STRUCTURED MODEL LEARNING (SS2019)
5. SEMINAR

Assignment 1. (*Random walks on graphs*)
Consider a random walk on an undirected graph $(V; E)$ defined by

$$p(s_{t+1} = i \mid s_t = j) = \begin{cases} w_{ij} / \sum_{k \in \mathcal{N}_j} w_{kj} & \text{if } \{i,j\} \in E \\ 0 & \text{otherwise} \end{cases}$$

and some given marginal probability $p(s_1)$ for the start vertex. The (not necessarily symmetric) weights $w_{ij} \in \mathbb{R}_+$ are known.
a) Which conditions on the graph ensure existence of a unique limiting distribution?
b) Can these conditions be weakened if the transition probabilities allow the walker to stay in the same vertex with some non-zero probability?
c) Does the transition probability satisfies the detailed balance condition?

Assignment 2. Prove that the Gibbs sampler (for a Gibbs random field) does not satisfy the detailed balance condition. Prove that the elementary transitions (resampling the state of a single node) do indeed satisfy this condition. *Hint:* consider for simplicity a “field” that consists of two random variables only.

Assignment 3. Consider an Ising model on an undirected graph (V, E), i.e. a binary valued random field with joint distribution

$$p(x) = \frac{1}{Z} \exp\Big[- \sum_{\{i,j\} \in E} |x_i - x_j|\Big],$$

where $x_i = 0, 1$.
a) What are the marginal probabilities $p(x_i)$, $x_i = 0, 1$ of this model? *Hint:* use an symmetry argument.
b) Prove that the mean field approximation provides the same solution.