Combinatorial Optimization
Lab 02: Examples

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Abstract: During the second lab of the course, we went through several simple examples of the ILP programs. This document provides short descriptions of the problems. The source codes in C++, Java and Python programming languages are located in the respective folders. Even though the problems are simple, they are $\mathcal{NP}$-hard (except for the LP example). Students should be able to formulate these problems formally and write the corresponding ILP models.

For a short description of the programming interfaces see, e.g., C++, Java, Python. We also encourage you to go through the Gurobi documentation (`http://www.gurobi.com/documentation/`) – you can find there descriptions of the individual methods, as well as many useful examples.

1 Two partition problem

Given multiset S of positive integers, we ask if it can be partitioned to two subsets S_1, S_2, such that sum of the numbers in S_1 equals to the sum of the numbers in S_2, and each number is assigned (either to S_1 or to S_2).

Example 1 Multiset $S = \{1, 1, 1, 2, 2, 3\}$ can be partitioned, e.g., to $S_1 = \{2, 3\}$, $S_2 = \{1, 1, 1, 2\}$. Note that it is not a unique solution.

Example 2 Multiset $S = \{2, 4\}$ cannot be partitioned.

2 Maximum independent set

Given graph $G = (V, E)$, we are trying to find subset $V^\star \subseteq V$, such that $V^\star$ is as large as possible and for each pair of vertices $u, v \in V^\star$, $u \neq v$, it holds that there is no edge $e = \{u, v\}$ in the original graph.

3 Minimum vertex cover

Given graph $G = (V, E)$, we are looking for subset $V^\star \subseteq V$, such that $V^\star$ is as small as possible and each edge $e = \{u, v\}$ is 'covered' by at least one vertex $x \in V^\star$, i.e. $\forall e = \{u, v\} \in E\ \exists x \in V^\star$ such that $x = u$ or $x = v$.

4 SAT (boolean satisfiability problem)

Given formula φ (in CNF), we are looking for truth assignment (True / False) to each variable, such that φ is satisfied (evaluates to True).

Example 1 Formula $\varphi = (x_1 \lor x_2) \land (\neg x_2 \lor x_3)$ can be satisfied by $x_1 :=$ True, $x_3 :=$ True, $x_2 :=$ False.

Example 2 Formula $\varphi = (x_1) \land (\neg x_1)$ cannot be satisfied.