STRUCTURED MODEL LEARNING (SS2019)
2. SEMINAR

Assignment 1. Let $x = \{x_i \mid i \in V\}$ be a set of binary valued variables, i.e. $x_i = 0, 1$.
a) Prove by induction over the number of variables that every function $f(x)$ can be written as a polynomial

$$f(x) = \sum_{C \subset V} a_C \prod_{i \in C} x_i,$$

where the sum is over all subsets of V and a_C are some coefficients.
b) Conclude that the distribution for a binary valued Gibbs random field on a graph (V, E) can be written as

$$p(x) = \frac{1}{Z(u)} \exp\Big[\sum_{i \in V} u_i x_i + \sum_{ij \in E} u_{ij} x_i x_j\Big]$$

with some real numbers u_i, u_{ij}.

Assignment 2. Consider the discrete optimisation task

$$\arg\max_{s \in K^V} \sum_{ij \in E} u_{ij}(s_i, s_j),$$

where $s = \{s_i \mid i \in V\}$ is a collection of K-valued variables, (V, E) is an undirected graph and $u_{ij} \colon K^2 \to \mathbb{R}$ are some functions. Prove that the task is NP-complete by reducing the max-clique problem to it.

Assignment 3. Let $X \in \mathbb{R}$ be a normally distributed random variable, i.e.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}}.$$

a) Prove the equality

$$\frac{\partial}{\partial \mu} \mathbb{E}_{\mathcal{N}(\mu,\sigma)} f(x) = \mathbb{E}_{\mathcal{N}(\mu,\sigma)} f'(x),$$

where $f'(x)$ denotes the derivative of f. Hint: use the substitution $\tilde{x} = (x - \mu)/\sigma$ in the integral for the expectation.
b*) Prove the equality

$$\frac{\partial}{\partial \sigma} \mathbb{E}_{\mathcal{N}(\mu,\sigma)} f(x) = \mathbb{E}_{\mathcal{N}(\mu,\sigma)} f''(x).$$

Hint: use the same substitution as in a) and integration by parts.