Bayesian decision making

Z. Straka, P. Švarný, J. Kostlivá

Today two examples:

1. Bayesian decision making basics
2. Power outage
3. Prior probabilities in practice

Bayesian decision making basics

Bayesian decision making basics

What is correct?

- A: $P(X = x_i) = \sum_j \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)}$
- B: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- C: $P(X = x_i) = \sum_i P(X = x_i, Y = y_j)$
- D: $P(Y = y_i) = \sum_j P(X = x_i, Y = y_j)$

Bayesian decision making basics

What is correct?

A: $P(X = x_i) = \sum_j \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)}$

B: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$

C: $P(X = x_i) = \sum_i P(X = x_i, Y = y_j)$

D: $P(Y = y_i) = \sum_j P(X = x_i, Y = y_j)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$

What is correct?

A: $P(X = x_i | Y = y_j) = P(Y = y_j, X = x_i) P(X = x_i)$

B: $P(X = x_i, Y = y_j) = \frac{P(Y = y_j | X = x_i)}{P(X = x_i)}$

C: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(Y = y_i)$

D: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(X = x_i)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$

What is correct?

A: $P(X = x_i | Y = y_j) = P(Y = y_j, X = x_i)P(X = x_i)$

B: $P(X = x_i, Y = y_j) = \frac{P(Y=y_j|X=x_i)}{P(X=x_i)}$

C: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i)P(Y = y_i)$

D: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i)P(X = x_i)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(X = x_i)$

What is correct?

A: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

B: $P(Y = y_i, X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

C: $P(Y = y_i | X = x_j) = P(X = x_i | Y = y_j) P(Y = y_j)$

D: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{\sum_l P(X=x_l, Y=y_i)}$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i)P(X = x_i)$

What is correct?

A: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

B: $P(Y = y_i, X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

C: $P(Y = y_i | X = x_j) = P(X = x_i | Y = y_j)P(Y = y_j)$

D: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{\sum_l P(X=x_l, Y=y_l)}$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(X = x_i)$

What is correct?

A: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

B: $P(Y = y_i, X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

C: $P(Y = y_i | X = x_j) = P(X = x_i | Y = y_j) P(Y = y_j)$

D: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{\sum_l P(X=x_l, Y=y_l)}$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(X = x_i)$
- Bayes' theorem: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$

What is correct?

A: $\delta^* = \arg\max_\delta \sum_x \sum_s l(s, \delta(x)) P(x, s)$

B: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x)) P(x|s)$

C: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x)) P(x, s)$

D: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, x) P(x, s)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i)P(X = x_i)$
- Bayes' theorem: $P(Y = y_i | X = x_j) = \frac{P(X=x_i | Y=y_j)P(Y=y_j)}{P(X=x_j)}$

What is correct?

A: $\delta^* = \arg\max_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$

B: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x|s)$

C: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$

D: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, x)P(x, s)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j|X = x_i)P(X = x_i)$
- Bayes' theorem: $P(Y = y_i|X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$
- Bayes optimal strategy: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$

What is correct?

A: $\delta^*(x) = \arg\min_d \sum_s l(s, d)P(s|x)$

B: $\delta^*(x) = \arg\min_d \sum_s l(s, d)P(s, x)$

C: $\delta^*(x) = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$

D: $\delta^*(x) = \arg\min_s \sum_d l(s, d)P(s|x)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i) P(X = x_i)$
- Bayes' theorem: $P(Y = y_i | X = x_j) = \frac{P(X=x_i | Y=y_j) P(Y=y_j)}{P(X=x_j)}$
- Bayes optimal strategy: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x)) P(x, s)$

What is correct?

A: $\delta^*(x) = \arg\min_d \sum_s l(s, d) P(s|x)$

B: $\delta^*(x) = \arg\min_d \sum_s l(s, d) P(s, x)$

C: $\delta^*(x) = \arg\min_\delta \sum_x \sum_s l(s, \delta(x)) P(x, s)$

D: $\delta^*(x) = \arg\min_s \sum_d l(s, d) P(s|x)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j|X = x_i)P(X = x_i)$
- Bayes' theorem: $P(Y = y_i|X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$
- Bayes optimal strategy: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$
- BOS solution: $\delta^*(x) = \arg\min_d \sum_s l(s, d)P(s|x)$

Assume $l(s, d) = 1, \texttt{if } d \neq s, l(s, d) = 0 \texttt{ otherwise.}$ What is correct?

A: $\delta^*(x) = \arg\min_d P(d|x)$

B: $\delta^*(x) = \arg\max_d P(d|x)$

C: $\delta^*(x) = \arg\max_d P(d|x)P(x)$

D: $\delta^*(x) = \arg\max_d P(d|x)P(s)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j | X = x_i)P(X = x_i)$
- Bayes' theorem: $P(Y = y_i | X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$
- Bayes optimal strategy: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$
- BOS solution: $\delta^*(x) = \arg\min_d \sum_s l(s, d)P(s|x)$

Assume $l(s, d) = 1, \texttt{if } d \neq s, l(s, d) = 0 \texttt{ otherwise.}$ What is correct?

A: $\delta^*(x) = \arg\min_d P(d|x)$

B: $\delta^*(x) = \arg\max_d P(d|x)$

C: $\delta^*(x) = \arg\max_d P(d|x)P(x)$

D: $\delta^*(x) = \arg\max_d P(d|x)P(s)$

Bayesian decision making basics

- Sum rule of probability: $P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$
- Product rule of probability: $P(X = x_i, Y = y_j) = P(Y = y_j|X = x_i)P(X = x_i)$
- Bayes' theorem: $P(Y = y_i|X = x_j) = \frac{P(X=x_i|Y=y_j)P(Y=y_j)}{P(X=x_j)}$
- Bayes optimal strategy: $\delta^* = \arg\min_\delta \sum_x \sum_s l(s, \delta(x))P(x, s)$
- BOS solution: $\delta^*(x) = \arg\min_d \sum_s l(s, d)P(s|x)$
- $L_{0,1}$ classification: $\delta^*(x) = \arg\max_d P(d|x)$

Power outage

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

Formalize:

A: $p(x = female | y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65 | x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

Formalize:

A: $p(x = female|y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65|x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

Formalize:

A: $p(x = female|y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65|x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞ kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

- $p(x = female, y < 65)$

Calculate:

A: $\frac{14}{24}$

B: $\frac{8}{58}$

C: $\frac{14}{58}$

D: $\frac{24}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

- $p(x = female, y < 65)$

Calculate:

A: $\frac{14}{24}$

B: $\frac{8}{58}$

C: $\frac{14}{58}$

D: $\frac{24}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 1: One person is randomly selected and has to leave the café. Estimate the probability that this person is a woman that weighs less than 65kg.

- $p(x = female, y < 65)$

Calculate:

A: $\frac{14}{24}$

B: $\frac{8}{58}$

C: $\frac{14}{58}$

D: $\frac{24}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- Estimate the probability of that person being a man.

Calculate $p(x = male)$:

A: $\frac{34}{58}$

B: $\frac{24}{58}$

C: $\frac{18}{24}$

D: $\frac{29}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- Estimate the probability of that person being a man.

Calculate $p(x = male)$:

A: $\frac{34}{58}$

B: $\frac{24}{58}$

C: $\frac{18}{24}$

D: $\frac{29}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- Estimate the probability of that person being a man.

Calculate $p(x = male)$:

A: $\frac{34}{58}$

B: $\frac{24}{58}$

C: $\frac{18}{24}$

D: $\frac{29}{58}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- You weigh this person and estimate that he/she weighs no more than 65kg. Estimate the probability of this person to be a woman.

Formalize:

A: $p(x = female | y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65 | x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- You weigh this person and estimate that he/she weighs no more than 65kg. Estimate the probability of this person to be a woman.

Formalize:

A: $p(x = female|y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65|x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞ kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- You weigh this person and estimate that he/she weighs no more than 65kg. Estimate the probability of this person to be a woman.

Formalize:

A: $p(x = female|y < 65)$

B: $p(x = female, y < 65)$

C: $p(y < 65|x = female)$

D: $p(x = male, y < 65)$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞ kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- You weigh this person and estimate that he/she weighs no more than 65kg. Estimate the probability of this person to be a woman.
- $p(x = female | y < 65)$

Calculate:

A: $\frac{8}{14}$

B: $\frac{9}{12}$

C: $\frac{7}{12}$

D: $\frac{7}{14}$

Power outage

Imagine you're spending your evening in a café and going around people with a scale and convincing them to weigh themselves. You noted the results into the following frequency table.

	40–55 kg	55–65 kg	65–75 kg	75-∞kg
male	4	6	9	15
female	6	8	7	3

Task 2: Let's assume the person in the first question didn't leave the café yet. Suddenly, the lights go out and the café is in complete darkness. You notice by sound that someone sat next to you.

- You weigh this person and estimate that he/she weighs no more than 65kg. Estimate the probability of this person to be a woman.
- $p(x = female|y < 65)$

Calculate:

A: $\frac{8}{14}$

B: $\frac{9}{12}$

C: $P(x = female| < 65) = \frac{P(x=female, y<65)}{P(y<65)} = \frac{P(y<65|x=female) \cdot P(x=female)}{P(y<65)} = \frac{\frac{14}{24} \cdot \frac{24}{58}}{\frac{24}{58}} = \frac{14}{24} = \frac{7}{12}$

D: $\frac{7}{14}$

Prior probabilities in practice

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female.

A: Male

B: Female

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female.

A: Male

B: Female

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{\text{cm}}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female.

A: Male

B: Female (if we assume that there are same the number of men and women.)

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{\text{cm}}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female.

A: Male

B: Female (if we assume that there are same the number of men and women.)

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{\text{cm}}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

Right step?

A: $P(X = male, Y = L) = P(X = female, Y = L)$

B: $P(X = male|Y = L) = P(X = female|Y = L)$

C: $P(X = male|Y > L) = P(X = female|Y < L)$

D: $P(X = male|Y > L) > P(X = female|Y < L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{\text{cm}}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

Right step?

A: $P(X = male, Y = L) = P(X = female, Y = L)$

B: $P(X = male|Y = L) = P(X = female|Y = L)$

C: $P(X = male|Y > L) = P(X = female|Y < L)$

D: $P(X = male|Y > L) > P(X = female|Y < L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

Right step?

A: $P(X = male, Y = L) = P(X = female, Y = L)$

B: $P(X = male|Y = L) = P(X = female|Y = L)$

C: $P(X = male|Y > L) = P(X = female|Y < L)$

D: $P(X = male|Y > L) > P(X = female|Y < L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{cm}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x \mid \text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x \mid \text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X = male|Y = L) = P(X = female|Y = L)$

From the equation get value of?

A: $P(X = male)$

B: $P(X = male|Y = L)$

C: $P(X = female|Y < L)$

D: $P(X = male|Y > L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

$\frac{x}{\text{cm}}$	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x \mid \text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x \mid \text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X = male \mid Y = L) = P(X = female \mid Y = L)$

From the equation get value of?

A: $P(X = male)$

B: $P(X = male \mid Y = L)$

C: $P(X = female \mid Y < L)$

D: $P(X = male \mid Y > L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X = male|Y = L) = P(X = female|Y = L)$

From the equation get value of? $P(X = male)$
Calculate $P(X = male)$:

A: $\frac{5}{11}$

B: $\frac{6}{11}$

C: $\frac{6}{10}$

D: $\frac{7}{12}$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 1: Estimate whether a 168cm tall (i.e. L) person is male or female. **Female**
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X = male|Y = L) = P(X = female|Y = L)$

From the equation get value of? $P(X = male)$
Calculate $P(X = male)$:

B: $\frac{6}{11}$

$P(X = male|Y = L) = P(X = female|Y = L)$
$\frac{P(L|male) \cdot P(male)}{P(L)} = \frac{P(L|female) \cdot P(female)}{P(L)}, P(female) = 1 - P(male)$
$P(L|male) \cdot P(male) = P(L|female) \cdot (1 - P(male))$
$0.25 \cdot P(male) = 0.3 - 0.3 \cdot P(male) \Rightarrow P(male) = \frac{6}{11}$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 3: Assuming there are 70% men and 30% women, consider the loss function l (s - state, d - decision): $l(s = \text{female}, d = \text{male}) = 2, l(s = \text{male}, d = \text{female}) = 1$,
$l(s = \text{male}, d = \text{male}) = l(s = \text{female}, d = \text{female}) = 0$.
How do you classify a person under consideration of L?
How?

- A: $\delta^*(X = L) = \text{argmin}_s \sum_s l(s, d) \cdot P(s|X = L)$
- B: $\delta^*(X = L) = \text{argmin}_d l(s, d) \cdot P(s|X = L)$
- C: $\delta^*(X = L) = \text{argmin}_s \sum_d l(s, d) \cdot P(s|X = L)$
- D: $\delta^*(X = L) = \text{argmin}_d \sum_s l(s, d) \cdot P(s|X = L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 3: Assuming there are 70% men and 30% women, consider the loss function L (s = state, d = decision): $l(s = \text{female}, d = \text{male}) = 2, l(s = \text{male}, d = \text{female}) = 1,$
$l(s = \text{male}, d = \text{male}) = l(s = \text{female}, d = \text{female}) = 0.$
How do you classify a person under consideration of L?
How?

D: $\delta^*(X = L) = \text{argmin}_d \sum_s l(s, d) \cdot P(s|X = L)$

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 3: Assuming there are 70% men and 30% women, consider the loss function L (s = state, d = decision): $l(s = \text{female}, d = \text{male}) = 2, l(s = \text{male}, d = \text{female}) = 1$,
$l(s = \text{male}, d = \text{male}) = l(s = \text{female}, d = \text{female}) = 0$.
How do you classify a person under consideration of L?
How? $\delta^*(X = L) = \text{argmin}_d \sum_s l(s, d) \cdot P(s|X = L)$
Result?

A: female

B: male

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 3: Assuming there are 70% men and 30% women, consider the loss function L (s = state, d = decision): $l(s = \text{female}, d = \text{male}) = 2, l(s = \text{male}, d = \text{female}) = 1,$
$l(s = \text{male}, d = \text{male}) = l(s = \text{female}, d = \text{female}) = 0.$
How do you classify a person under consideration of L?
How? $\delta^*(X = L) = \text{argmin}_d \sum_s l(s, d) \cdot P(s|X = L)$
Result?

A: female

B: male

Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

x cm	XS (0–100)	S (100–125)	M (125–150)	L (150–175)	XL (175–200)	XXL (200-∞)	$\sum$
$P(x\|\text{male})$	0.05	0.15	0.2	0.25	0.3	0.05	**1**
$P(x\|\text{female})$	0.05	0.1	0.3	0.3	0.25	0.0	**1**

Task 3: Assuming there are 70% men and 30% women, consider the loss function L (s = state, d = decision): $l(s = \text{female}, d = \text{male}) = 2, l(s = \text{male}, d = \text{female}) = 1,$
$l(s = \text{male}, d = \text{male}) = l(s = \text{female}, d = \text{female}) = 0.$
How do you classify a person under consideration of L?
$P(male|L) = \frac{P(L|male) \cdot P(male)}{P(L)} = \frac{P(L|male) \cdot P(male)}{P(L|male) \cdot P(male) + P(L|female) \cdot P(female)} = \frac{0.25 \cdot 0.7}{0.25 \cdot 0.7 + 0.3 \cdot 0.3} = 0.66$
$P(female|L) = 1 - 0.66 = 0.34$
$\delta^*(X) = \text{argmin}_d \left(l(female, d) \cdot P(female|L) + l(male, d) \cdot P(male|L)\right)$

$$\delta^*(X) = \text{argmin}_d \left\{ \begin{array}{c} d = female : 0 \cdot 0.34 + 1 \cdot 0.66 = 0.66 \\ d = male : 2 \cdot 0.34 + 0 \cdot 0.66 = 0.68 \end{array} \right\} \Rightarrow d = female$$