PUI: Notes on Classical Planning

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1. Representations

Definition 1. A STRIPS **planning task** Π is specified by a tuple $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, where $\mathcal{F} = \{f_1, ..., f_n\}$ is a set of facts, $\mathcal{O} = \{o_1, ..., o_m\}$ is a set of operators, and c is a cost function mapping each operator to a non-negative real number. A **state** $s \subseteq \mathcal{F}$ is a set of facts, $s_{init} \subseteq \mathcal{F}$ is an **initial state** and $s_{goal} \subseteq \mathcal{F}$ is a **goal** specification. An **operator** o is a triple $o = \langle \mathrm{pre}(o), \mathrm{add}(o), \mathrm{del}(o) \rangle$, where $\mathrm{pre}(o) \subseteq \mathcal{F}$ is a set of preconditions, and $\mathrm{add}(o) \subseteq \mathcal{F}$ and $\mathrm{del}(o) \subseteq \mathcal{F}$ are sets of add and delete effects, respectively. All operators are well-formed, i.e., $\mathrm{add}(o) \cap \mathrm{del}(o) = \emptyset$ and $\mathrm{pre}(o) \cap \mathrm{add}(o) = \emptyset$. An operator o is **applicable** in a state s if $\mathrm{pre}(o) \subseteq s$. The **resulting state** of applying an applicable operator o in a state s is the state $\mathrm{res}(o, s) = (s \setminus \mathrm{del}(o)) \cup \mathrm{add}(o)$. A state s is a **goal state** iff $s_{goal} \subseteq s$.

A **sequence of operators** $\pi = \langle o_1, ..., o_n \rangle$ is applicable in a state s_0 if there are states $s_1, ..., s_n$ such that o_i is applicable in s_{i-1} and $s_i = \mathrm{res}(o_i, s_{i-1})$ for $1 \leq i \leq n$. The resulting state of this application is $\mathrm{res}(\pi, s_0) = s_n$ and the cost of the plan is $\mathrm{c}(\pi) = \sum_{o \in \pi} \mathrm{c}(o)$. A sequence of operators π is called a **plan** iff $s_{goal} \subseteq \mathrm{res}(\pi, s_{init})$, and an **optimal plan** is a plan with the minimal cost over all plans.

A state s is called a **reachable state** if there exists an applicable operator sequence π such that $\mathrm{res}(\pi, s_{init}) = s$. A set of all reachable states is denoted by $\mathcal{R}_\Pi$.

Definition 2. An FDR planning task P is specified by a tuple $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, where $\mathcal{V}$ is a finite set of **variables**. Each variable $V \in \mathcal{V}$ has a finite domain D_V. A (partial) **state** s is a (partial) variable assignement over $\mathcal{V}$. We write $\mathrm{vars}(s)$ for the set of variables defined in s and $s[V]$ for the value of V in s. The notation $s[V] = \bot$ means that $V \notin \mathrm{vars}(s)$. A partial state s is **consistent** with a partial state s' if $s[V] = s'[V]$ for all $V \in \mathrm{vars}(s')$. We say that **atom** $V = v$ is true in a (partial) state s iff $s[V] = v$. By c we denote a cost function mapping each operator to a non-negative real number. An **operator** $o \in \mathcal{O}$ is a pair $o = \langle \mathrm{pre}(o), \mathrm{eff}(o) \rangle$, where precondition $\mathrm{pre}(o)$ and effect $\mathrm{eff}(o)$ are partial assignements over $\mathcal{V}$. We require that $V = v$ cannot be both a precondition and an effect. The (complete) state s_{init} is the **initial state** of the task and the partial state s_{goal} describes its **goal**.

An operator o is **applicable** in a state s if s is consistent with $\mathrm{pre}(o)$. The **resulting state** of applying an applicabe operator o in the state s is the state $\mathrm{res}(o, s)$ with

$$\mathrm{res}(o, s) = \begin{cases} \mathrm{eff}(o)[V] & \text{if } V \in \mathrm{vars}(\mathrm{eff}(o)), \\ s[V] & \text{otherwise.} \end{cases}$$

A **sequence of operators** $\pi = \langle o_1, ..., o_n \rangle$ is applicable in a state s_0 if there are states $s_1, ..., s_n$ such that o_i is applicable in s_{i-1} and $s_i = \mathrm{res}(o_i, s_{i-1})$ for $1 \leq i \leq n$. The resulting

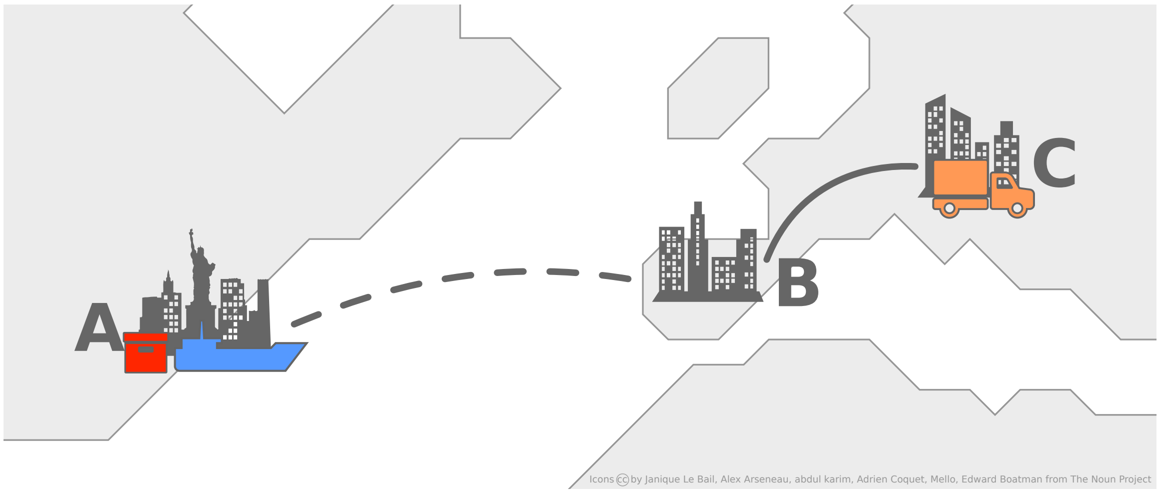

Figure 1: Example problem.

state of this application is $\mathrm{res}(\pi, s_0) = s_n$ and the cost of the plan is $\mathrm{c}(\pi) = \sum_{o \in \pi} \mathrm{c}(o)$. A sequence of operators π is called a **plan** iff $\mathrm{res}(\pi, s_{init})$ is consistent with s_{goal}, and an **optimal plan** is a plan with the minimal cost over all plans.

Definition 3. Let Π denote a STRIPS planning task. A sequence of operators π is called an s-**plan** iff π is applicable in a state s and $\mathrm{res}(\pi, s)$ is a goal state.

A **heuristic** $h : \mathcal{R}_\Pi \mapsto \mathbb{R} \cup \{\infty\}$ estimates the cost of optimal s-plans. The **optimal heuristic** $h^\star(s)$ maps each reachable state s to the cost of the optimal s-plan or to ∞ if there is no s-plan. A heuristic h is called (a) **admissible** iff $h(s) \leq h^\star(s)$ for every reachable state $s \in \mathcal{R}_\Pi$; (b) **goal-aware** iff $h(s) \leq 0$ for every reachable goal state s; (c) **safe** iff $h(s) = \infty$ implies $h^\star(s) = \infty$; and (d) **consistent** iff $h(s) \leq h(\mathrm{res}(o, s)) + \mathrm{c}(o)$ for all reachable states $s \in \mathcal{R}_\Pi$ and operators $o \in \mathcal{O}$ applicable in s.

Exercises

Ex. 1.1 — Let h denote a heuristic function. Which of the following statements hold?

1. If h is both goal-aware and safe, then h is admissible.
2. If h is both goal-aware and consistent, then h is admissible.
3. If h is both safe and consistent, then h is admissible.

Ex. 1.2 — Model the problem from Fig. 1 in STRIPS.

Ex. 1.3 — Model the problem from Fig. 1 in FDR.

2. $\mathrm{h}^{\max}$ Heuristic

Definition 4. Given a STRIPS planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, $\Pi^+ = \langle \mathcal{F}, \mathcal{O}^+, s_{init}, s_{goal}, \mathrm{c} \rangle$ denotes a **relaxed** STRIPS planning task, where $\mathcal{O}^+ = \{o_i^+ = \langle \mathrm{pre}(o_i), \mathrm{add}(o_i), \emptyset \rangle \mid o_i \in \mathcal{O}\}$.

Definition 5. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a STRIPS planning task. The heuristic function $\mathrm{h}^{\mathrm{add}}(s)$ gives an estimate of the distance from s to a node that satisfies the goal s_{goal} as $\mathrm{h}^{\mathrm{add}}(s) = \Sigma_{f \in s_{goal}} \Delta_0(s, f)$, where:

$$\Delta_0(s, o) = \Sigma_{f \in \mathrm{pre}(o)} \Delta_0(s, f), \quad \forall o \in \mathcal{O},$$

and

$$\Delta_0(s, f) = \begin{cases} 0 & \text{if } f \in s, \\ \infty & \text{if } \forall o \in \mathcal{O} : f \notin \mathrm{add}(o), \\ \min\{\mathrm{c}(o) + \Delta_0(s, o) \mid o \in \mathcal{O}, f \in \mathrm{add}(o)\} & \text{otherwise.} \end{cases}$$

Definition 6. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a STRIPS planning task. The heuristic function $\mathrm{h}^{\max}(s)$ gives an estimate of the distance from s to a node that satisfies the goal s_{goal} as $\mathrm{h}^{\max}(s) = \max_{f \in s_{goal}} \Delta_1(s, f)$, where:

$$\Delta_1(s, o) = \max_{f \in \mathrm{pre}(o)} \Delta_1(s, f), \quad \forall o \in \mathcal{O},$$

and

$$\Delta_1(s, f) = \begin{cases} 0 & \text{if } f \in s, \\ \infty & \text{if } \forall o \in \mathcal{O} : f \notin \mathrm{add}(o), \\ \min\{\mathrm{c}(o) + \Delta_1(s, o) \mid o \in \mathcal{O}, f \in \mathrm{add}(o)\} & \text{otherwise.} \end{cases}$$

Exercises

Ex. 2.1 — Modify Algorithm 1 to compute $\mathrm{h}^{\mathrm{add}}$ instead of $\mathrm{h}^{\max}$.

Ex. 2.2 — Compute $\mathrm{h}^{\max}(s_{init})$, $\mathrm{h}^{\mathrm{add}}(s_{init})$, $\mathrm{h}^{+}(s_{init})$, and $\mathrm{h}^{\star}(s_{init})$ for the following problem $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$:

$\mathcal{F} = \{a, b, c, d, e, f, g\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{a\}$	$\{c, d\}$	$\{a\}$	1
o_2	$\{a, b\}$	$\{e\}$	$\emptyset$	1
o_3	$\{b, e\}$	$\{d, f\}$	$\{a, e\}$	1
o_4	$\{b\}$	$\{a\}$	$\emptyset$	1
o_5	$\{d, e\}$	$\{g\}$	$\{e\}$	1

$s_{init} = \{a, b\}$, $s_{goal} = \{f, g\}$

Algorithm 1: Algorithm for computing $\mathrm{h}^{\max}(s)$.

Input: $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, state s

Output: $\mathrm{h}^{\max}(s)$

for each $f \in s$ **do** $\Delta_1(s, f) \leftarrow 0$;
for each $f \in \mathcal{F} \setminus s$ **do** $\Delta_1(s, f) \leftarrow \infty$;
for each $o \in \mathcal{O}, pre(o) = \emptyset$ **do**
    for each $f \in add(o)$ **do** $\Delta_1(s, f) \leftarrow \min\{\Delta_1(s, f), \mathrm{c}(o)\}$;
end
for each $o \in \mathcal{O}$ **do** $U(o) \leftarrow |\mathrm{pre}(o)|$;
$C \leftarrow \emptyset$;
while $s_{goal} \not\subseteq C$ **do**
    $k \leftarrow \arg\min_{f \in \mathcal{F} \setminus C} \Delta_1(s, f)$;
    $C \leftarrow C \cup \{k\}$;
    for each $o \in \mathcal{O}, k \in pre(o)$ **do**
        $U(o) \leftarrow U(o) - 1$;
        if $U(o) = 0$ **then**
            for each $f \in add(o)$ **do**
                $\Delta_1(s, f) \leftarrow \min\{\Delta_1(s, f), \mathrm{c}(o) + \Delta_1(s, k)\}$;
            end
        end
    end
end
$\mathrm{h}^{\max}(s) = \max_{f \in s_{goal}} \Delta_1(s, f)$;

3. LM-Cut Heuristic

Definition 7. A **disjunctive operator landmark** $L \subseteq \mathcal{O}$ is a set of operators such that every plan contains at least one operator from L.

Definition 8. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a planning task, let Δ_1 denote the function from Definition 6 for Π, and let $\mathrm{supp}(o) = \arg\max_{f \in \mathrm{pre}(o)} \Delta_1(s, f)$ denote a function mapping each operator to its **supporter**, where s is the state for which we want to compute the heuristic estimate.

A **justification graph** $G = (N, E)$ is a directed labeled multigraph with a set of nodes $N = \{n_f \mid f \in \mathcal{F}\}$ and a set of edges $E = \{(n_s, n_t, o) \mid o \in \mathcal{O}, s = \mathrm{supp}(o), t \in \mathrm{add}(o)\}$, where the triple (a, b, l) denotes an edge from a to b with the label l.

An **s-t-cut** $\mathcal{C}(G, s, t) = (N^0, N^\star \cup N^b)$ is a partitioning of nodes from the justification graph $G = (N, E)$ such that $N^\star$ contains all nodes from which t can be reached with a zero-cost path, N^0 contains all nodes reachable from s without passing through any node from $N^\star$, and $N^b = N \setminus (N^0 \cup N^\star)$.

Algorithm 2: Algorithm for computing $\mathrm{h}^{\text{lm-cut}}(s)$.

Input: $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, state s
Output: $\mathrm{h}^{\text{lm-cut}}(s)$

if $\mathrm{h}^{\max}(\Pi, s_{init}) = \infty$ **then**
  $\mathrm{h}^{\text{lm-cut}}(s) \leftarrow \infty$ and terminate;
end
$\mathrm{h}^{\text{lm-cut}}(s) \leftarrow 0$;
$\Pi_1 = \langle \mathcal{F}' = \mathcal{F} \cup \{I, G\}, \mathcal{O}' = \mathcal{O} \cup \{o_{init}, o_{goal}\}, s'_{init} = \{I\}, s'_{goal} = \{G\}, \mathrm{c}_1 \rangle$, where $\mathrm{pre}(o_{init}) = \{I\}$, $\mathrm{add}(o_{init}) = s$, $\mathrm{del}(o_{init}) = \emptyset$, $\mathrm{pre}(o_{goal}) = s_{goal}$, $\mathrm{add}(o_{goal}) = \{G\}$, $\mathrm{del}(o_{goal}) = \emptyset$, $\mathrm{c}_1(o_{init}) = 0$, $\mathrm{c}_1(o_{goal}) = 0$, and $\mathrm{c}_1(o) = \mathrm{c}(o)$ for all $o \in \mathcal{O}$;
$i \leftarrow 1$;
while $\mathrm{h}^{\max}(\Pi_i, s'_{init}) \neq 0$ **do**
  Construct a justification graph G_i from Π_i;
  Construct an s-t-cut $\mathcal{C}_i(G_i, n_I, n_G) = (N_i^0, N_i^\star \cup N_i^b)$;
  Create a landmark L_i as a set of labels of edges that cross the cut $\mathcal{C}_i$, i.e., they lead from N_i^0 to $N_i^\star$;
  $m_i \leftarrow \min_{o \in L_i} \mathrm{c}_i(o)$;
  $\mathrm{h}^{\text{lm-cut}}(s) \leftarrow \mathrm{h}^{\text{lm-cut}}(s) + m_i$;
  Set $\Pi_{i+1} = \langle \mathcal{F}', \mathcal{O}', s'_{init}, s'_{goal}, \mathrm{c}_{i+1} \rangle$, where $\mathrm{c}_{i+1}(o) = \mathrm{c}_i(o) - m_i$ if $o \in L_i$, and $\mathrm{c}_{i+1}(o) = \mathrm{c}_i(o)$ otherwise;
  $i \leftarrow i + 1$;
end

Exercises

Ex. 3.1 — Modify Algorithm 1 to compute $\mathrm{h}^{\max}$ and to find supporters from Definition 8 at the same time.

Ex. 3.2 — Compute $h^{\text{lm-cut}}(s_{init})$ for the following problem $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, c \rangle$:
$\mathcal{F} = \{s, t, q_1, q_2, q_3\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{s\}$	$\{q_1, q_2\}$	$\emptyset$	1
o_2	$\{s\}$	$\{q_1, q_3\}$	$\emptyset$	1
o_3	$\{s\}$	$\{q_2, q_3\}$	$\emptyset$	1
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	$\emptyset$	0

$s_{init} = \{s\}, s_{goal} = \{t\}$

Ex. 3.3 — Compute $h^{\max}(s_{init})$, $h^{\text{lm-cut}}(s_{init})$, $h^{+}(s_{init})$, and $h^{\star}(s_{init})$ for the following problem $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, c \rangle$:
$\mathcal{F} = \{a, b, c, d, e, i, g\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{i\}$	$\{a, b\}$	$\emptyset$	2
o_2	$\{i\}$	$\{b, c\}$	$\emptyset$	3
o_3	$\{a, c\}$	$\{d\}$	$\{c\}$	1
o_4	$\{b, d\}$	$\{e\}$	$\{b\}$	3
o_5	$\{a, c, e\}$	$\{g\}$	$\{c, d\}$	1
o_6	$\{a\}$	$\{e\}$	$\{a, c\}$	5

$s_{init} = \{i\}, s_{goal} = \{g\}$

Ex. 3.4 — Decide dominance for the following cases: $h^{\max} \succcurlyeq h^{\text{add}}$, $h^{\max} \succcurlyeq h^{\text{lm-cut}}$, $h^{\max} \succcurlyeq h^{+}$, $h^{\text{lm-cut}} \preccurlyeq h^{+}$, $h^{\text{lm-cut}} \succcurlyeq h^{\max}$.

4. Merge And Shrink Heuristic

Definition 9. A **transition system** is a tuple $\mathcal{T} = \langle S, L, T, I, G \rangle$, where S is a finite set of **states**, L is a finite set of **labels**, each label has **cost** $\mathrm{c}(l) \in \mathbb{R}^+_0$, $T \subseteq S \times L \times S$ is a **transition relation**, $I \subseteq S$ is a set of initial states, and $G \subseteq S$ is a set of goal states.

Definition 10. Given an FDR planning task $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, $\mathcal{T}(P) = \langle S, L, T, I, G \rangle$ denote a **state space of** P, where S is a set of states over $\mathcal{V}$, $L = \mathcal{O}$, $T = \{(s, o, t) \mid \mathrm{res}(o, s) = t\}$, $I = \{s_{init}\}$, and $G = \{s \mid s \in S, s \text{ is consistent with } s_{goal}\}$.

Definition 11. Let $\mathcal{T}^1 = \langle S^1, L, T^1, I^1, G^1 \rangle$ and $\mathcal{T}^2 = \langle S^2, L, T^2, I^2, G^2 \rangle$ denote two transition systems with the same set of labels, and let $\alpha : S^1 \mapsto S^2$. We say that S^2 is an **abstraction of** S^1 with **abstraction function** α if for every $s \in I^1$ it holds that $\alpha(s) \in I^2$ and for every $s \in G^1$ it holds that $\alpha(s) \in G^2$ and for every $(s, l, t) \in T^1$ it holds that $(\alpha(s), l, \alpha(t)) \in T^2$.

Definition 12. Let P denote an FDR planning task, let $\mathcal{A}$ denote an abstraction of a transition system $\mathcal{T}(P) = \langle S, L, T, I, G \rangle$ with the abstraction function α. The **abstraction heuristic** induced by $\mathcal{A}$ and α is the function $\mathrm{h}^{\mathcal{A},\alpha}(s) = \mathrm{h}^\star_{\mathcal{A}}(\alpha(s))$ for all $s \in S$.

Definition 13. Given two transition systems $\mathcal{T}^1 = \langle S^1, L, T^1, I^1, G^1 \rangle$ and $\mathcal{T}^2 = \langle S^2, L, T^2, I^2, G^2 \rangle$ with the same set of labels, the **synchronized product** $\mathcal{T}^1 \otimes \mathcal{T}^2 = \mathcal{T}$ is a transition system $\mathcal{T} = \langle S, L, T, I, G \rangle$, where $S = S^1 \times S^2$, $T = \{((s_1, s_2), l, (t_1, t_2)) \mid (s_1, l, s_2) \in T^1, (s_2, l, t_2) \in T^2\}$, $I = I^1 \times I^2$, and $G = G^1 \times G^2$.

Algorithm 3: Algorithm for computing merge-and-shrink.

Input: $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$
Output: Abstraction $\mathcal{M}$

$\mathcal{A} \leftarrow$ Set of (atomic) abstractions $(\alpha_i, \mathcal{T}_i)$ of $\mathcal{T}(P)$;
while $|\mathcal{A}| > 1$ **do**
    $A_1 = (\alpha_1, \mathcal{T}_1), A_2 = (\alpha_2, \mathcal{T}_2) \leftarrow$ Select two abstractions from $\mathcal{A}$;
    Shrink A_1 and/or A_2 until they are "small enough";
    $\mathcal{A} \leftarrow (\mathcal{A} \setminus \{A_1, A_2\}) \cup (A_1 \otimes A_2)$ `// Merge`
end
$\mathcal{M} \leftarrow$ The only element of $\mathcal{A}$;

Exercises

Ex. 4.1 — Compute the synchronized product of $\mathcal{T}^1 = \langle S^1, L, T^1, I^1, G^1 \rangle$ and $\mathcal{T}^2 = \langle S^2, L, T^2, I^2, G^2 \rangle$, where $L = \{a, b, c, d, e\}$, $S^1 = \{A, B, C, D\}$, $T^1 = \{(A, a, B), (B, b, C), (C, c, A), (A, d, A), (A, e, D)\}$, $I^1 = \{A\}$, $G^1 = \{A, C\}$, $S^2 = \{X, Y, Z\}$, $T^2 = \{(X, a, Y), (X, a, Z), (Y, b, Z), (Z, c, Y), (Z, d, Y), (Z, e, Z)\}$, $I^2 = \{X\}$, and $G^2 = \{X\}$.

Ex. 4.2 — Study merge and shrink strategies proposed by Helmert, Haslum, and Hoffmann (2007) and compute $\mathrm{h}^{\mathrm{m\&s}}(s_{init})$ for the problem in Fig. 1 (Ex. 1.3).

5. LP-Based Heuristics

Definition 14. Let $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote an FDR planning task. For every variable $V \in \mathcal{V}$ and every value $v \in D_V$, we define:

- A set of operators producing $\langle V, v \rangle$: $\mathrm{prod}(\langle V, v \rangle) = \{o \mid o \in \mathcal{O}, V \in \mathrm{vars}(\mathrm{eff}(o)), \mathrm{eff}(o)[V] = v\}$, and
- a set of operators consuming $\langle V, v \rangle$: $\mathrm{cons}(\langle V, v \rangle) = \{o \mid o \in \mathcal{O}, V \in \mathrm{vars}(\mathrm{pre}(o)) \cap \mathrm{vars}(\mathrm{eff}(o)), \mathrm{pre}(o)[V] = v, \mathrm{pre}(o)[V] \neq \mathrm{eff}(o)[V]\}$.

Definition 15. Let $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote an FDR planning task, and s a state reachable from s_{init}. Given the following linear program with real-valued variables x_o for each operator $o \in \mathcal{O}$:

$$\begin{aligned} \text{minimize} \quad & \sum_{o \in \mathcal{O}} c(o) x_o \\ \text{subject to} \quad & LB_{V,v} \leq \sum_{o \in \mathrm{prod}(\langle V, v \rangle)} x_o - \sum_{o \in \mathrm{cons}(\langle V, v \rangle)} x_o \quad \forall V \in \mathcal{V}, \forall v \in D_V, \end{aligned}$$

where

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \mathrm{vars}(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \mathrm{vars}(s_{goal}) \text{ and } s_{goal}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \mathrm{vars}(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \mathrm{vars}(s_{goal}) \text{ or } s_{goal}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

then the value of the **flow heuristic** $\mathrm{h}^{\mathrm{flow}}(s)$ for the state s is

$$\mathrm{h}^{\mathrm{flow}}(s) = \begin{cases} \left\lceil \sum_{o \in \mathcal{O}} c(o) x_o \right\rceil & \text{if the solution is feasible,} \\ \infty & \text{if the solution is not feasible.} \end{cases}$$

(Bonet, 2013; Bonet & van den Briel, 2014)

Definition 16. Let $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote an FDR planning task and s a state reachable from s_{init}. Given the following linear program with real-valued variables $P_{V,v}$ for each variable $V \in \mathcal{V}$ and each value $v \in D_V$, and real-valued variables M_V for each variable $V \in \mathcal{V}$:

$$\begin{aligned} \text{maximize} \quad & \sum_{V \in \mathcal{V}} P_{V, s_{init}[V]} \\ \text{subject to} \quad & P_{V,v} \leq M_V && \forall V \in \mathcal{V}, \forall v \in D_V \\ & \sum_{V \in \mathcal{V}} maxpot(V, s_{goal}) \leq 0 \\ & \sum_{V \in \mathrm{vars}(\mathrm{eff}(o))} (maxpot(V, \mathrm{pre}(o)) - P_{V, \mathrm{eff}(o)[V]}) \leq \mathrm{c}(o) && \forall o \in \mathcal{O}, \end{aligned}$$

where

$$maxpot(V,p) = \begin{cases} P_{V,p[V]} & \text{if } V \in \mathrm{vars}(p), \\ M_V & \text{otherwise .} \end{cases}$$

then the value of the **potential heuristic** $\mathrm{h}^{\mathrm{pot}}(s)$ for the state s is

$$\mathrm{h}^{\mathrm{pot}}(s) = \begin{cases} \sum_{V\in\mathcal{V}} P_{V,s[V]} & \text{if the solution is feasible,} \\ \infty & \text{if the solution is not feasible.} \end{cases}$$

(Pommerening, Helmert, Röger, & Seipp, 2015; Seipp, Pommerening, & Helmert, 2015)

Exercises

Ex. 5.1 — Compute the $\mathrm{h}^{\mathrm{flow}}(s_{init})$ and $\mathrm{h}^{\mathrm{pot}}(s_{init})$ for the following FDR planning task $P = \langle \mathcal{V}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$:
$\mathcal{V} = \{A, B, C\}$,
$D_A = \{D, E\}$, $D_B = \{F, G\}$, $D_C = \{H, J, K\}$,
$s_{init} = \{A = D, B = F, C = H\}$, $s_{goal} = \{A = D, C = K\}$
$\mathcal{O} = \{o_1, o_2, o_3, o_4, o_5\}$,
$o_1 : A = D, C = H \mapsto A = E, C = J$, $\mathrm{c}(o_1) = 2$,
$o_2 : A = D \mapsto B = G$, $\mathrm{c}(o_2) = 1$,
$o_3 : B = G, C = J \mapsto C = K$, $\mathrm{c}(o_3) = 1$,
$o_4 : A = E \mapsto A = D$, $\mathrm{c}(o_4) = 2$,
$o_5 : C = H \mapsto C = J$, $\mathrm{c}(o_5) = 5$.

Ex. 5.2 — How can be flow heuristic improved with landmarks (e.g., from the LM-Cut heuristic)?

Ex. 5.3 — How can we modify objective of the LP for the potential heuristic so we still obtain admissible estimate for all reachable states?

6. Mutual Exclusion Invariants

Definition 17. A **mutex** $M \subseteq \mathcal{F}$ is a set of facts such that for every reachable state s it holds that $M \not\subseteq s$.

Definition 18. A **mutex group** $M \subseteq \mathcal{F}$ is a set of facts such that for every reachable state s it holds that $|M \cap s| \leq 1$. A mutex group that is not subset of any other mutex group is called a **maximal mutex group**.

Definition 19. A **fact-alternating mutex group** (fam-group) $M \subseteq \mathcal{F}$ is a set of facts such that $|M \cap s_{init}| \leq 1$ and $|M \cap \mathrm{add}(o)| \leq |M \cap \mathrm{pre}(o) \cap \mathrm{del}(o)|$ for every operator $o \in \mathcal{O}$. A fam-group that is not subset of any other fam-group is called a **maximal fam-group**.

Proposition 20. *Every fam-group is a mutex group.*

Algorithm 4: Inference of fact-alternating mutex groups using ILP.

Input: STRIPS planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$
Output: A set of fam-groups $\mathcal{M}$

Create ILP with a binary variable $x_i \in \{0, 1\}$ for every fact $f_i \in \mathcal{F}$;
Add constraint $\sum_{f_i \in s_{init}} x_i \leq 1$;
For each operator $o \in \mathcal{O}$ add constraint $\sum_{f_i \in \mathrm{add}(o)} x_i \leq \sum_{f_i \in \mathrm{del}(o) \cap \mathrm{pre}(o)} x_i$;
Set objective function of ILP to maximize $\sum_{f_i \in \mathcal{F}} x_i$;
$M \leftarrow \emptyset$;
Solve ILP and if a solution was found, save $\{f_i \mid f_i \in \mathcal{F}, x_i = 1\}$ into M;
while $|M| \geq 1$ **do**
    Add M to the output set $\mathcal{M}$;
    Add constraint $\sum_{f_i \notin M} x_i \geq 1$;
    $M \leftarrow \emptyset$;
    Solve ILP and if a solution was found, save $\{f_i \mid f_i \in \mathcal{F}, x_i = 1\}$ into M;
end

Theorem 21. *Algorithm 4 is complete with respect to the maximal fam-groups.*

(Haslum & Geffner, 2000; Haslum, Bonet, & Geffner, 2005; Haslum, 2009; Fišer & Komenda, 2018)

Exercises

Ex. 6.1 — Translate the FDR planning task from Ex. 5.1 into STRIPS.

Ex. 6.2 — Translate the following STRIPS planning task into FDR: $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$:
$\mathcal{F} = \{a, b, c, d, e, f\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{a\}$	$\{b\}$	$\{a\}$	1
o_2	$\{b\}$	$\{a\}$	$\{b\}$	1
o_3	$\{b\}$	$\{c\}$	$\{b\}$	1
o_4	$\{a, d\}$	$\{f\}$		1
o_5	$\{c, d, f\}$	$\{e\}$	$\{d, f\}$	1

$s_{init} = \{b, d\}, s_{goal} = \{e\}$
Try to guess mutex groups.

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