Problem solving by search II

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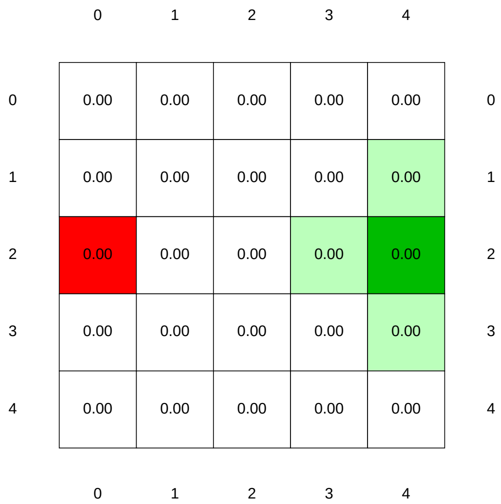
Notes

Outline

- Graph search
- Heuristics (how to search faster)
- Greedy
- A^*. A-star search.

Notes

A Maze, what could possibly go wrong?



Notes

Analyze the demo run (BFS). What happened? Why did it take that long?
Because it is TREE_SEARCH...
Many loops are created and all nodes with depth < 7 need to be expanded first. Goal is at depth 8.
Notes for teacher:
Working note for demo:

```
python3 easy_search_agents.py
'n' for next
's' for skip
```

code settings:

```
MAP = 'maps/easy/easy2.bmp'
TREE_SEARCH = True
node_type = 'BFS'
```

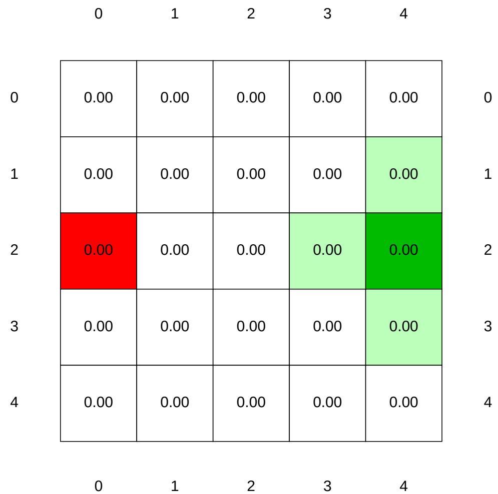
How to decode printout on command line:

- Every iteration ends with: `print('End of while loop: length of the frontier:',len(frontier), 'length of the expanded:', len(expanded_states), frontier, frontier.is_empty())`
- But note that the algo is written in a general way (like UCS), stopping after expanding the goal node – that is why you see also depth 9 in the frontier notes at the end.

Tree search the maze

function TREE_SEARCH(env) **return** a solution or failure
- initialize the frontier
- **while** frontier **do**
  - node = frontier.pop()
  - **if** goal in node **then**
    - break
  - **end if**
  - nodes = env.expand(node.state)
  - Add nodes to frontier
- **end while**

end function



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Make a `frontier` and `expand` columns on a paper and follow the algorithm by putting and removing (scratching out) nodes from the list.

Note that there are many more nodes than states (*search tree* vs. *state space*).

Tree search seems hugely ineffective. Note that this is (also) because of the state space. It's a maze with undirected egdes. If we had directed edges, there would be much much fewer cycles.

A graph search

```
function GRAPH_SEARCH(env) return a solution or failure
    init frontier by the start state
    initialize the explored set to be empty
    while frontier do
        node = frontier.pop()
        if goal in node then break
        end if
        nodes = env.expand(node.state)
        add node.state to explored
        for all nodes do
            if node.state not in explored (or in frontier) then
                add nodes to frontier
            end if
        end for
    end while
end function
```


⚠ **Do not forget:** node is not the same as state!

Notes

Think about what is node and what state. What is main difference? How are they connected? Where do they appear? What is node/state in the maze problem?

The main idea: Do not expand a state twice.

What would be a good data structure to implement the *explored* set? Yes, it would be a *set* ;) – where every element is present only once. Unlike *list*.

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The BFS graph search

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function BFS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← FIFOqueue(node)
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        explored.add(node.state)                        ▷ adding state not node!
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored or in frontier then
                if child_node contains Goal then return child_node
                end if
                frontier.insert(child_node)
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        end for
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end function
```


Notes

Why adding/checking `state` and not `node` in `explored` data structure? Can I do the simple presence check for all kind of graph search algorithms?

Run demo again with BFS graph search.

Notes for teacher:

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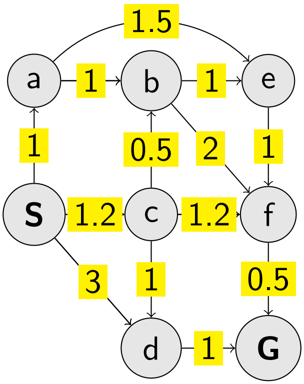
What about actual costs graph search?



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When following the algorithm (animation) use the paper list of `frontier` and `explored`
Note the extra features of UCS vs. BFS in action:

1. Update of cost:
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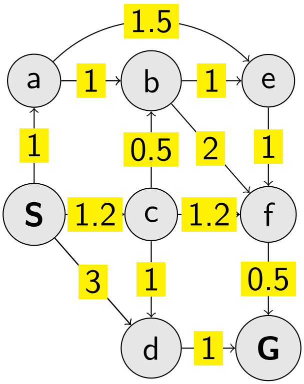

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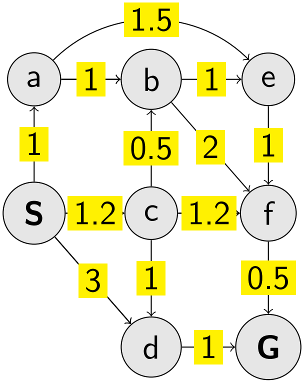

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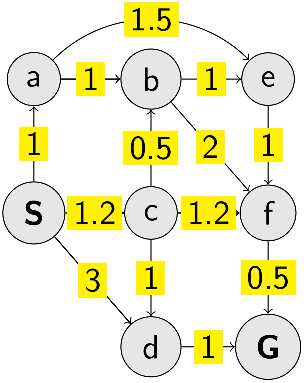

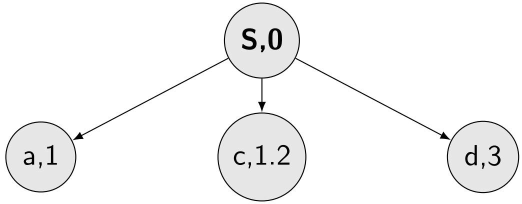

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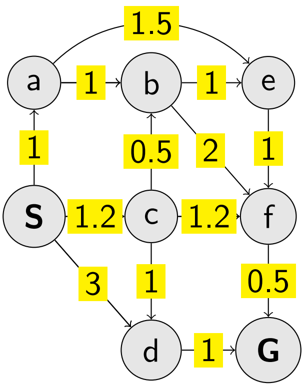

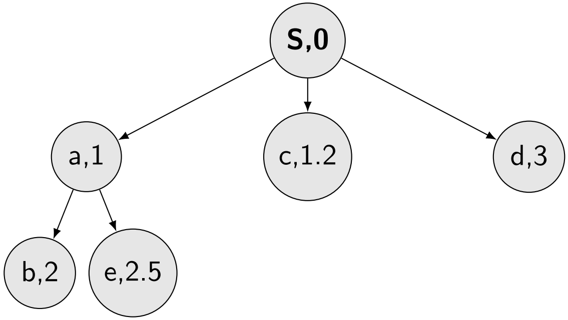

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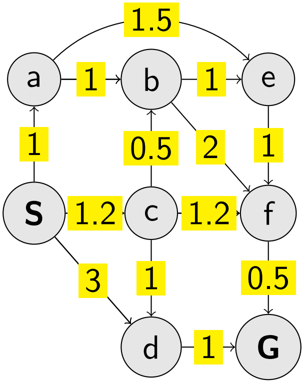

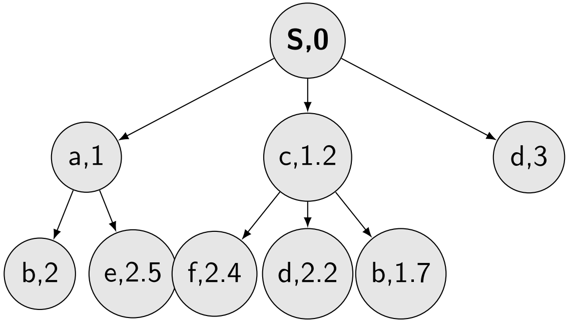

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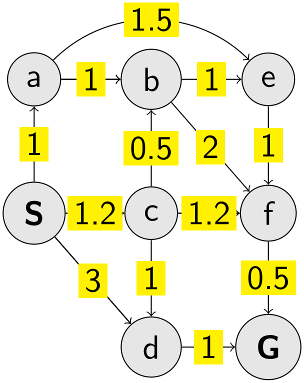

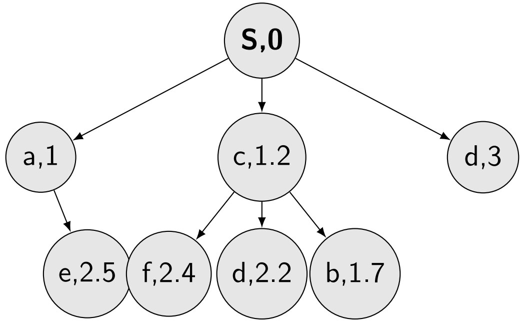

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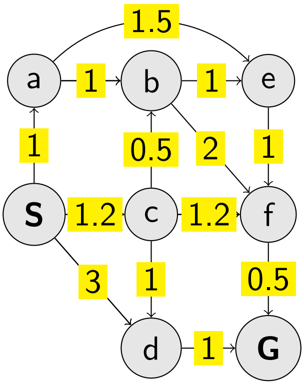

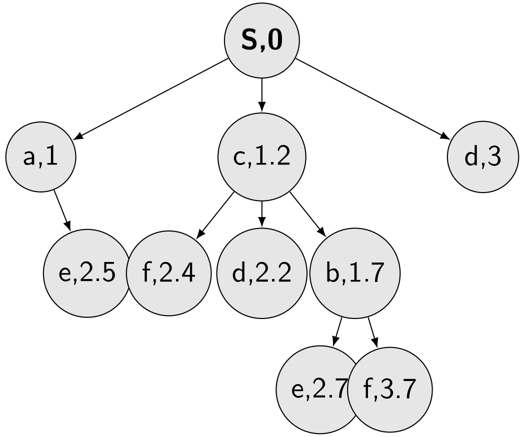

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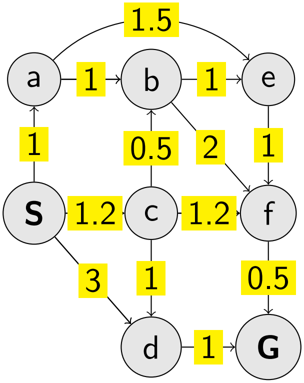

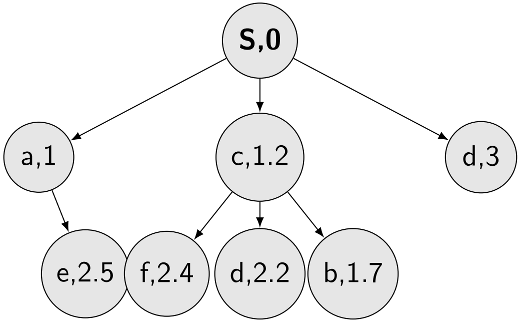

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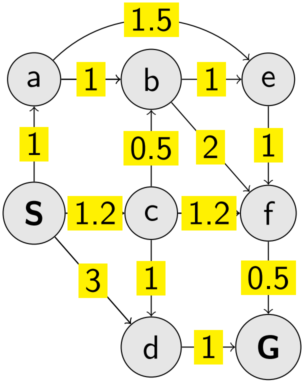

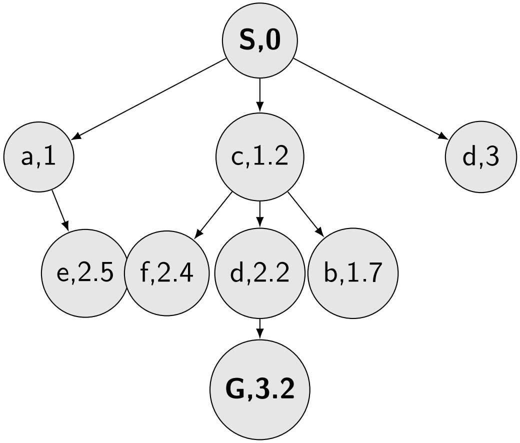

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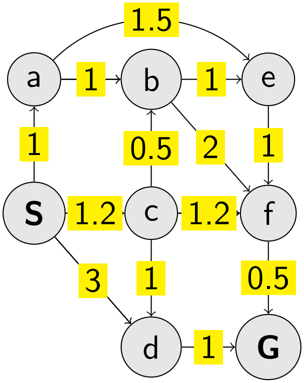

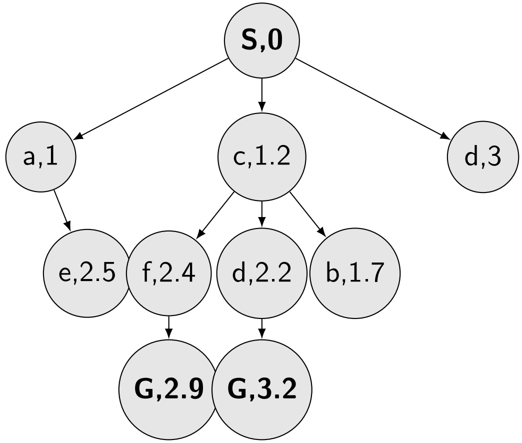

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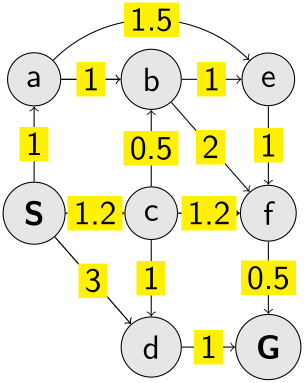
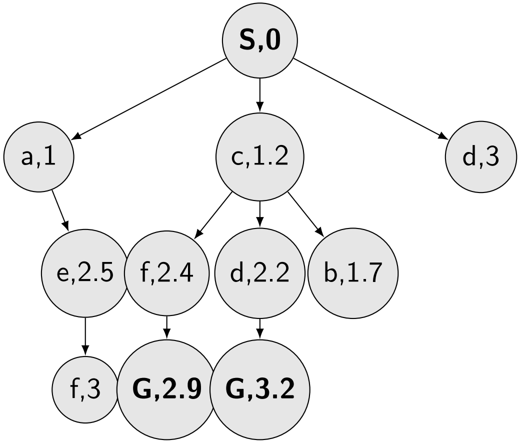
2. Termination only after expanding node with goal state.

What about actual costs graph search?





Notes

When following the algorithm (animation) use the paper list of `frontier` and `explored`
Note the extra features of UCS vs. BFS in action:

1. Update of cost:
   - "b,2" disappears as "b,1.7" appears – update with lower cost.
   - Similarly, "e,2.7" and "f,3.7" appear to immediately disappear again – their cost is higher than already available for those states.
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The UCS graph search

```
function UCS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← priority_queue(node)                          ▷ path_cost for ordering
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        if node contains Goal then return node                        ▷ check here!
        end if
        explored.add(node.state)
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored or in frontier then
                frontier.insert(child_node)
            else if child_node.state in frontier with higher cost then
                replace that node with the child_node
            end if
        end for
    end while
end function
```


Notes

Does the algorithm always find the best (cheapest) path? Are there any requirements for the path optimality function?

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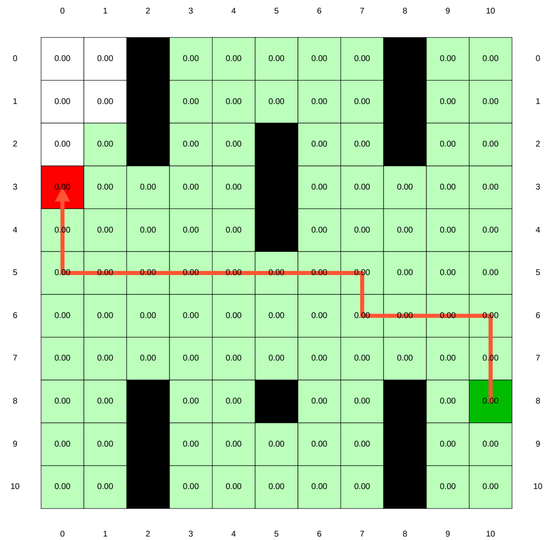
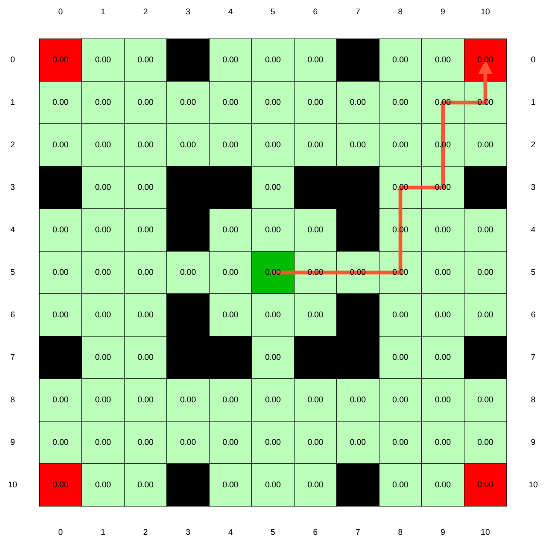
Few examples of search strategies so far



Run the demos.

Notes

What is wrong with UCS and other strategies?



Run the demo.

Notes

Node selection, take argmin $f(n)$

Selecting next node to expand/visit:

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{frontier}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS: $f(n) = -n.depth$
- BFS: $f(n) = n.depth$
- UCS: $f(n) = n.path\_cost$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the cost from n to the start - only backward cost; cost-to-come (to n).

Notes

Do humans look back when planing path? Is looking back important at all? If yes, when?

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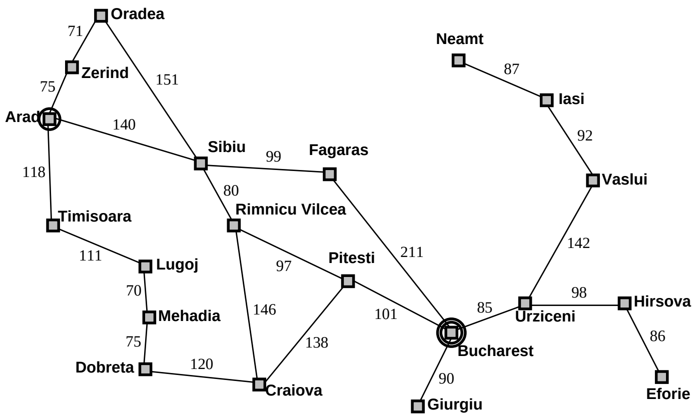
How far are we from the goal cost-to-go ? – Heuristics

- A function that estimates how close a state is to the goal.
- Designed for a particular problem.
- We will use $h(n)$ – heuristic value of node n.

Notes

What happens if $h(n) = \text{true cost}$?

Example of heuristics



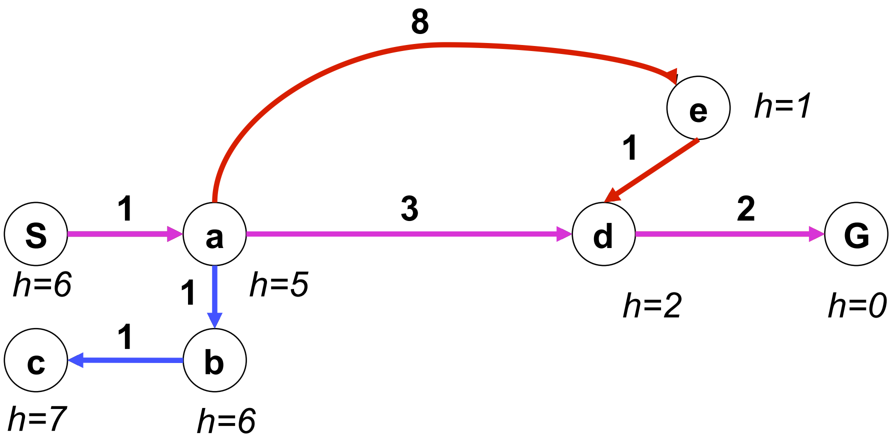
Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Notes

Straight-line distance to Bucharest.

Illustration of *greedy* failing: Imagine going from Iasi to Fagaras. Neamt will be chosen for expansion. This will add Iasi back. Iasi is closer to Fagaras than Vaslui is and will be expanded again. Infinite loop... (3.5.1. in [2])

Greedy, take the node argmin $h(n)$



[1]

What is wrong (and nice) with the Greedy?

[1] Graph example: Ted Grenager

Notes

Also called "Greedy best-first search" [2].
What will happen in this example:

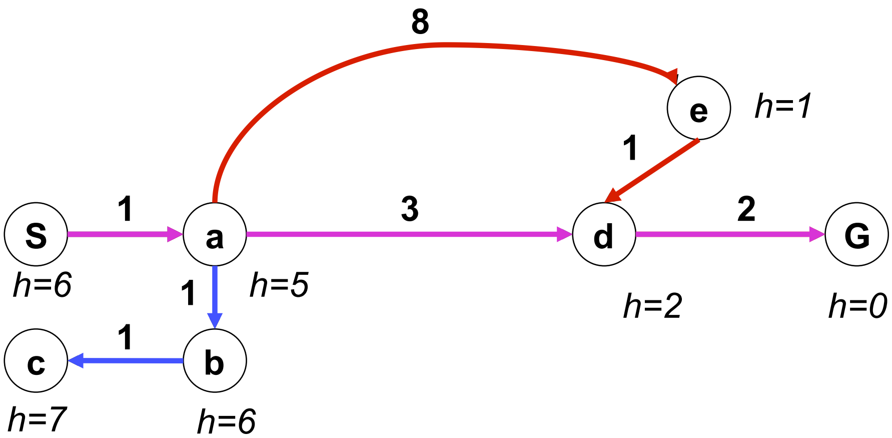
1. Expand "S". Add "a" to frontier.
2. Expand "a". Add "b", "d", "e".
3. Expand "e" ($h = 1$). We already have "d".
4. Expand "d". Get "G".

Wrong:

- not optimal
- not complete (tree search version) (Can be shown on the Romania example – go back.)
- (graph search version is complete only in finite state spaces)

Nice: it is simple.

Greedy, take the node argmin $h(n)$



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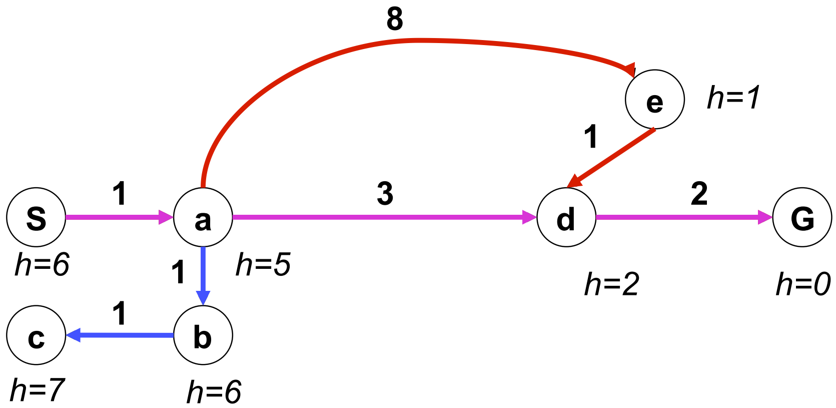
A* combines UCS and Greedy

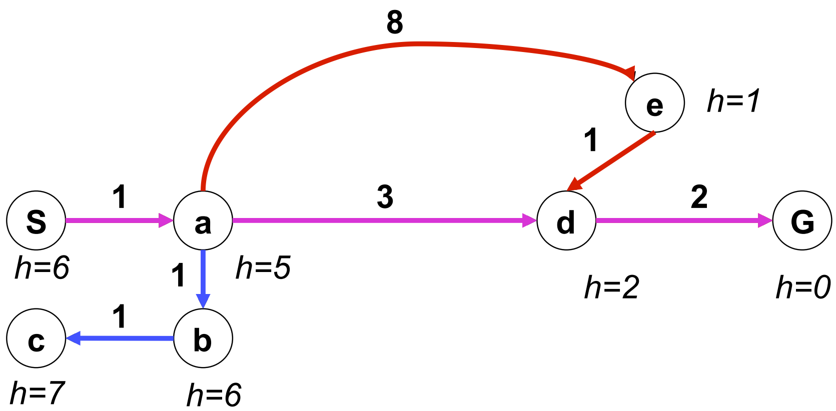


UCS orders by backward (path) cost $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

Notes

A* combines UCS and Greedy



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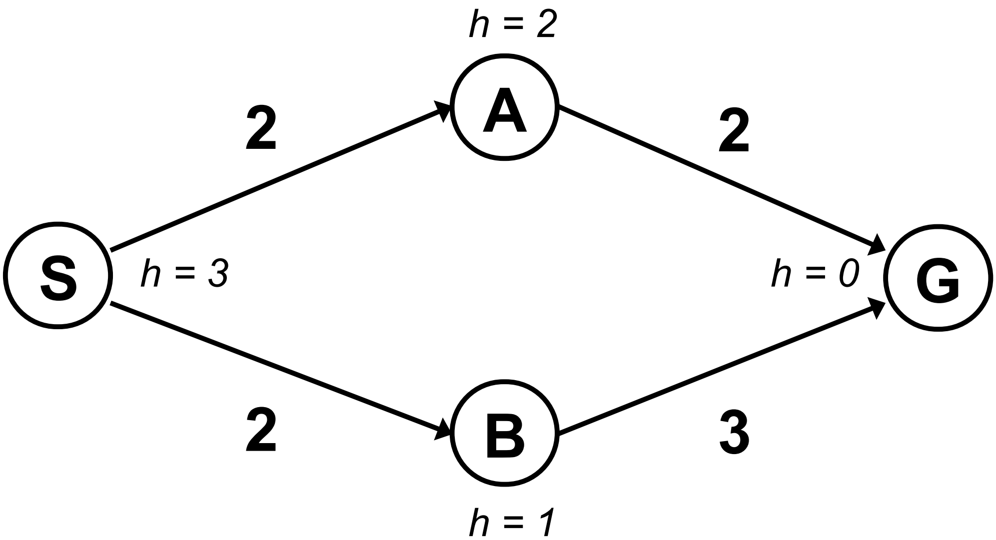
A* orders nodes by: $f(n) = g(n) + h(n)$

Notes

When to stop A*?



2

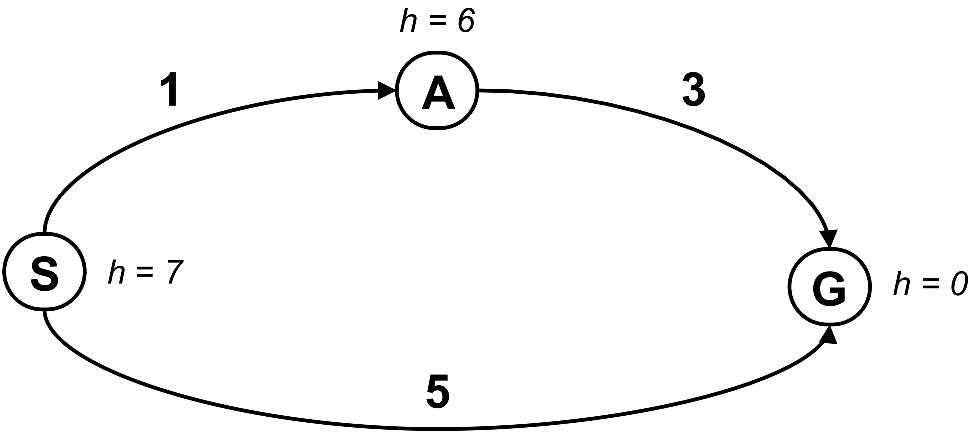
[2] Graph example: Dan Klein and Pieter Abbeel

Notes

1. S
   - $f(S) = g(S) + h(S) = 0 + 3 = 3$
   - expanding/poping this one and crossing out (removing from frontier)
2. $S \to A$
   - $f(A) = g(A) + h(A) = 2 + 2 = 4$
3. $S \to B$
   - $f(B) = g(B) + h(B) = 2 + 1 = 3$
   - expanding this one and crossing out
4. $S \to B \to G$
   - $f(G) = g(G) + h(G) = 5 + 0 = 5$
   - Should I stop now? No. Pop $S \to A$ with $f = 4$.
5. $S \to A \to G$
   - $f(G) = g(G) + h(G) = 4 + 0 = 4$
   - This is now cheapest on the frontier. I pop/expand and I'm done.

Note: h is a function of the state. g is a function of a node (the path matters).

Is A* optimal?



[3]

What is the problem?

[3] Graph example: Dan Klein and Pieter Abbeel

Notes

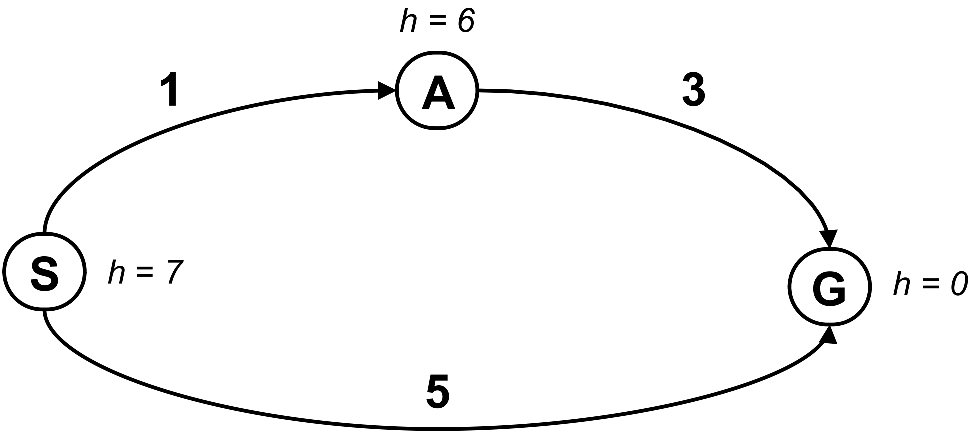
Try to answer the question before going to the next slide.

1. S
   - $f(S) = g(S) + h(S) = 0 + 7 = 7$
   - expanding/poping this one and crossing out (removing from frontier)
2. $S \rightarrow A$
   - $f(A) = g(A) + h(A) = 1 + 6 = 7$
3. $S \rightarrow G$
   - $f(B) = g(B) + h(B) = 5 + 0 = 5$
   - This is now cheapest on the frontier. I pop/expand and I'm done.

Ooops! That's not cheapest! What went wrong?

What follows – keep for next slide. Problem with $h(A) = 6$. Overestimating the expense. Estimates need to be $\leq$ actual costs. C is correct.

Is A* optimal?



3

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Ooops! That's not cheapest! What went wrong?

What follows – keep for next slide. Problem with $h(A) = 6$. Overestimating the expense. Estimates need to be $\leq$ actual costs. C is correct.

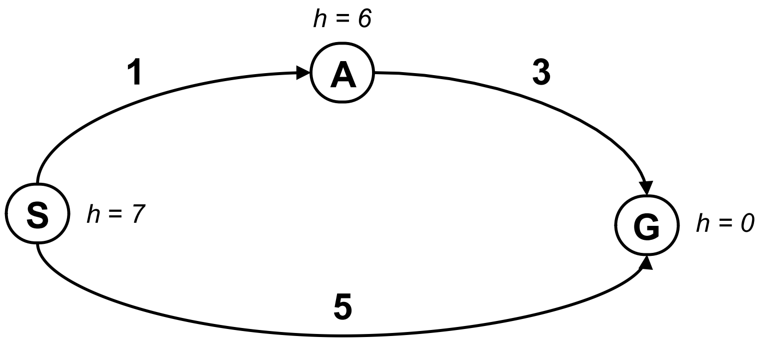
What is the right $h(A)$?

A: $0 \leq h(A) \leq 4$

B: $h(A) \leq 3$

C: $0 \leq h(A) \leq 3$

D: $0 \leq h(A)$



Notes

$h(A) \leq 3$ it means less than the actual cost of going from A to goal. Heuristic must not be overly pesimistic.

Admissible heuristics

A heuristic function h is admissible if:

$$h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost of going from n to the nearest goal.

Notes

Optimality of A^* tree search

A* is optimal if h(n) is admissible.

Notes

A* graph search

```
function GRAPH_SEARCH(env)
    frontier.insert(startnode)
    explored = set()
    while frontier do
        node = frontier.pop()
        if goal in node then break
        end if
        nodes = env.expand(node.state)
        explored.add(node.state)
        for all nodes do
            if node.state not in explored then
                frontier.insert(node)
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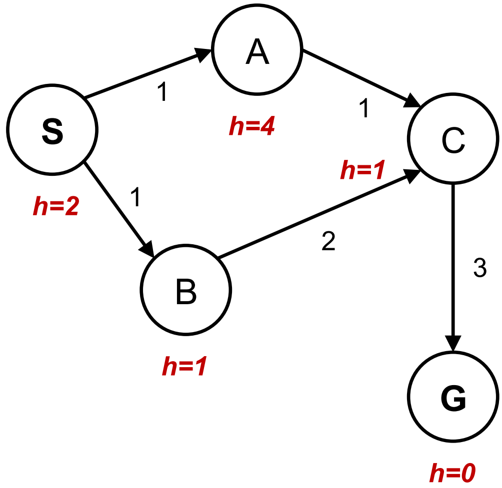
        end for
    end while
end function
```



Graph example: Dan Klein and Pieter Abbeel.

What went wrong?

Notes

1.
   - $f(S) = g(S) + h(S) = 0 + 2 = 2$
   - expanding/poping this one and crossing out (removing from frontier); *explored set*: S
2. $S \to A$; $f(A) = g(A) + h(A) = 1 + 4 = 5$
3. $S \to B$; $f(B) = g(B) + h(B) = 1 + 1 = 2$
4. B is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, B
5. $B \to C$; $f(C) = g(C) + h(C) = 3 + 1 = 4$
6. C is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, B, C
7. $C \to G$; $f(G) = f(G) + h(G) = 6 + 0 = 6$
8. A is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, A, B, C
9. $A \to C$; $f(C) = f(C) + h(C) = 2 + 1 = 3$
10. C is cheapest on the frontier. But, it's on *explored set*! Can't be expanded.
11. Moving on to G, expanding and finishing.

Ooops! That's not cheapest! $cost(S \to B \to C \to G) = 6$; $cost(S \to A \to C \to G) = 5$ What went wrong?

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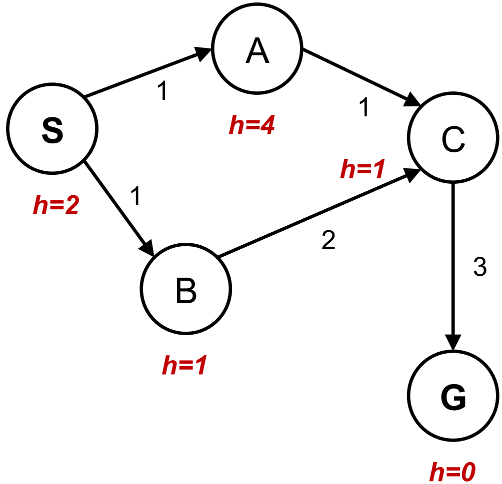

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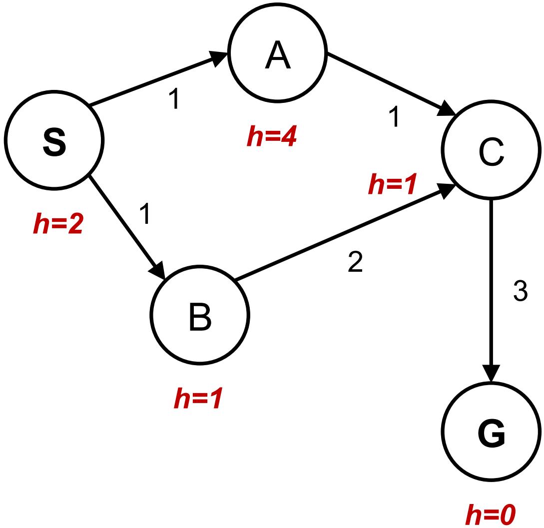
Ooops! That's not cheapest! $cost(S \rightarrow B \rightarrow C \rightarrow G) = 6$; $cost(S \rightarrow A \rightarrow C \rightarrow G) = 5$ What went wrong?

What is the proper $h(A)$?

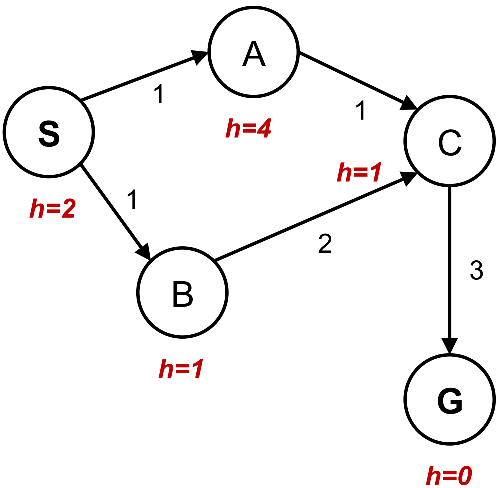
A: $h(A) = 1$
B: $h(A) = 2$
C: $1 \leq h(A) \leq 2$
D: $0 \leq h(A) \leq 1$



Notes

As it will be explained in the next slides: $h(A) \leq c(A, C) + h(C) = 2$
$h(S) \leq c(S, A) + h(A)$ it means $h(A) \geq h(S) - c(A, S) = 1$

Consistent heuristics



Admissible h:
$h(A) \leq$ true cost $A \rightarrow G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \rightarrow C$
in general:
$h(n) - h(s) \leq$ true cost $n \rightarrow s$ for any pair: node n and its successor s

$f(n) = g(n) + h(n)$ along a path never decreases!

Notes

Our heuristic was admissible.
With *tree search* it would have worked. It would have expanded C and found the alternative, cheaper path.
For graph search, the problem is the $A \rightarrow C \rightarrow G$ subgraph where the *consistent* heuristic condition is violated.
The general condition means we have two constraints for (A) for this particuar graph:
$h(S) - h(A) \leq c(S, A)$
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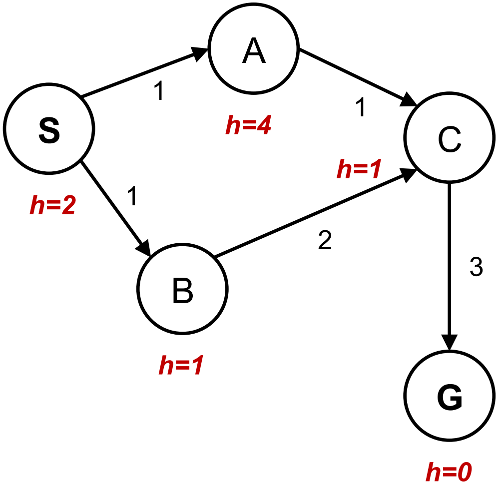

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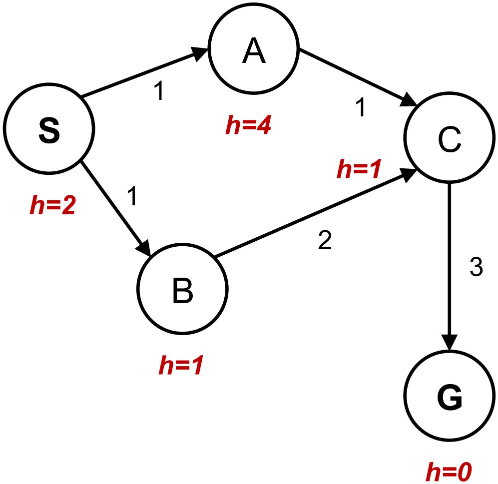

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Our heuristic was admissible.

With *tree search* it would have worked. It would have expanded C and found the alternative, cheaper path.

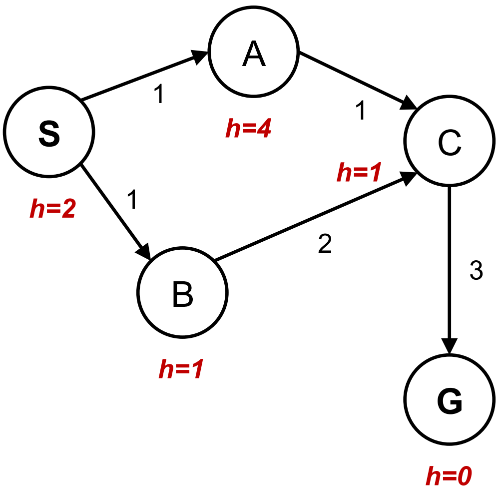
For graph search, the problem is the $A \rightarrow C \rightarrow G$ subgraph where the *consistent* heuristic condition is violated.

The general condition means we have two constraints for (A) for this particuar graph:

$h(S) - h(A) \leq c(S, A)$

$h(A) - h(C) \leq c(A, C)$

Consistent heuristics



Admissible h:
$h(A) \leq$ true cost $A \to G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(n) - h(s) \leq$ true cost $n \to s$ for any pair: node n and its successor s

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Optimality of A*

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- consistent h for graph search
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References, further reading

Some figures from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms. (State space) Search algoritmhs are ubiquitous, explanations in many (text)books about Algorithms.
Nice online course from UC Berkeley (CS 188 Into to AI):
http://ai.berkeley.edu/lecture_videos.html Lecture: Informed Search.

[1] Steven M. LaValle.
Planning Algorithms.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

Notes