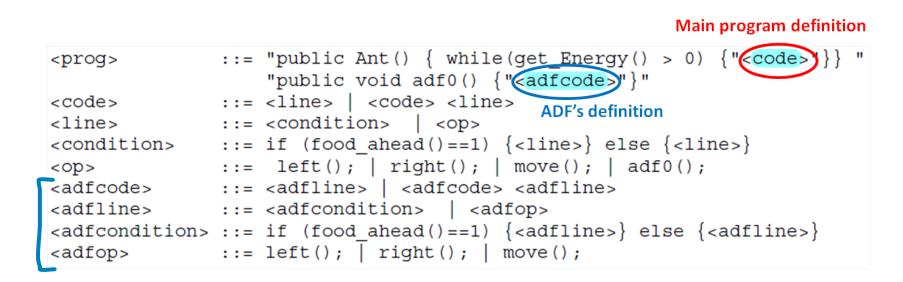


•  $adf \star$  () is a function call to a defined function.

A codon from the solution chromosome is used to select which function is called.

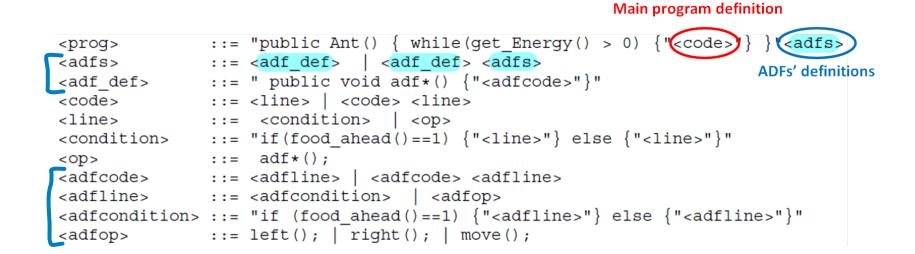
In a solution grammar the multiple defined ADFs are post-processed to make each function signature unique.

GE evolving a population of individuals, each individual has a single chromosome that encodes both the main program as well as the ADF.



A header of a single ADF, adf0(), is defined so only a body of this particular ADF can be generated using the chromosome.

## **GE Grammar with Multiple ADFs**

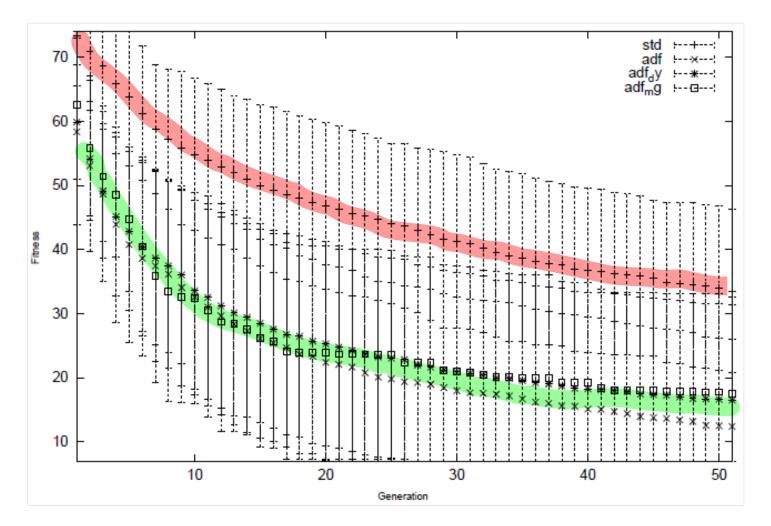


A number of ADFs can be generated via the non-terminal <adfs> using the chromosome.

adf\*() is expanded to enumerate all the allowed ADFs.

### **GE with ADFs: Results on Santa Fe Ant Trail**

Irrespective of the ADF representation, the presence of ADFs alone is sufficient to significantly improve performance of the GE.



## Reading

- Poli, R., Langdon, W., McPhee, N.F.: A Field Guide to Genetic Programming, 2008 http://www.gp-field-guide.org.uk/
- O'Neill, M., Ryan, C.: Grammatical Evolution. IEEE Transactions on Evolutionary Computation, VOL. 5, NO. 4, AUGUST 2001 http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=942529
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