



$$w^* \in \underset{\|w\|=1, i=1, \dots, \ell}{\operatorname{arg\,min}} \langle w, a^i \rangle$$

$$\gamma = \min_{i=1, \dots, \ell} \langle w^*, a^i \rangle$$

$$w_1^* = \phi$$

$$w_2 = a_1 + w_1$$

$$\langle a_1, w_1 \rangle \leq \phi$$

$$w_3 = a_2 + w_2$$

$$\langle a_2, w_2 \rangle \leq \phi$$

⋮

$$\begin{aligned} \textcircled{1} \quad \|w_{t+1}\|^2 &= \|w_t + a_t\|^2 = \|w_t\|^2 + 2 \underbrace{\langle w_t, a_t \rangle}_{\leq \phi} + \|a_t\|^2 \leq \|w_t\|^2 + A^2 \\ &\leq \|w_t\|^2 + A^2 \end{aligned}$$

$$\|w_{t+1}\|^2 \leq t \cdot A^2$$

$$\textcircled{2} \quad \underbrace{\|w_{t+1}\|}_{\geq 1} \cdot \|w^*\| \geq \langle w^*, w_{t+1} \rangle = \langle w^*, \sum_{i=1}^t a_i \rangle = \sum_{i=1}^t \underbrace{\langle w^*, a_i \rangle}_{\geq \gamma} \geq t \cdot \gamma$$

$$\|w_{t+1}\|^2 \geq t^2 \cdot \gamma^2$$

$$\textcircled{3} \quad t^2 \cdot \gamma^2 \leq \|w_{t+1}\|^2 \leq t \cdot A^2 \Rightarrow t \leq \frac{A^2}{\gamma^2}$$

