Heuristic (Informed) Search

(Where we try to choose smartly)

R&N: Chap. 4, Sect. 4.1–3

Search Algorithm #2

SEARCH#2

1. INSERT(initial-node,Open-List)
2. Repeat:
   a. If empty(Open-List) then return failure
   b. N ← REMOVE(Open-List)
   c. s ← STATE(N)
   d. If GOAL?(s) then return path or goal state
   e. For every state s' in SUCCESSORS(s)
      i. Create a node N' as a successor of N
      ii. INSERT(N',Open-List)

Recall that the ordering of Open List defines the search strategy

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Best-First Search

- It exploits state description to estimate how "good" each search node is
- An evaluation function f maps each node N of the search tree to a real number $f(N) \geq 0$
  [Traditionally, f(N) is an estimated cost; so, the smaller f(N), the more promising N]
- Best-first search sorts the Open List in increasing f
  [Arbitrary order is assumed among nodes with equal f]

Best-First Search

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- An evaluation function f maps each node N of the search t
  f(N) ≥ 0
  [Traditionally, f(N)
  f(N), the more prom

"Best" does not refer to the quality of the generated path
Best-first search does not generate optimal paths in general

- Best-first search sorts the Open List in increasing f
  [Arbitrary order is assumed among nodes with equal f]

How to construct f?

- Typically, f(N) estimates:
  - either the cost of a solution path through N

    Then $f(N) = g(N) + h(N)$, where
    - g(N) is the cost of the path from the initial node to N
    - h(N) is an estimate of the cost of a path from N to a goal node
  - or the cost of a path from N to a goal node

    Then $f(N) = h(N)$ → Greedy best-search

- But there are no limitations on f. Any function of your choice is acceptable.

  But will it help the search algorithm?

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    Then f(N) = h(N)

Heuristic function

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  But will it help the search algorithm?

Heuristic Function

- The heuristic function $h(N) \geq 0$ estimates the cost to go from STATE(N) to a goal state

  Its value is **independent of the current search tree**; it depends only on STATE(N) and the goal test GOAL?

- Example:

5		8
4	2	1
7	3	6

STATE(N)

1	2	3
4	5	6
7	8	

Goal state

$h_1(N)$ = number of misplaced numbered tiles = 6

[Why is it an estimate of the distance to the goal?]

Other Examples

5		8
4	2	1
7	3	6

STATE(N)

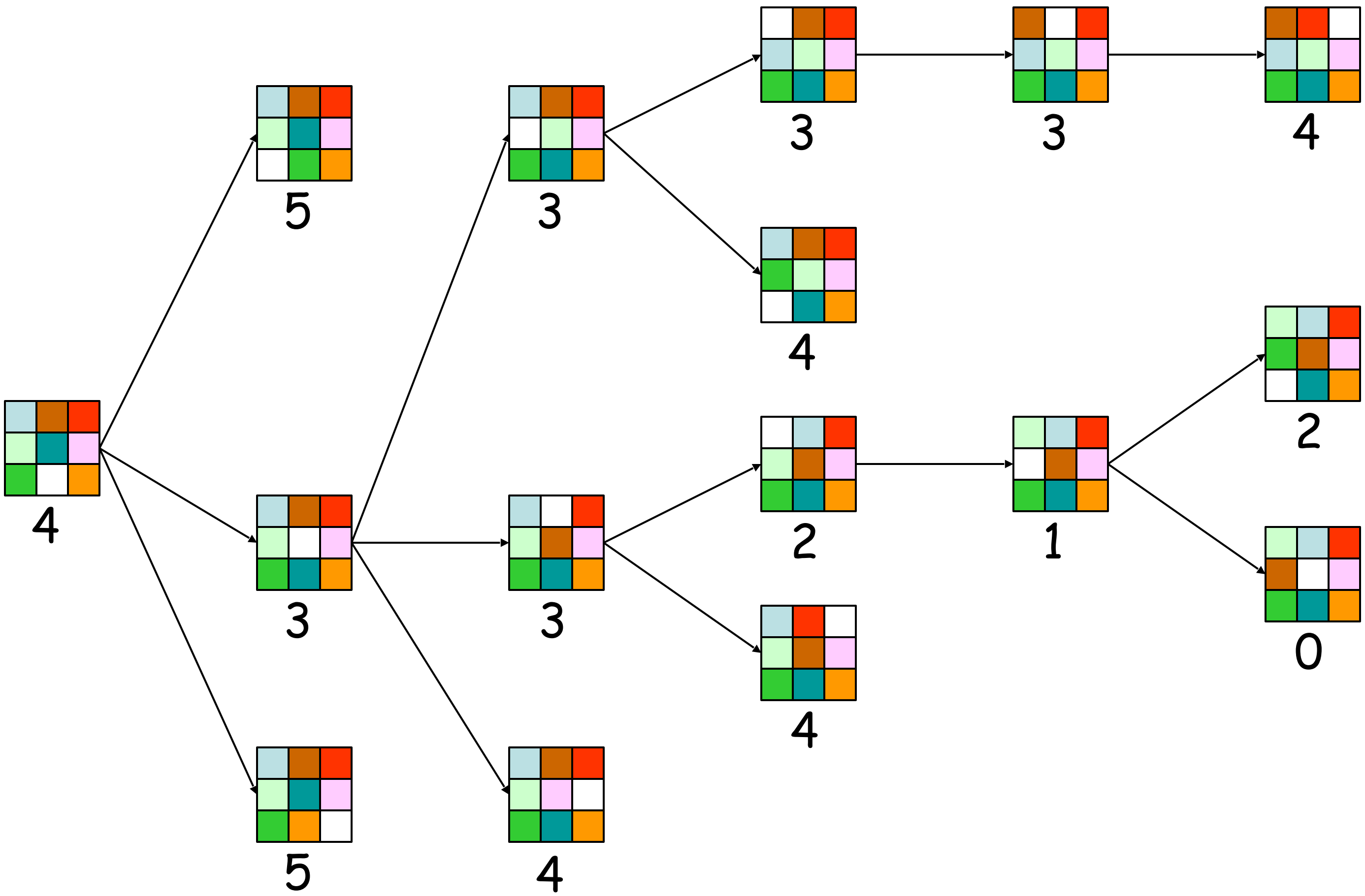
1	2	3
4	5	6
7	8	

Goal state

- $h_1(N)$ = number of misplaced numbered tiles = 6
- $h_2(N)$ = sum of the (Manhattan) distance of every numbered tile to its goal position
  = 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1 = 13
- $h_3(N)$ = sum of permutation inversions
  $= n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6$
  = 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0
  = 16

8-Puzzle

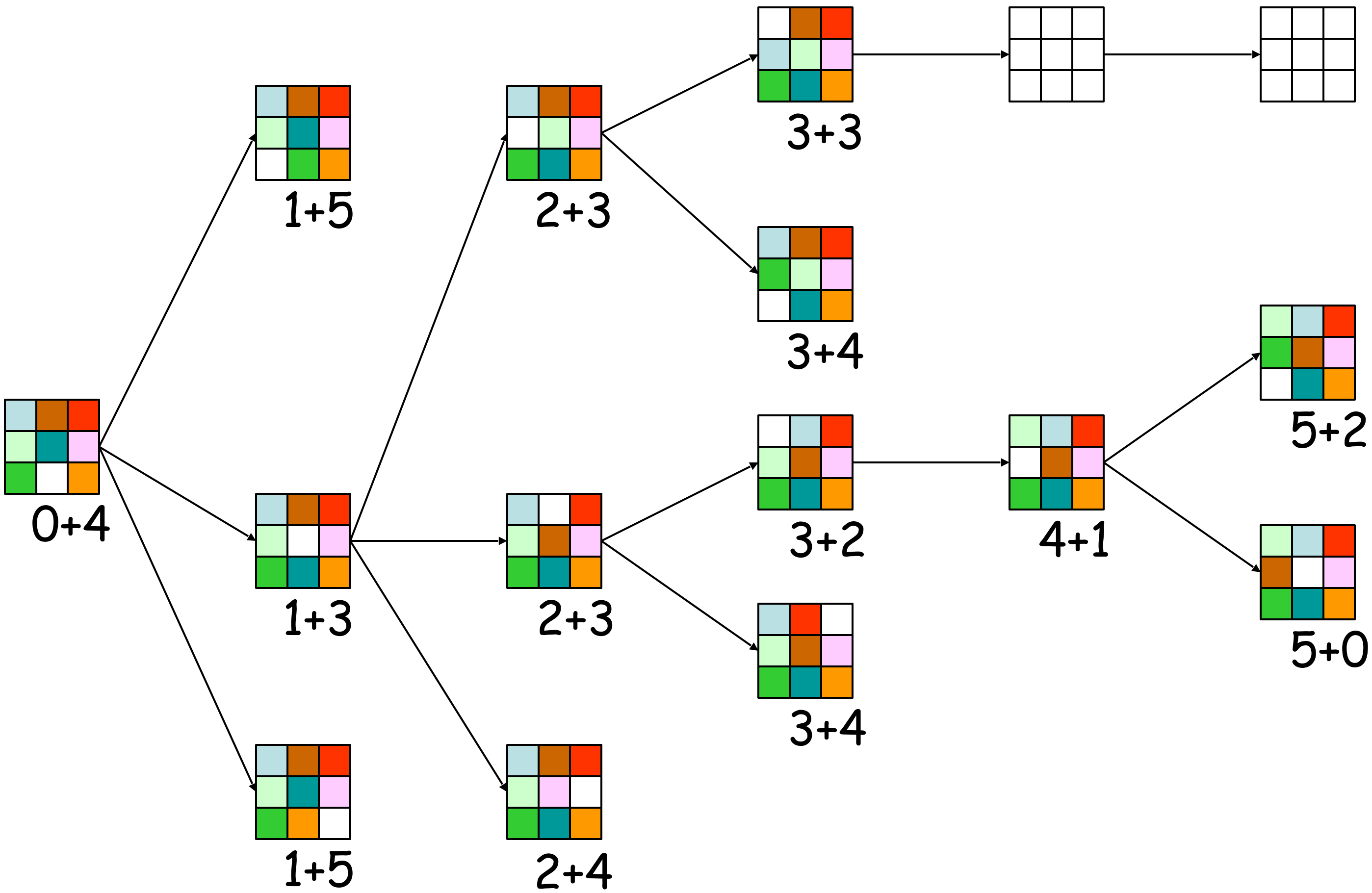
f(N) = h(N) = number of misplaced numbered tiles



The white tile is the empty tile

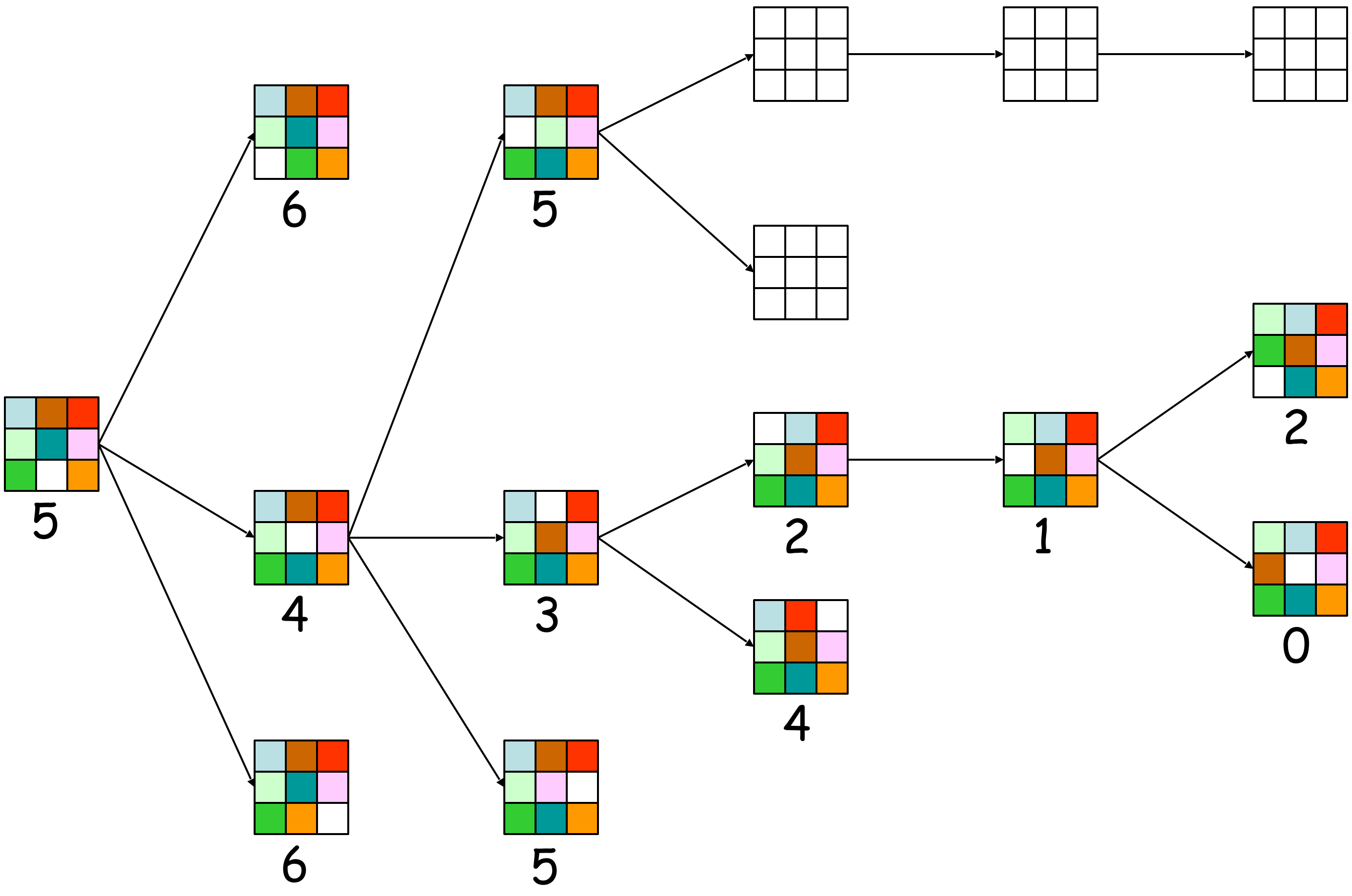
8-Puzzle

$f(N) = g(N) + h(N)$

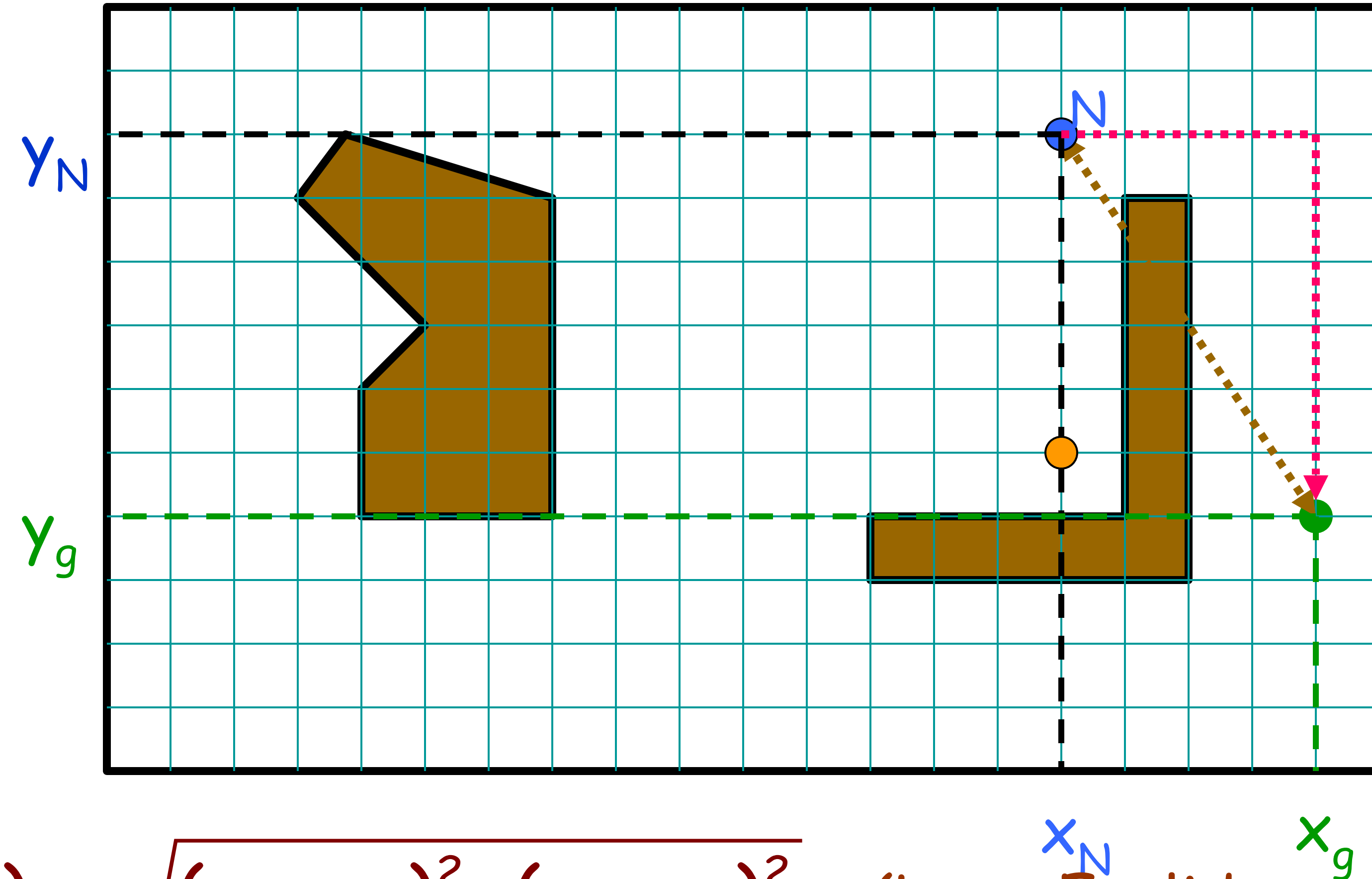


8-Puzzle

f(N) = h(N) = Σ distances of numbered tiles to their goals



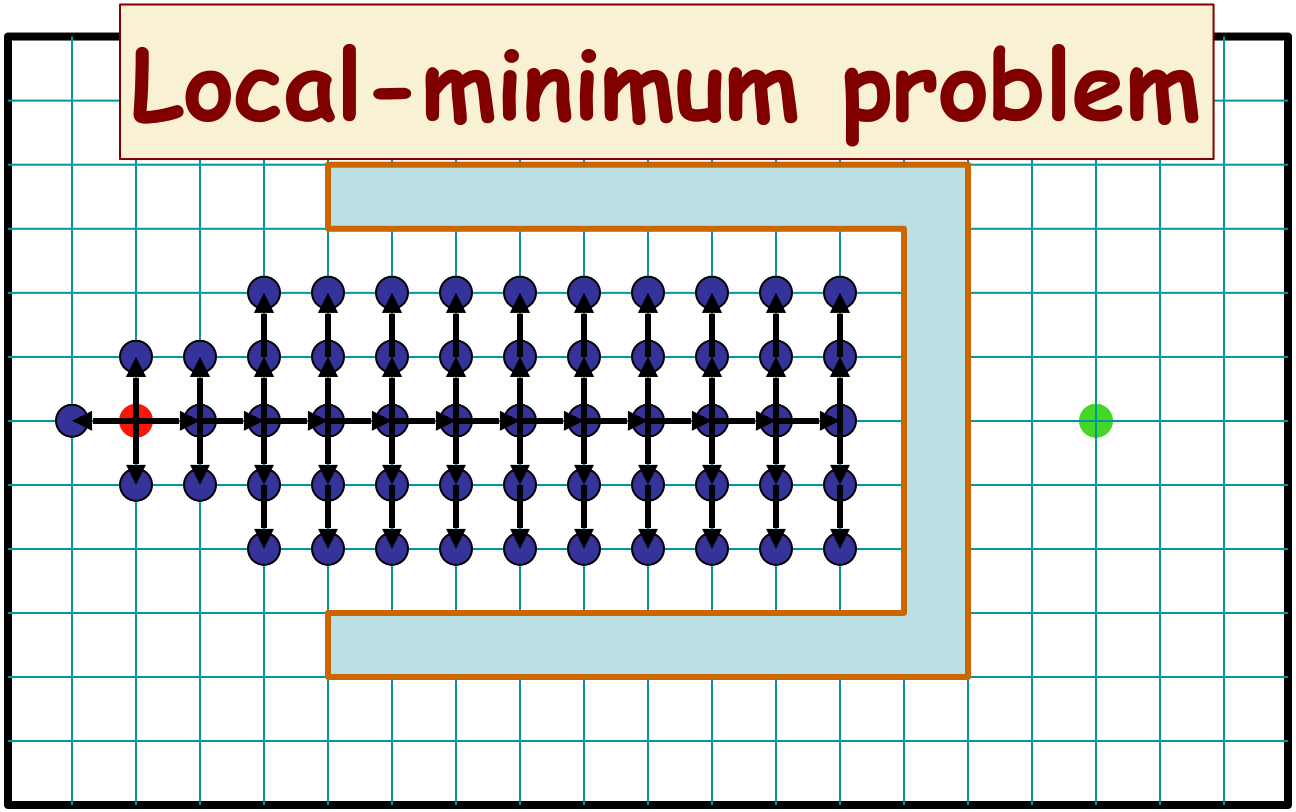
Robot Navigation



$$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$$ (L_2 or Euclidean distance)

$$h_2(N) = |x_N - x_g| + |y_N - y_g|$$ (L_1 or Manhattan distance)

Best-First ↛ Efficiency



f(N) = h(N) = straight distance to the goal

Can we prove anything?

- If the state space is infinite, in general the search is not complete
- If the state space is finite and we do not discard nodes that revisit states, in general the search is not complete
- If the state space is finite and we discard nodes that revisit states, the search is complete, but in general is not optimal

Admissible Heuristic

- Let $h^*(N)$ be the cost of the optimal path from N to a goal node
- The heuristic function $h(N)$ is admissible if:

$$0 \leq h(N) \leq h^*(N)$$

- An admissible heuristic function is always optimistic !

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G is a goal node → h(G) = 0

8-Puzzle Heuristics

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Goal state

- $h_1(N)$ = number of misplaced tiles = 6 is ???

8-Puzzle Heuristics

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  is admissible
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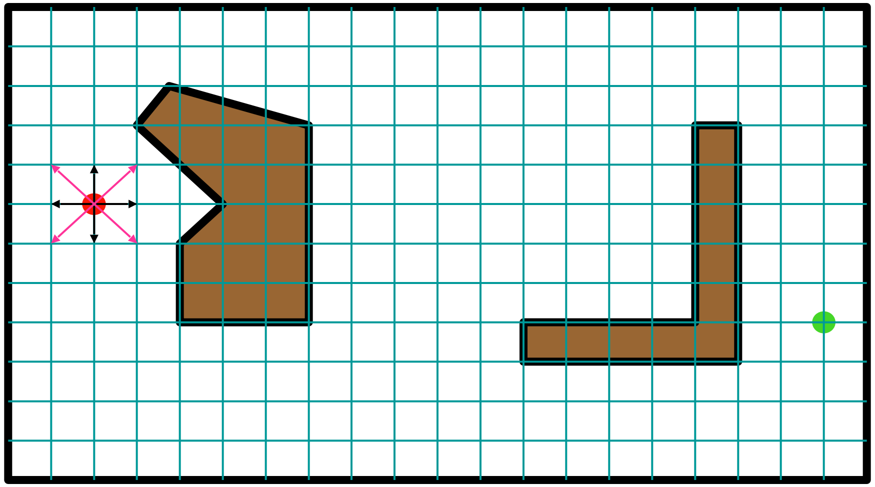
  is admissible
- $h_3(N)$ = sum of permutation inversions
  = 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0 = 16
  is not admissible

Robot Navigation Heuristics

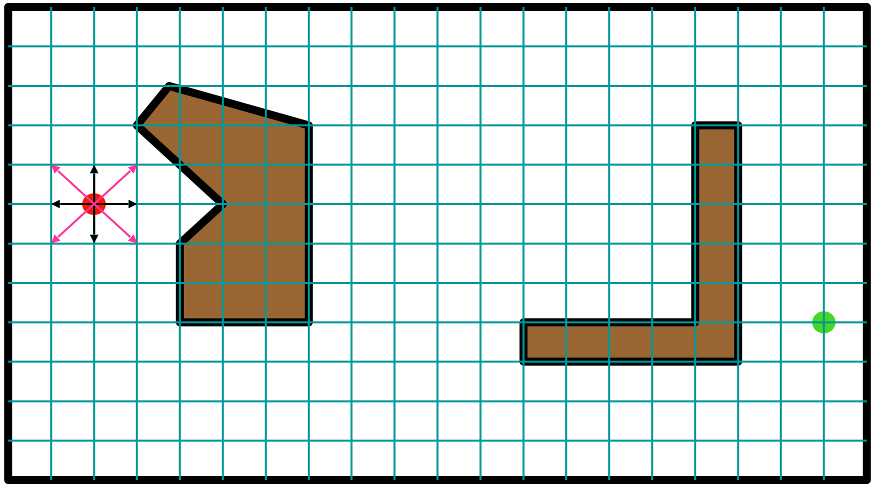


Cost of one horizontal/vertical step = 1

Cost of one diagonal step = $\sqrt{2}$

$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ is admissible

Robot Navigation Heuristics



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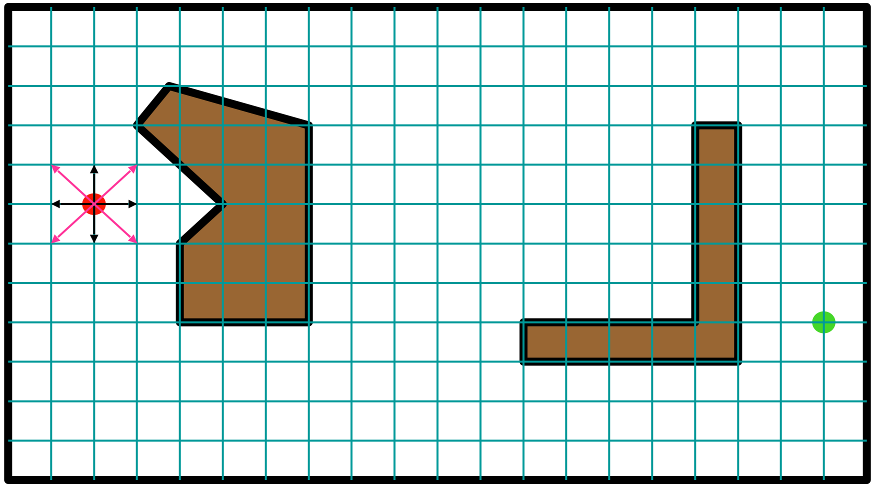
Robot Navigation Heuristics



Cost of one horizontal/vertical step = 1

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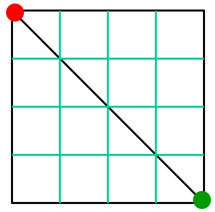
$h_2(N) = |x_N - x_g| + |y_N - y_g|$ is admissible if moving along diagonals is not allowed, and not admissible otherwise

$h^*(I) = 4\sqrt{2}$

$h_2(I) = 8$



How to create an admissible h?

- An admissible heuristic can usually be seen as the cost of an optimal solution to a relaxed problem (one obtained by removing constraints)
- In robot navigation:
  - The Manhattan distance corresponds to removing the obstacles
  - The Euclidean distance corresponds to removing both the obstacles and the constraint that the robot moves on a grid
- More on this topic later

A* Search
(most popular algorithm in AI)

1) $f(N) = g(N) + h(N)$, where:
   - $g(N)$ = cost of best path found so far to N
   - $h(N)$ = **admissible** heuristic function

2) for all arcs: $c(N,N') \geq \varepsilon > 0$

3) SEARCH#2 algorithm is used

→ Best-first search is then called *A* search*

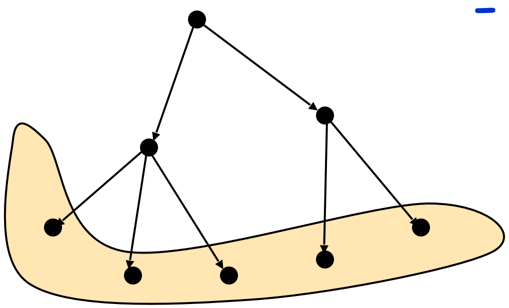
Result #1

A* is complete and optimal

[This result holds if nodes revisiting states are not discarded]

Proof (1/2)

1) If a solution exists, A* terminates and returns a solution



- For each node N on the Open List, $f(N) = g(N) + h(N) \geq g(N) \geq d(N) \times \varepsilon$, where d(N) is the depth of N in the tree

SEARCH#2

1. INSERT(initial-node,Open List)
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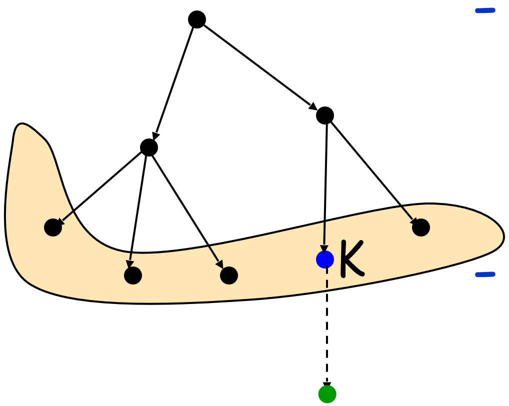

- For each node N on the Open List, $f(N) = g(N) + h(N) \geq g(N) \geq d(N) \times \varepsilon$, where d(N) is the depth of N in the tree
- As long as A* hasn't terminated, a node K on the Open List lies on a solution path

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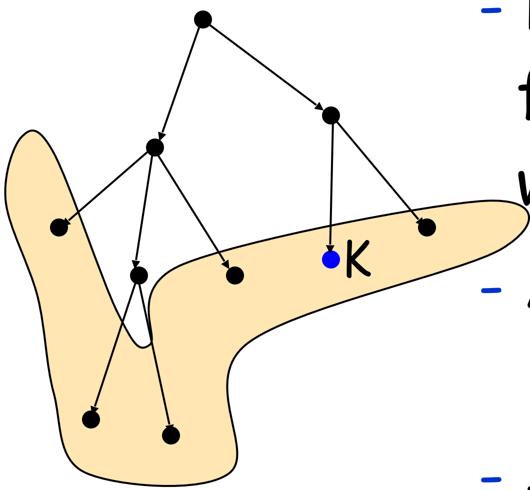

2. Repeat:
   a. If empty(Open List) then returr
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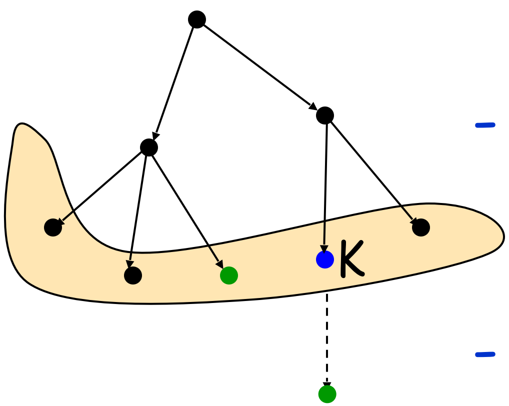
- For each node N on the Open List, $f(N) = g(N)+h(N) \geq g(N) \geq d(N)\times\varepsilon$, where d(N) is the depth of N in the tree
- As long as A* hasn't terminated, a node K on the Open List lies on a solution path
- Since each node expansion increases the length of one path, K will eventually be selected for expansion, unless a solution is found along another path

Proof (2/2)

2) Whenever A* chooses to expand a goal node, the path to this node is optimal



- C^*= cost of the optimal solution path

- G': non-optimal goal node in the Open List

  $f(G') = g(G') + h(G') = g(G') > C^*$

- A node K in the Open List lies on an optimal path:

  $f(K) = g(K) + h(K) \leq C^*$

- So, G' will not be selected for expansion

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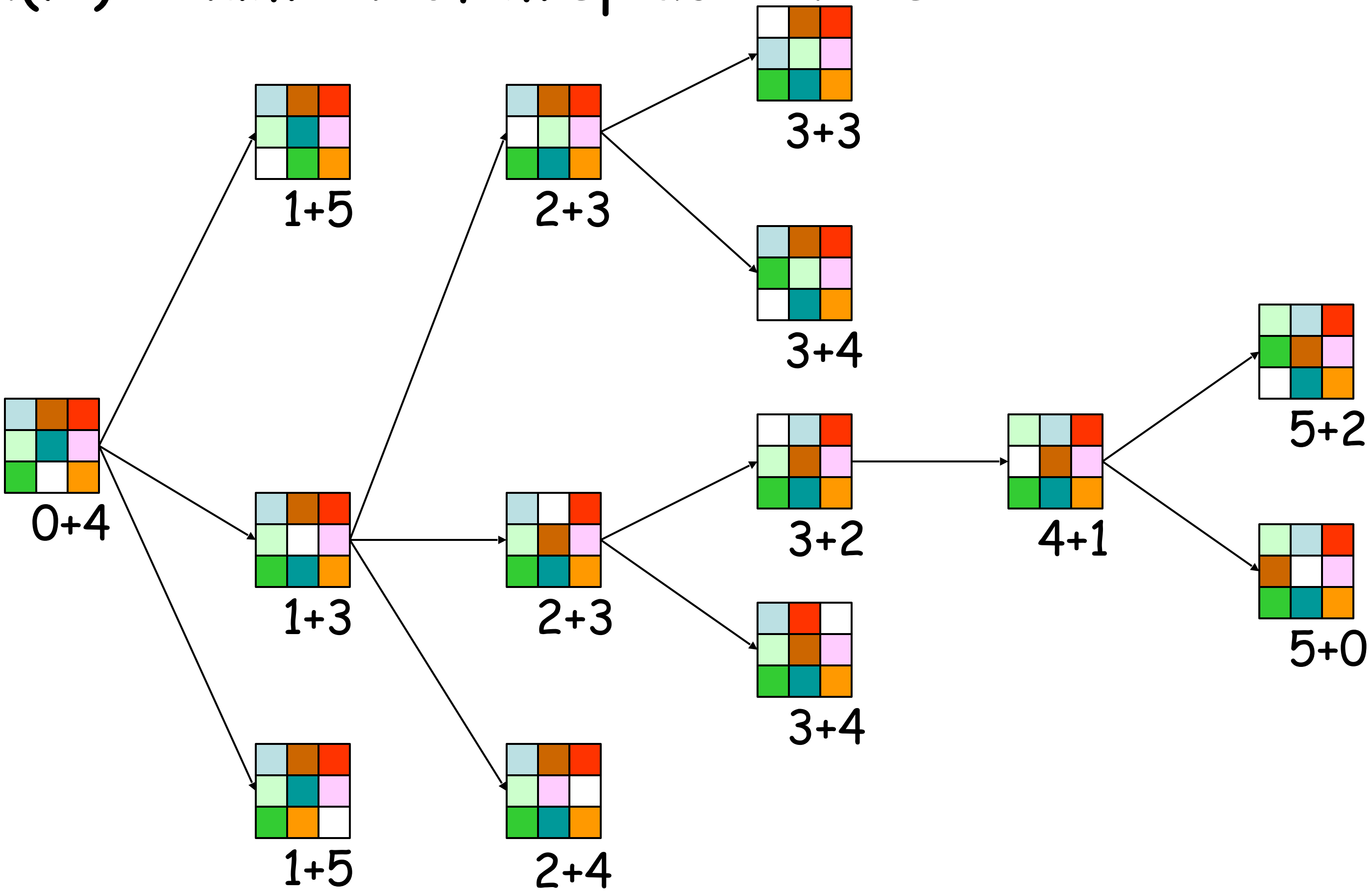
Time Limit Issue

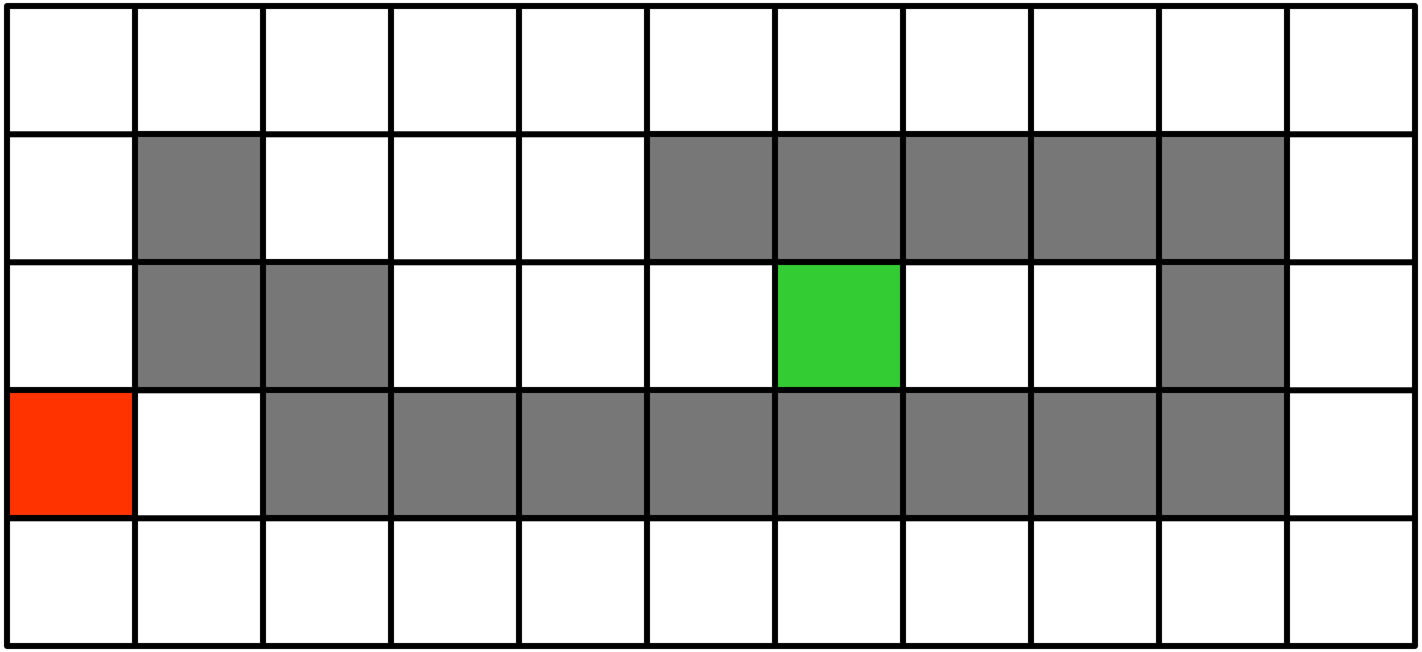
- When a problem has no solution, A* runs for ever if the state space is infinite. In other cases, it may take a huge amount of time to terminate
- So, in practice, A* is given a time limit. If it has not found a solution within this limit, it stops. Then there is no way to know if the problem has no solution, or if more time was needed to find it
- When AI systems are "small" and solving a single search problem at a time, this is not too much of a concern.
- When AI systems become larger, they solve many search problems concurrently, some with no solution.

8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



Robot Navigation



Robot Navigation

f(N) = h(N), with h(N) = Manhattan distance to the goal
(not *A**)

8	7	6	5	4	3	2	3	4	5	6
7		5	4	3						5
6			3	2	1	0	1	2		4
7	6									5
8	7	6	5	4	3	2	3	4	5	6

Robot Navigation

f(N) = h(N), with h(N) = Manhattan distance to the goal
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8	7	6	5	4	3	2	3	4	5	6
7		5	4	3						5
6			3	2	1	0	1	2		4
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8	7	6	5	4	3	2	3	4	5	6

Robot Navigation

f(N) = g(N)+h(N), with h(N) = Manhattan distance to goal
(A*)

8+3	7+4	6+3	5+6	4+7	3+8	2+9	3+10	4	5	6
7+2		5+6	4+7	3+8						5
6+1			3	2+9	1+10	0+11	1	2		4
7+0	6+1									5
8+1	7+2	6+3	5+4	4+5	3+6	2+7	3+8	4	5	6

Best-First Search

- An evaluation function f maps each node N of the search tree to a real number
  $f(N) \geq 0$
- Best-first search sorts the Open List in increasing f

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2) for all arcs: $c(N,N') \geq \varepsilon > 0$

3) SEARCH#2 algorithm is used

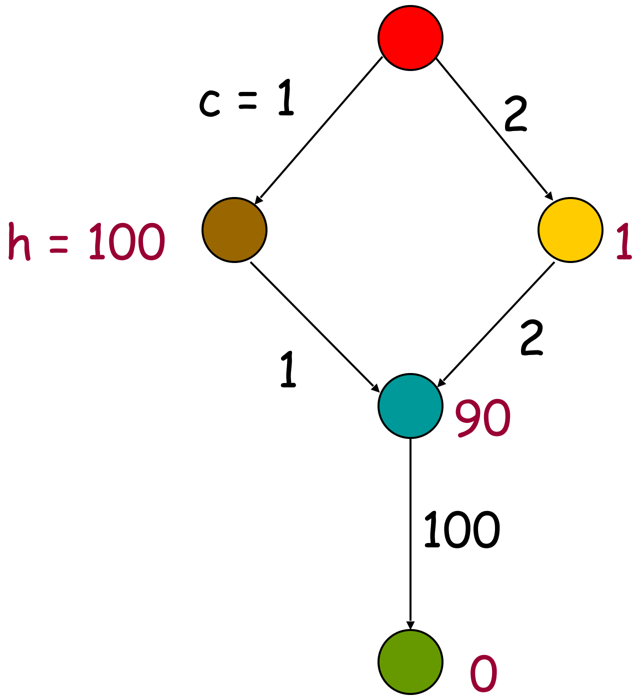
→ Best-first search is then called *A* search*

Result #1

A* is complete and optimal

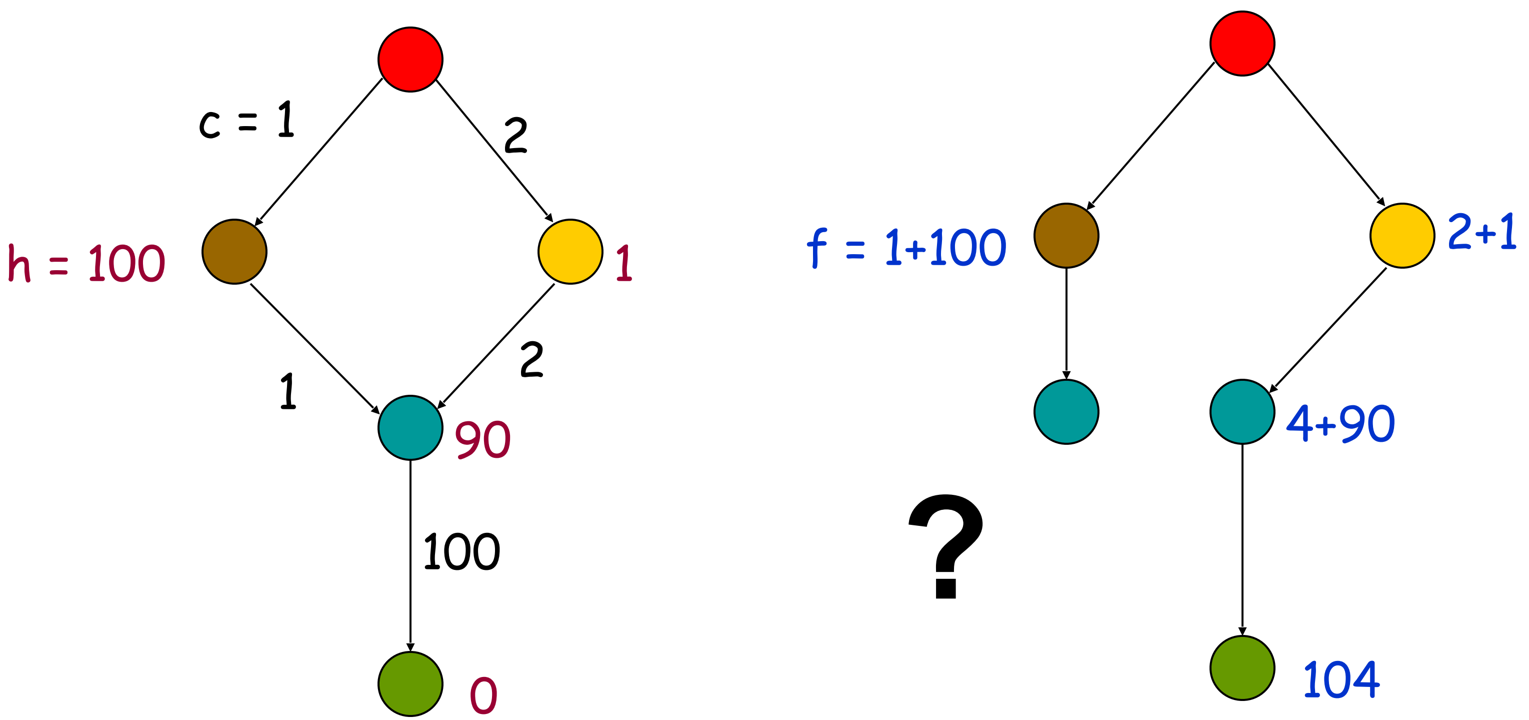
[This result holds if nodes revisiting states are not discarded]

What to do with revisited states?



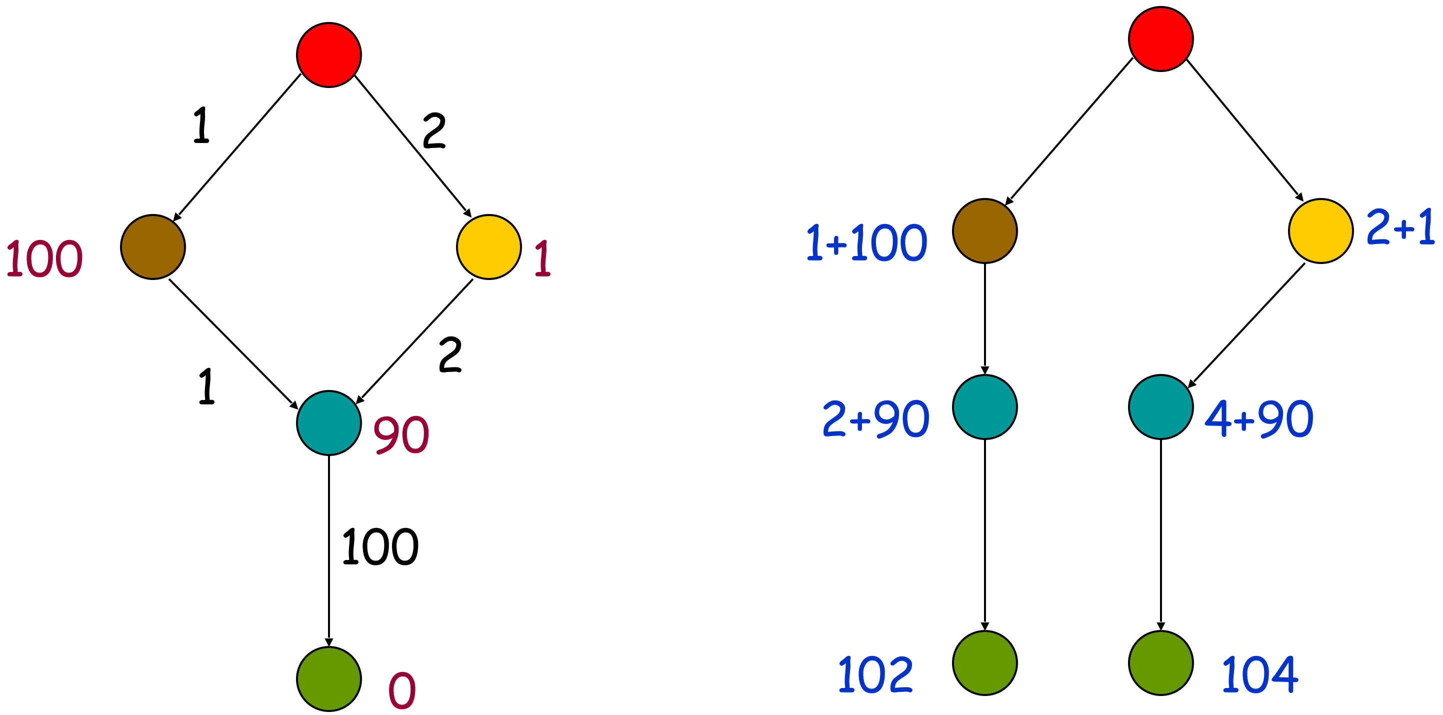
The heuristic h is clearly admissible

What to do with revisited states?



If we discard this new node, then the search algorithm expands the goal node next and returns a non-optimal solution

What to do with revisited states?



Instead, if we do not discard nodes revisiting states, the search terminates with an optimal solution

- It is not harmful to discard a node revisiting a state if the cost of the new path to this state is ≥ cost of the previous path
  [so, in particular, one can discard a node if it re-visits a state already visited by one of its ancestors]

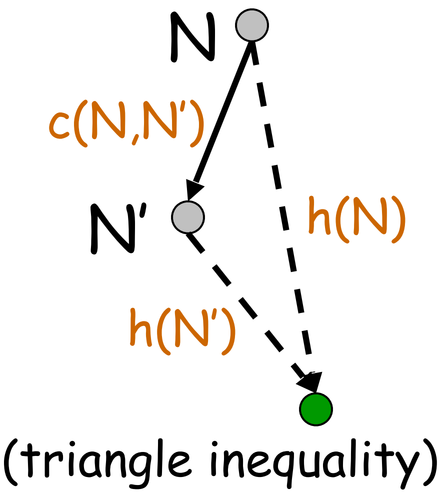
- A* remains optimal, but states can still be re-visited multiple times
  [the size of the search tree can still be exponential in the number of visited states]

- Fortunately, for a large family of admissible heuristics – consistent heuristics – there is a much more efficient way to handle revisited states

Consistent Heuristic

An admissible heuristic h is consistent (or monotone) if for each node N and each child N' of N:

$$h(N) \leq c(N,N') + h(N')$$

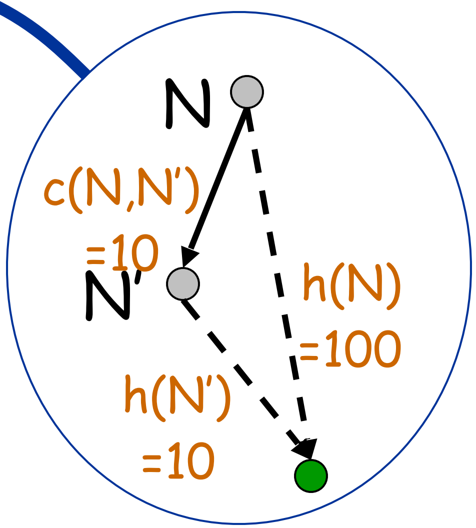


$$h(N) \leq C^*(N) \leq c(N,N') + h^*(N')$$
$$h(N) - c(N,N') \leq h^*(N')$$
$$h(N) - c(N,N') \leq h(N') \leq h^*(N')$$

→ Intuition: a consistent heuristics becomes more precise as we get deeper in the search tree

Consistency Violation

If h tells that N is 100 units from the goal, then moving from N along an arc costing 10 units should **not** lead to a node N' that h estimates to be 10 units away from the goal



(triangle inequality)

Consistent Heuristic

(alternative definition)

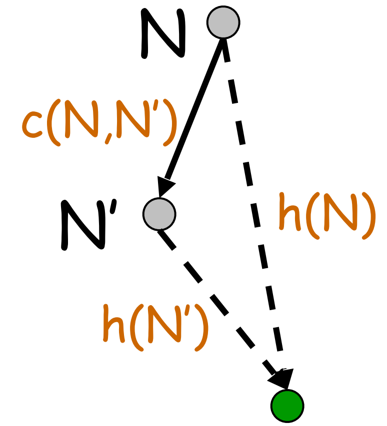
A heuristic h is consistent (or monotone) if

1) for each node N and each child N' of N:

$$h(N) \leq c(N,N') + h(N')$$

2) for each goal node G:

$$h(G) = 0$$



(triangle inequality)

A consistent heuristic is also admissible

Admissibility and Consistency

- A consistent heuristic is also admissible
- An admissible heuristic may not be consistent, but many admissible heuristics are consistent

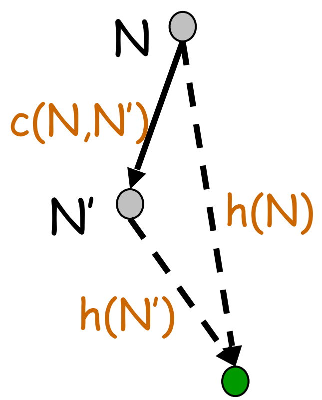
8-Puzzle

5		8
4	2	1
7	3	6

STATE(N)

1	2	3
4	5	6
7	8	

goal

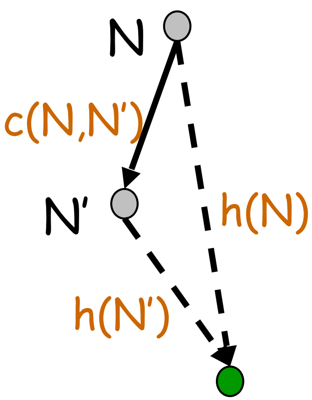


$h(N) \leq c(N,N') + h(N')$

- $h_1(N)$ = number of misplaced tiles
- $h_2(N)$ = sum of the (Manhattan) distances of every tile to its goal position

are both consistent (why?)

Robot Navigation



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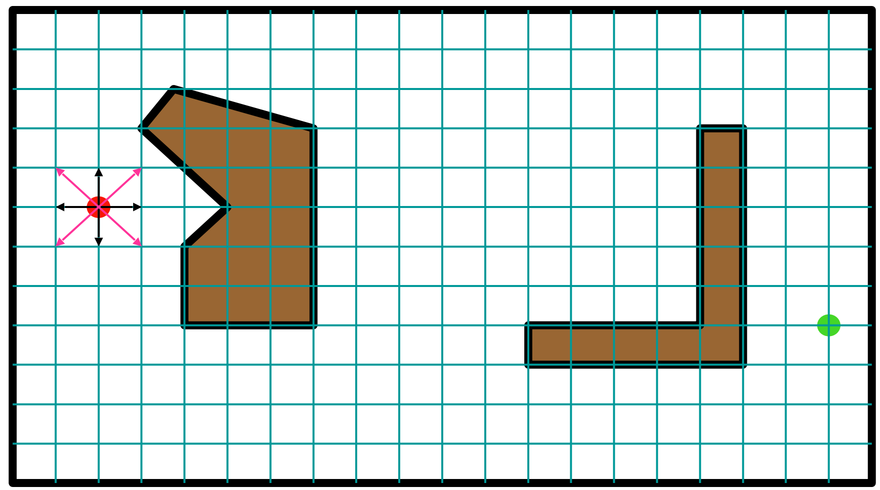


Cost of one horizontal/vertical step = 1

Cost of one diagonal step = $\sqrt{2}$

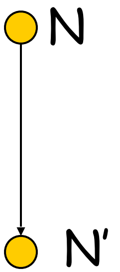
$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ is consistent

$h_2(N) = |x_N - x_g| + |y_N - y_g|$ is consistent if moving along diagonals is not allowed, and not consistent otherwise

Result #2

If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state

Proof (1/2)



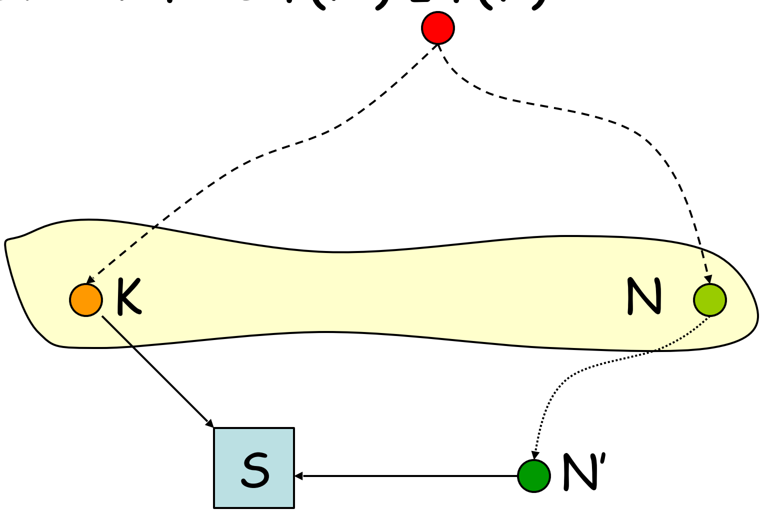
1) Consider a node N and its child N'
   Since h is consistent: $h(N) \leq c(N,N') + h(N')$

   $f(N) = g(N)+h(N) \leq g(N)+c(N,N')+h(N') = f(N')$

   So, f is non-decreasing along any path

Proof (2/2)

2) If a node K is selected for expansion, then any other node N in the Open List verifies $f(N) \geq f(K)$



If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

$f(N') \geq f(N) \geq f(K)$ and $h(N') = h(K)$

So, $g(N') \geq g(K)$

Proof (2/2)

2) If a node K is selected for expansion, then any other node N in the Open List verifies $f(N) \geq f(K)$

Result #2

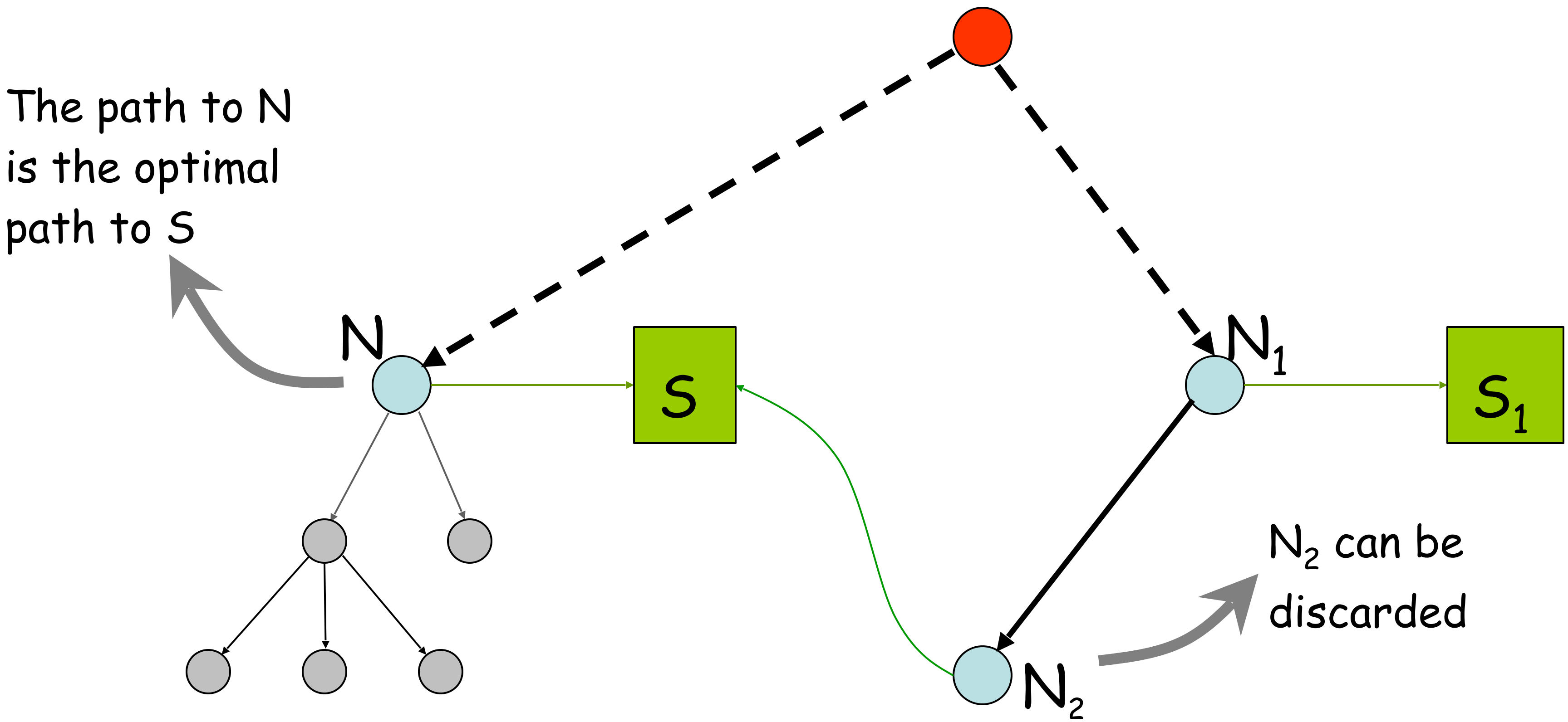
If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state

If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

$f(N') \geq f(N) \geq f(K)$ and $h(N') = h(K)$

So, $g(N') \geq g(K)$

Implication of Result #2



Revisited States with Consistent Heuristic

- When a node is expanded, store its state into CLOSED
- When a new node N is generated:
  - If STATE(N) is in CLOSED, discard N
  - If there exists a node N' in the Open List such that STATE(N') = STATE(N), discard the node – N or N' – with the largest f (or, equivalently, g)

Is A* with some consistent heuristic all that we need?

No !

There are **very dumb** consistent heuristic functions

For example: $h \equiv 0$

- It is consistent (hence, admissible) !
- A* with $h \equiv 0$ is uniform-cost search
- Breadth-first and uniform-cost are particular cases of A*

Heuristic Accuracy

Let h_1 and h_2 be two consistent heuristics such that for all nodes N:

$$h_1(N) \leq h_2(N)$$

h_2 is said to be more accurate (or more informed) than h_1

5		8
4	2	1
7	3	6

STATE(N)

1	2	3
4	5	6
7	8	

Goal state

- $h_1(N)$ = number of misplaced tiles
- $h_2(N)$ = sum of distances of every tile to its goal position

- h_2 is more accurate than h_1

Result #3

- Let h_2 be more accurate than h_1
- Let A_1^* be A^* using h_1
  and A_2^* be A^* using h_2
- Whenever a solution exists, all the nodes expanded by A_2^*, are also expanded by A_1^*
  - except possibly for some nodes such that $f_1(N) = f_2(N) = C^*$ (cost of optimal solution)

Proof

- C^* = h*(initial-node) [cost of optimal solution]
- Every node N such that f(N) < C^* is eventually expanded. No node N such that f(N) > C^* is ever expanded
- Every node N such that h(N) < C^*−g(N) is eventually expanded. So, every node N such that h2(N) < C^*−g(N) is expanded by A2*. Since h1(N) ≤ h2(N), N is also expanded by A1*

Effective Branching Factor

- It is used as a measure the effectiveness of a heuristic
- Let n be the total number of nodes expanded by A* for a particular problem and d the depth of the solution
- The effective branching factor b* is defined by $n = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$

Experimental Results

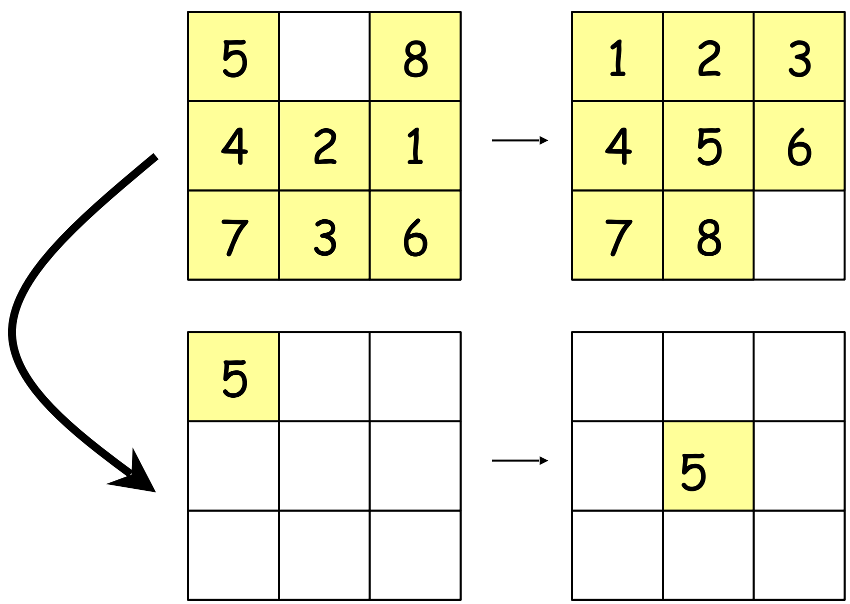
(see R&N for details)

- 8-puzzle with:
  - h_1 = number of misplaced tiles
  - h_2 = sum of distances of tiles to their goal positions
- Random generation of many problem instances
- Average effective branching factors (number of expanded nodes):

d	IDS	A_1^*	A_2^*
2	2.45	1.79	1.79
6	2.73	1.34	1.30
12	2.78 (3,644,035)	1.42 (227)	1.24 (73)
16	--	1.45	1.25
20	--	1.47	1.27
24	--	1.48 (39,135)	1.26 (1,641)

How to create good heuristics?

- By solving relaxed problems at each node
- In the 8-puzzle, the sum of the distances of each tile to its goal position (h_2) corresponds to solving 8 simple problems:



d_i is the length of the shortest path to move tile i to its goal position, ignoring the other tiles, e.g., $d_5 = 2$

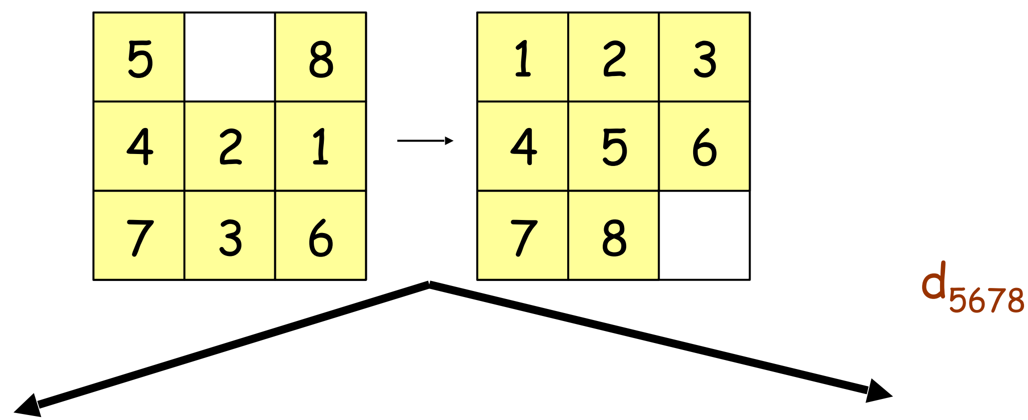
$$h_2 = \Sigma_{i=1,...8}\, d_i$$

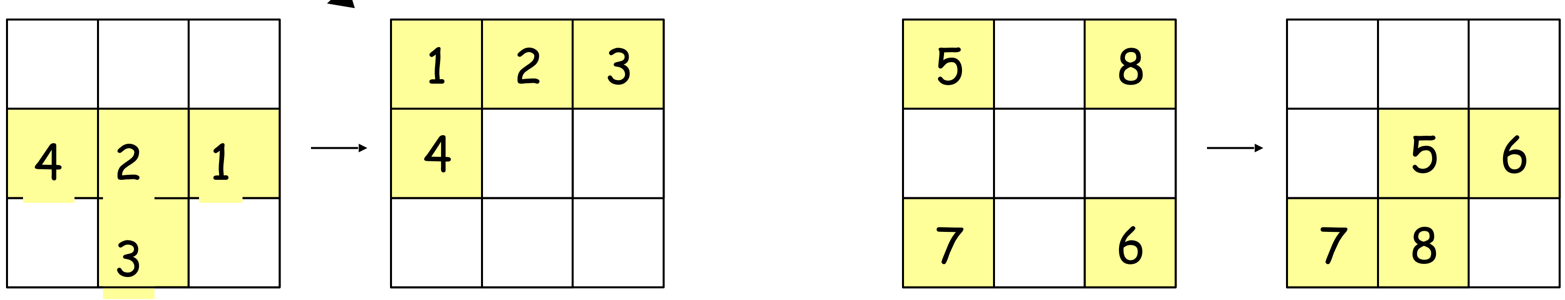
- It ignores negative interactions among tiles

Can we do better?

- For example, we could consider two more complex relaxed problems:

d_{1234} = length of the shortest path to move tiles 1, 2, 3, and 4 to their goal positions, ignoring the other tiles





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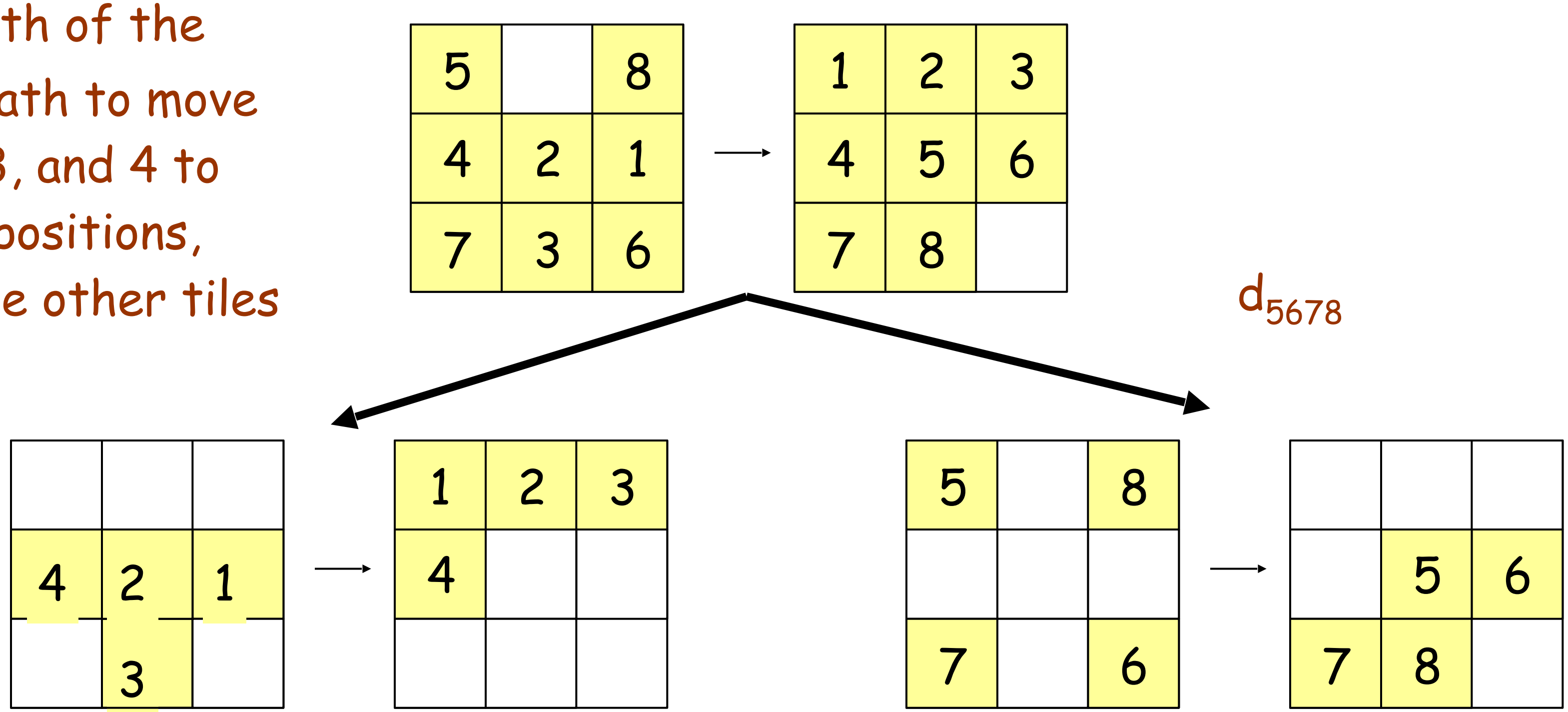


→ Several order-of-magnitude speedups for the 15- and 24-puzzle (see R&N)

On Completeness and Optimality

- A* with a consistent heuristic function has nice properties: completeness, optimality, no need to revisit states
- Theoretical completeness does not mean "practical" completeness if you must wait too long to get a solution (remember the time limit issue)
- So, if one can't design an accurate consistent heuristic, it may be better to settle for a non-admissible heuristic that "works well in practice", even through completeness and optimality are no longer guaranteed

Iterative Deepening A* (IDA*)

- Idea: Reduce memory requirement of A* by applying cutoff on values of f
- Consistent heuristic function h
- Algorithm IDA*:
  1. Initialize cutoff to f(initial-node)
  2. Repeat:
     a. Perform depth-first search by expanding all nodes N such that $f(N) \leq$ cutoff
     b. Reset cutoff to smallest value f of non-expanded (leaf) nodes

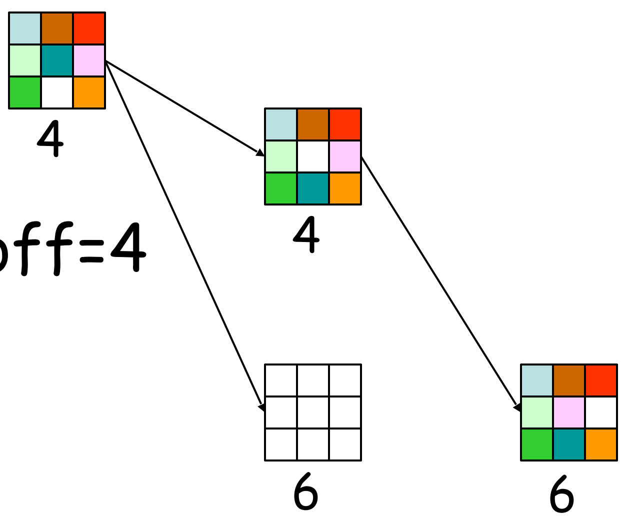
8-Puzzle

$f(N) = g(N) + h(N)$
with $h(N)$ = number of misplaced tiles

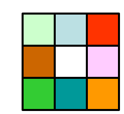
8-Puzzle

$f(N) = g(N) + h(N)$

with $h(N)$ = number of misplaced tiles

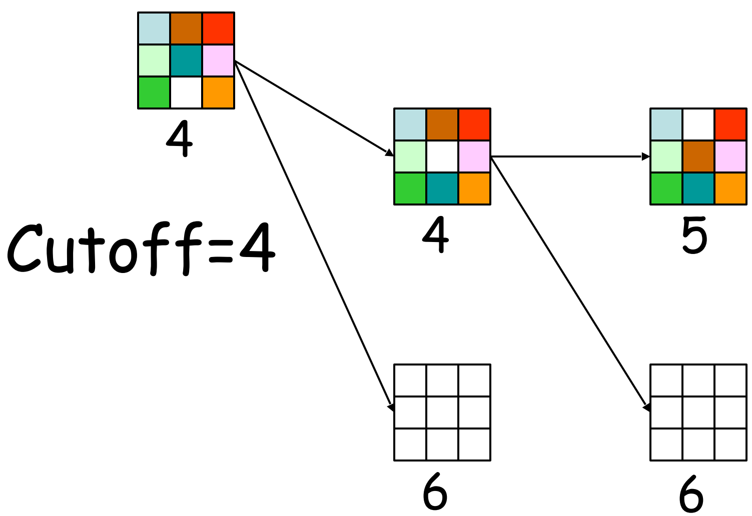


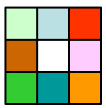
Cutoff=4



8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles





8-Puzzle

f(N) = g(N) + h(N)
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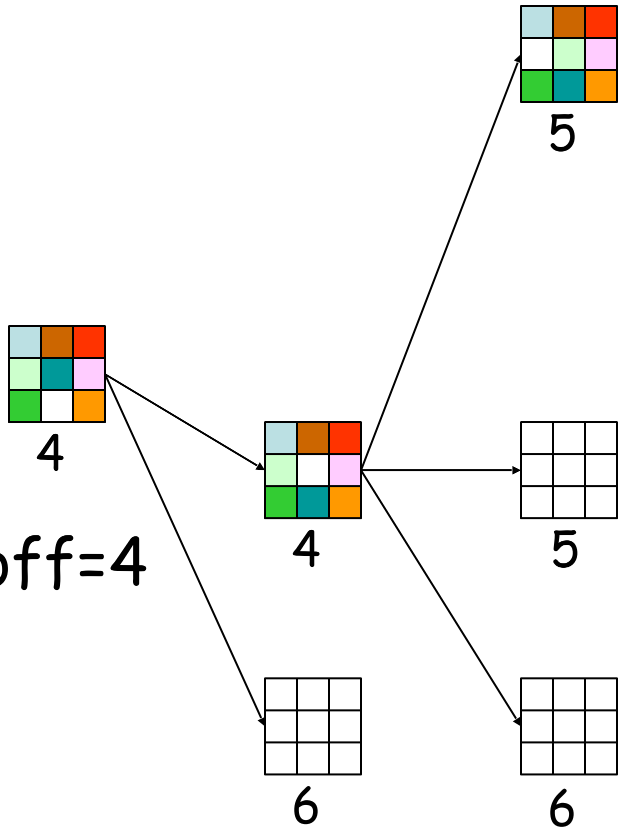
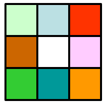
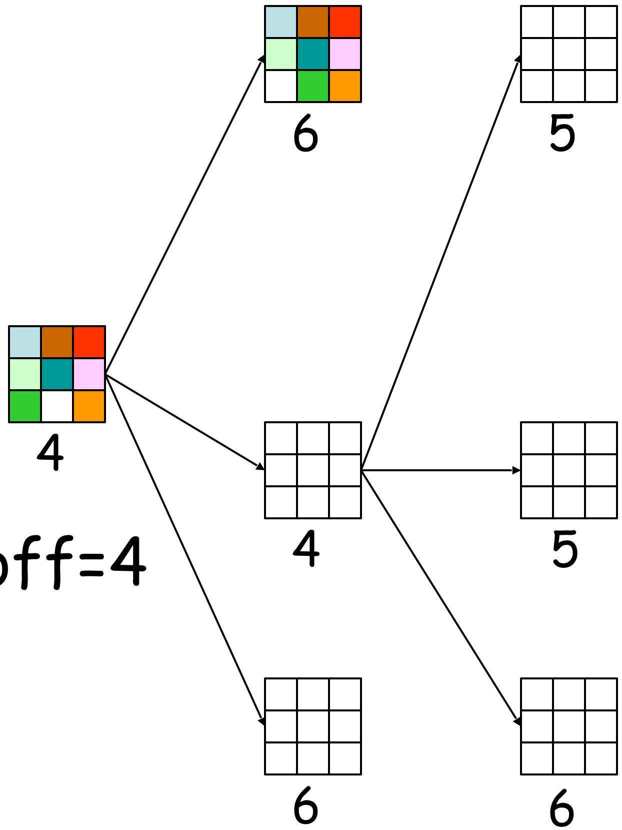


Cutoff=4

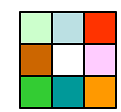


8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



Cutoff=4



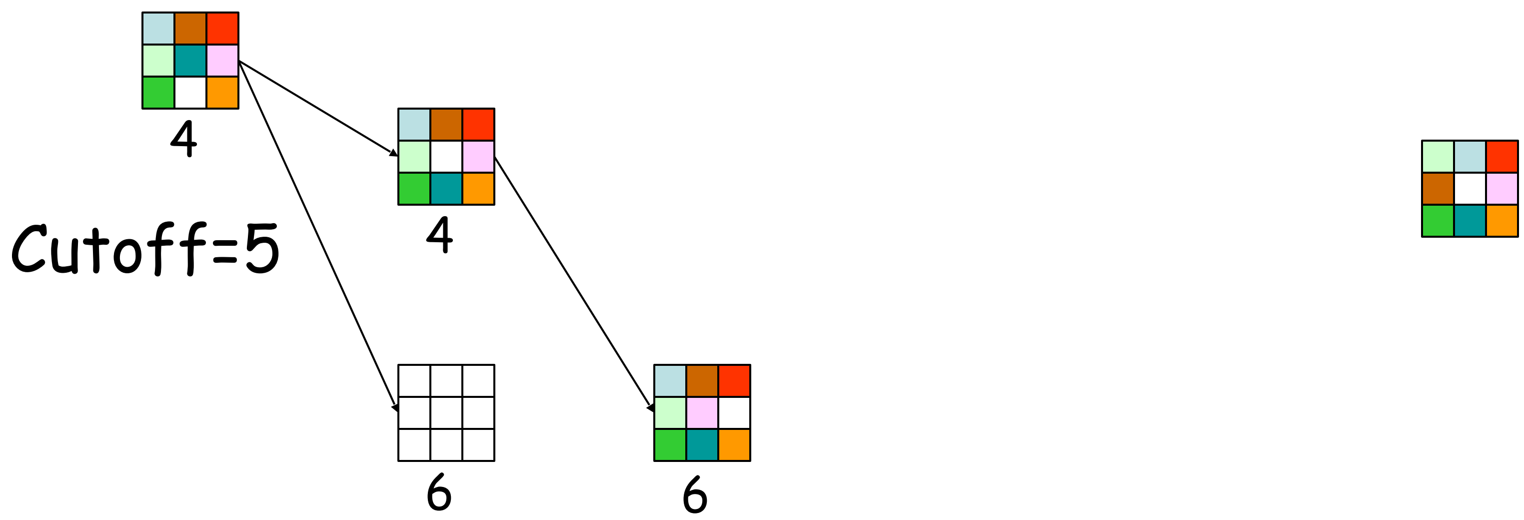
8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

8-Puzzle

$f(N) = g(N) + h(N)$

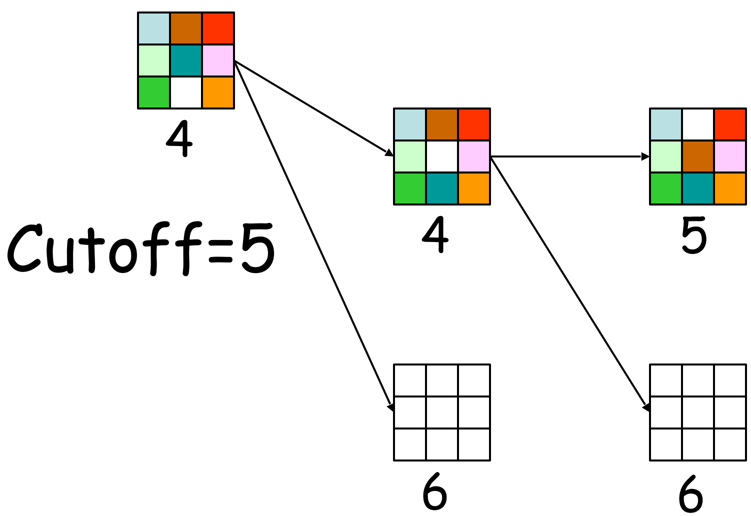
with $h(N)$ = number of misplaced tiles

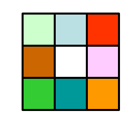


8-Puzzle

$f(N) = g(N) + h(N)$

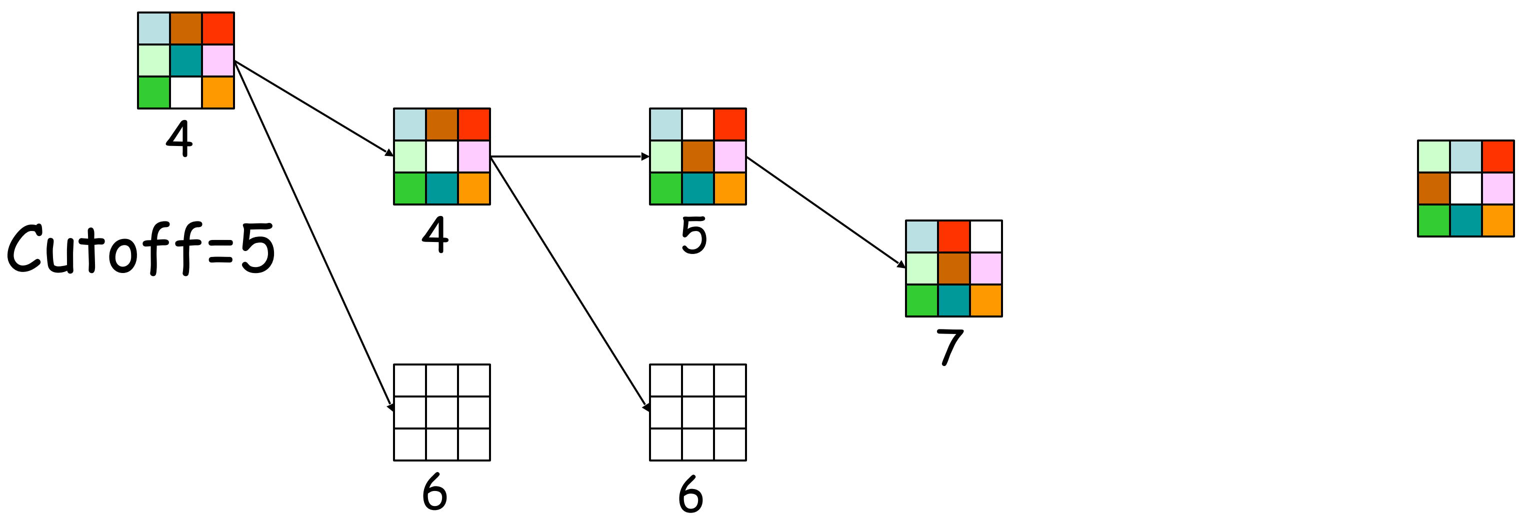
with $h(N)$ = number of misplaced tiles





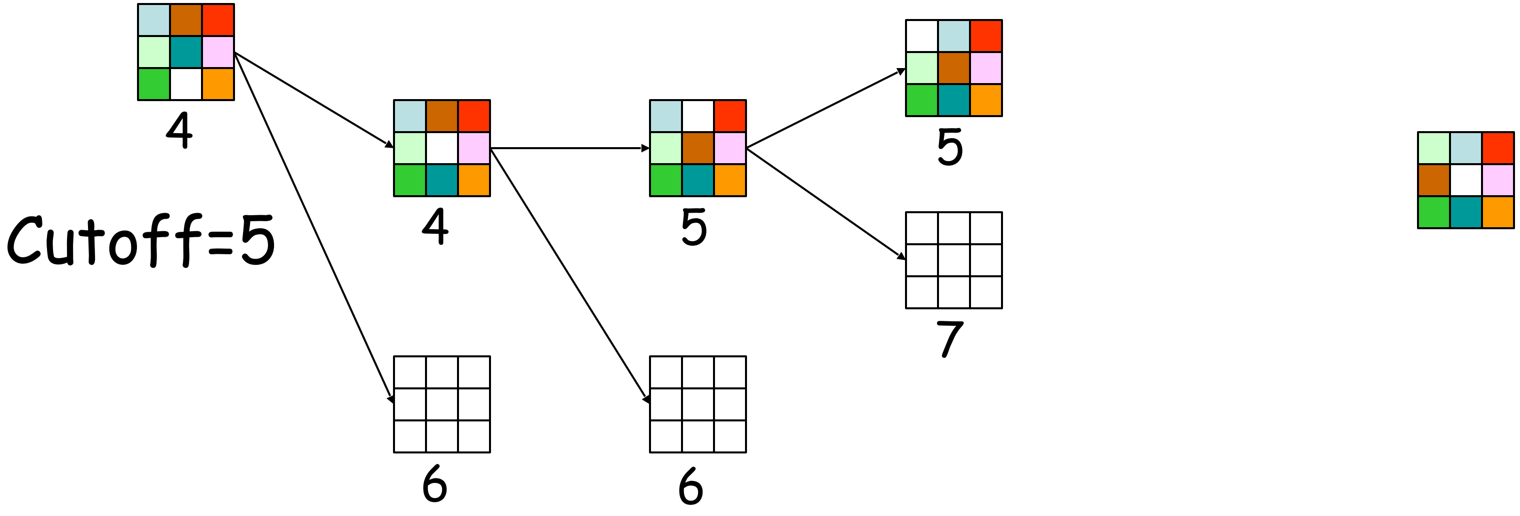
8-Puzzle

$f(N) = g(N) + h(N)$
with $h(N)$ = number of misplaced tiles



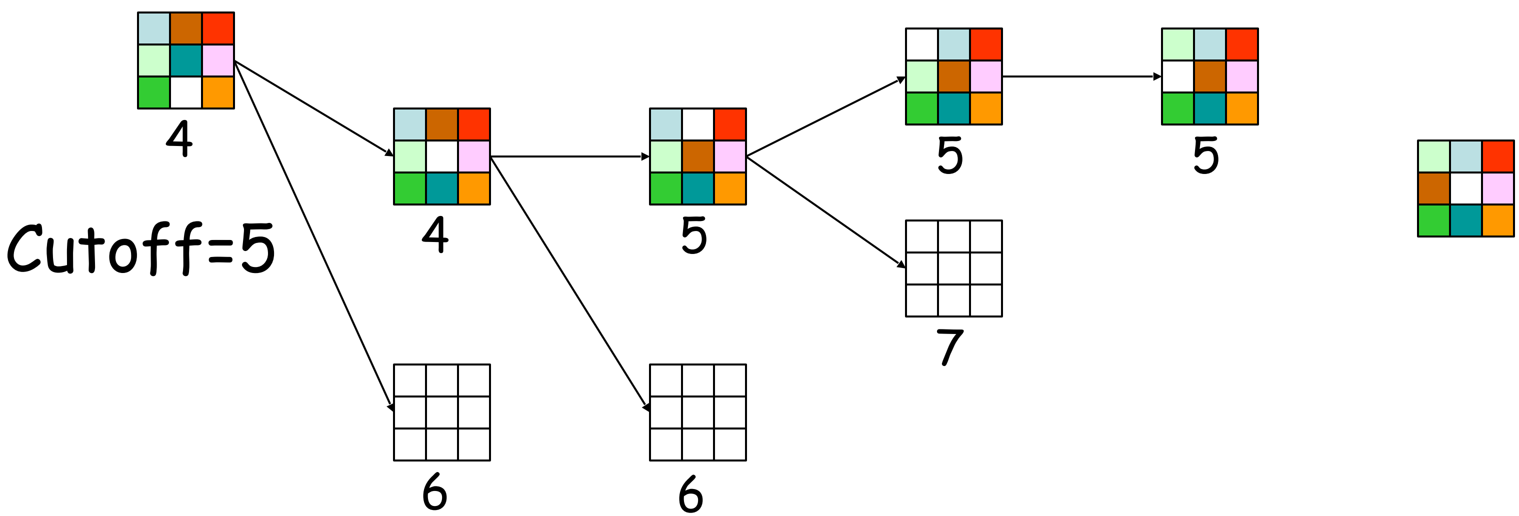
8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



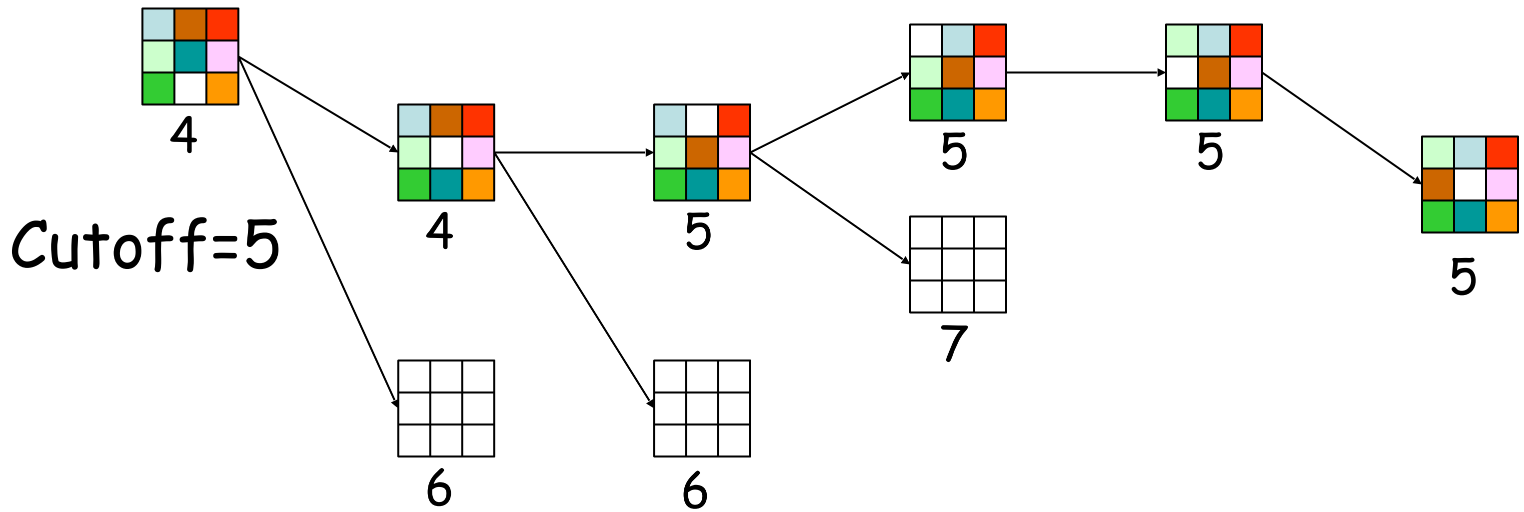
8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



Experimental Results of IDA*

- IDA* is asymptotically same time as A* but only O(d) in space - versus O(bd) for A*
  - Also avoids overhead of sorted queue of nodes
- IDA* is simpler to implement - no closed lists (limited open list).
- In Korf's 15-puzzle experiments IDA*: solved all problems, ran faster even though it generated more nodes than A*.

Advantages/Drawbacks of IDA*

- Advantages:
  - Still complete and optimal
  - Requires less memory than A*
  - Avoid the overhead to sort the Open List
- Drawbacks:
  - Can't avoid revisiting states not on the current path
  - Available memory is poorly used
    (→ memory-bounded search, see R&N p. 101-104)

Local Search

- Light-memory search method
- No search tree; only the current state is represented!
- Only applicable to problems where the path is irrelevant (e.g., 8-queen), unless the path is encoded in the state
- Many similarities with optimisation techniques

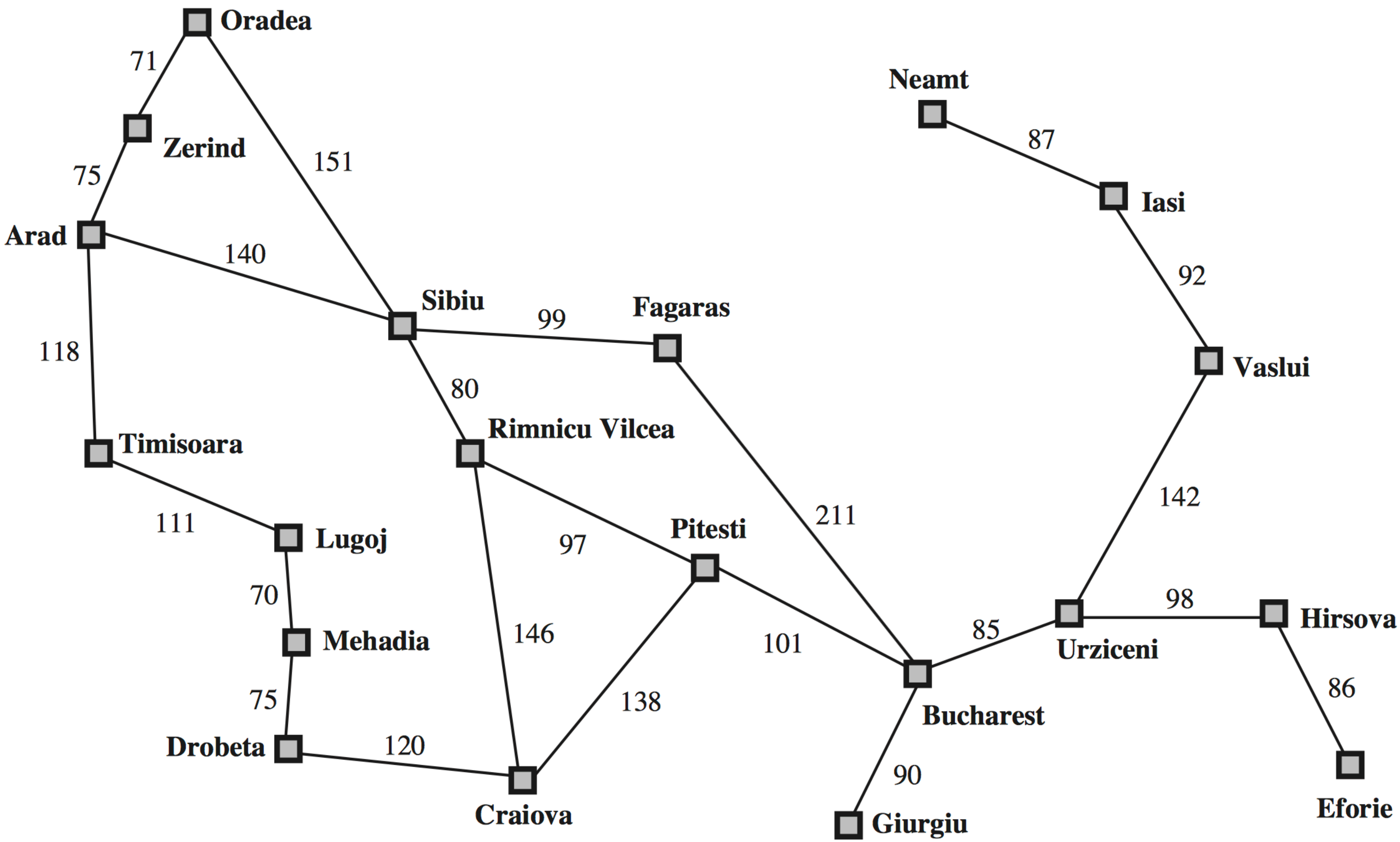


Figure 3.2 **FILES: figures/romania-distances.eps (Tue Nov 3 16:23:37 2009).** A simplified road map of part of Romania.

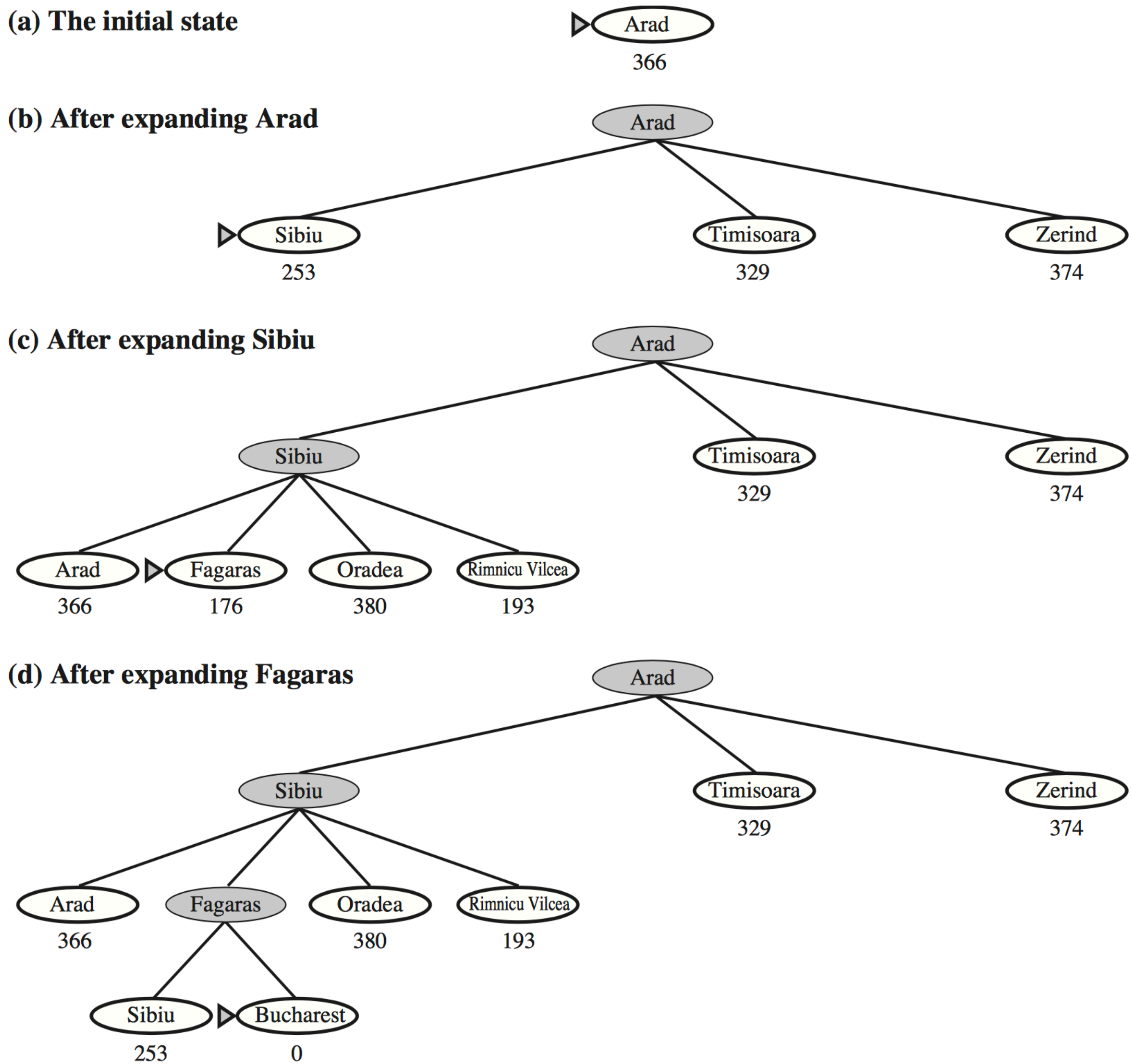


Figure 3.23 FILES: figures/greedy-progress.eps (Tue Nov 3 16:22:55 2009). Stages in a greedy best-first tree search for Bucharest with the straight-line distance heuristic h_{SLD}. Nodes are labeled with their h-values.

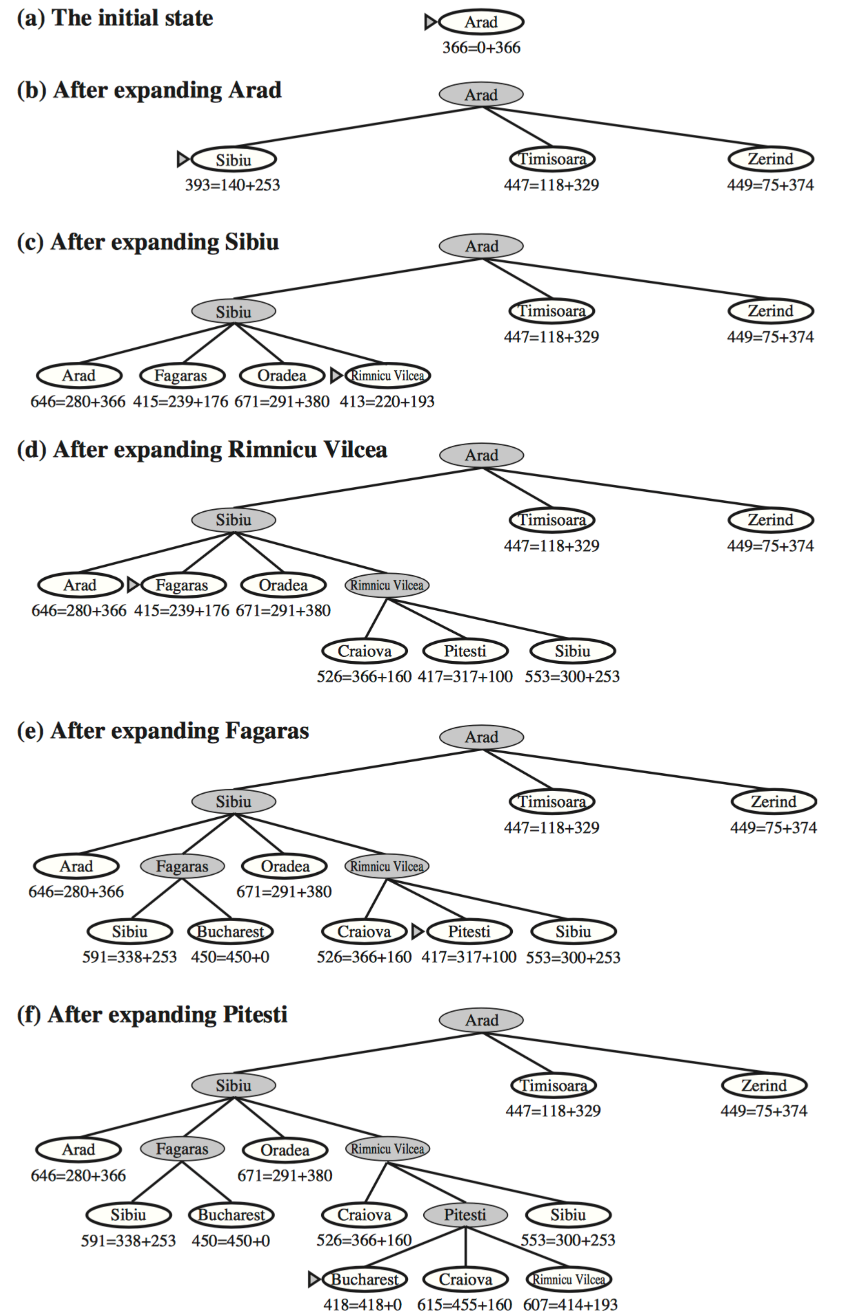
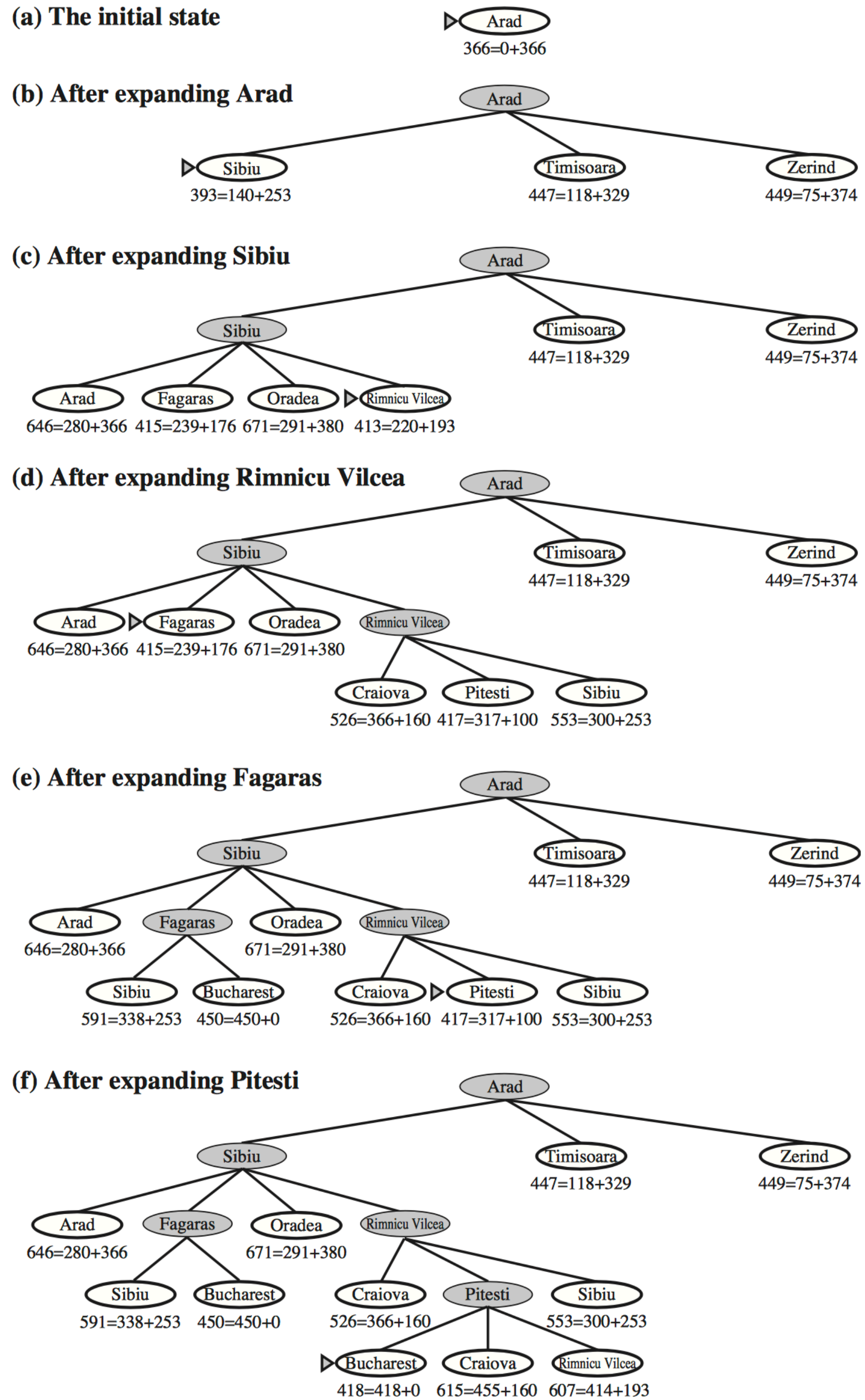


Figure 3.24 **FILES: figures/astar-progress.eps (Tue Nov 3 16:22:24 2009).** Stages in an A* search for Bucharest. Nodes are labeled with $f = g + h$. The h values are the straight-line distances to Bucharest taken from Figure 3.20.


(a) The initial state
Arad
366=0+366
(b) After expanding Arad
Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374
(c) After expanding Sibiu
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193
(d) After expanding Rimnicu Vilcea
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253
(e) After expanding Fagaras
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
591=338+253
Bucharest
450=450+0
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253
(f) After expanding Pitesti
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
591=338+253
Bucharest
450=450+0
Craiova
526=366+160
Pitesti
Sibiu
553=300+253
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

RBFS - Recursive Best-First Search

- Mimics best-first search with linear space
- Similar to recursive depth-first
  - Limits recursion by keeping track of the f-value of the best alternative path from any ancestor node – one step look-ahead
  - If current node exceeds this value, recursion unwinds back to the alternative path – same idea as contour
- As recursion unwinds, replaces f-value of node with best f- value of children
  - Allows to remember whether to re-expand path at later time
- Exploits information gathered from previous searches about minimum f so as to focus further searches

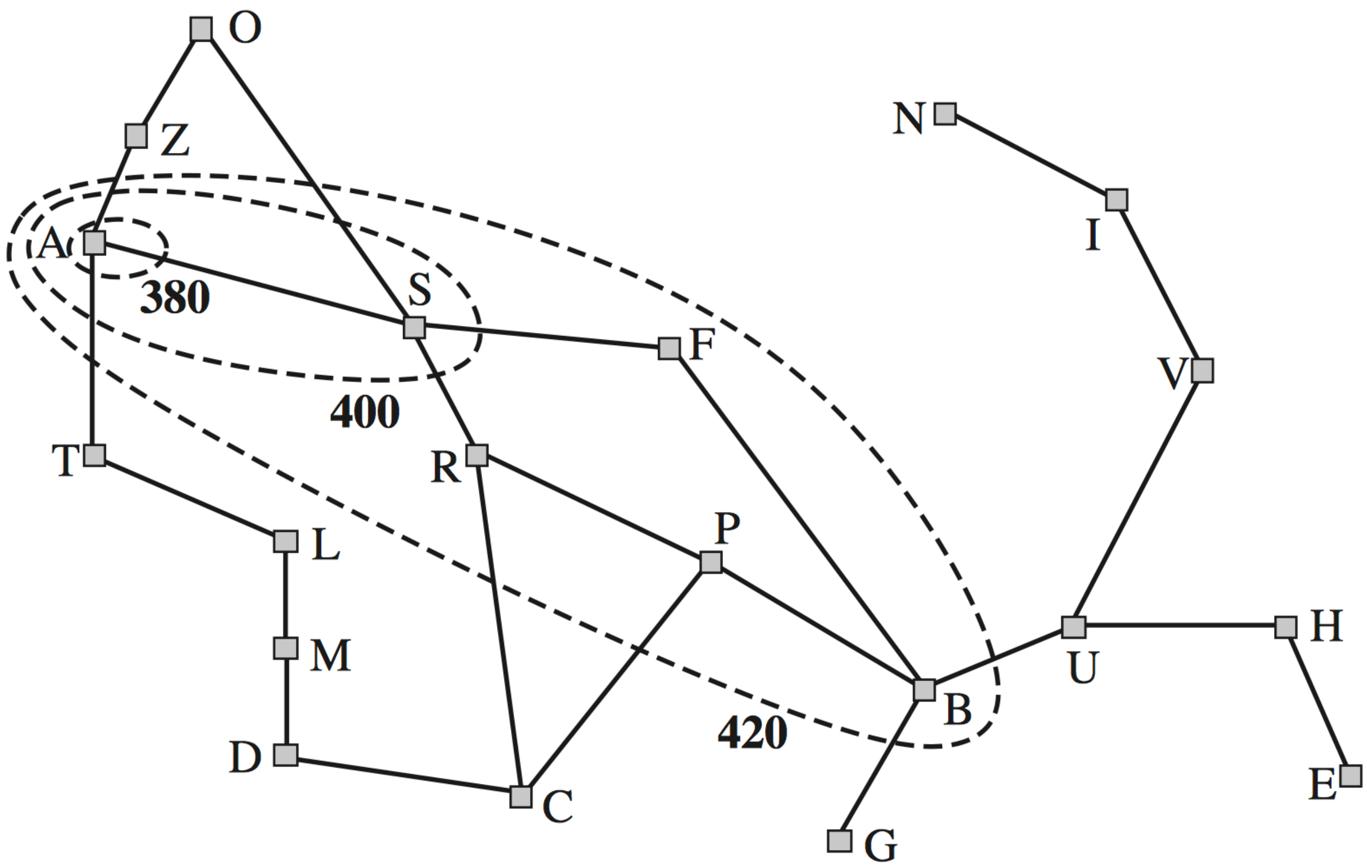


Figure 3.25 **FILES: figures/f-circles.eps (Tue Nov 3 16:22:45 2009).** Map of Romania showing contours at $f = 380$, $f = 400$, and $f = 420$, with Arad as the start state. Nodes inside a given contour have f-costs less than or equal to the contour value.

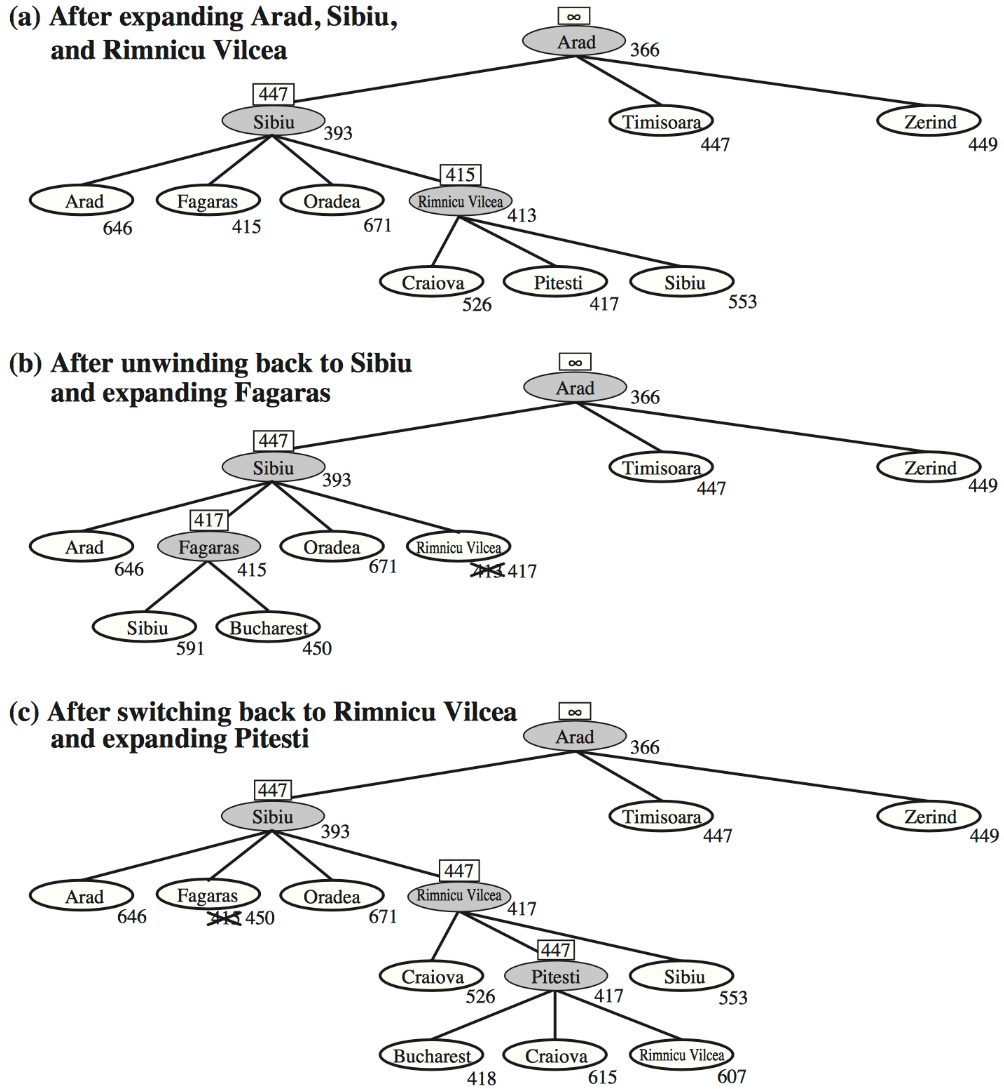
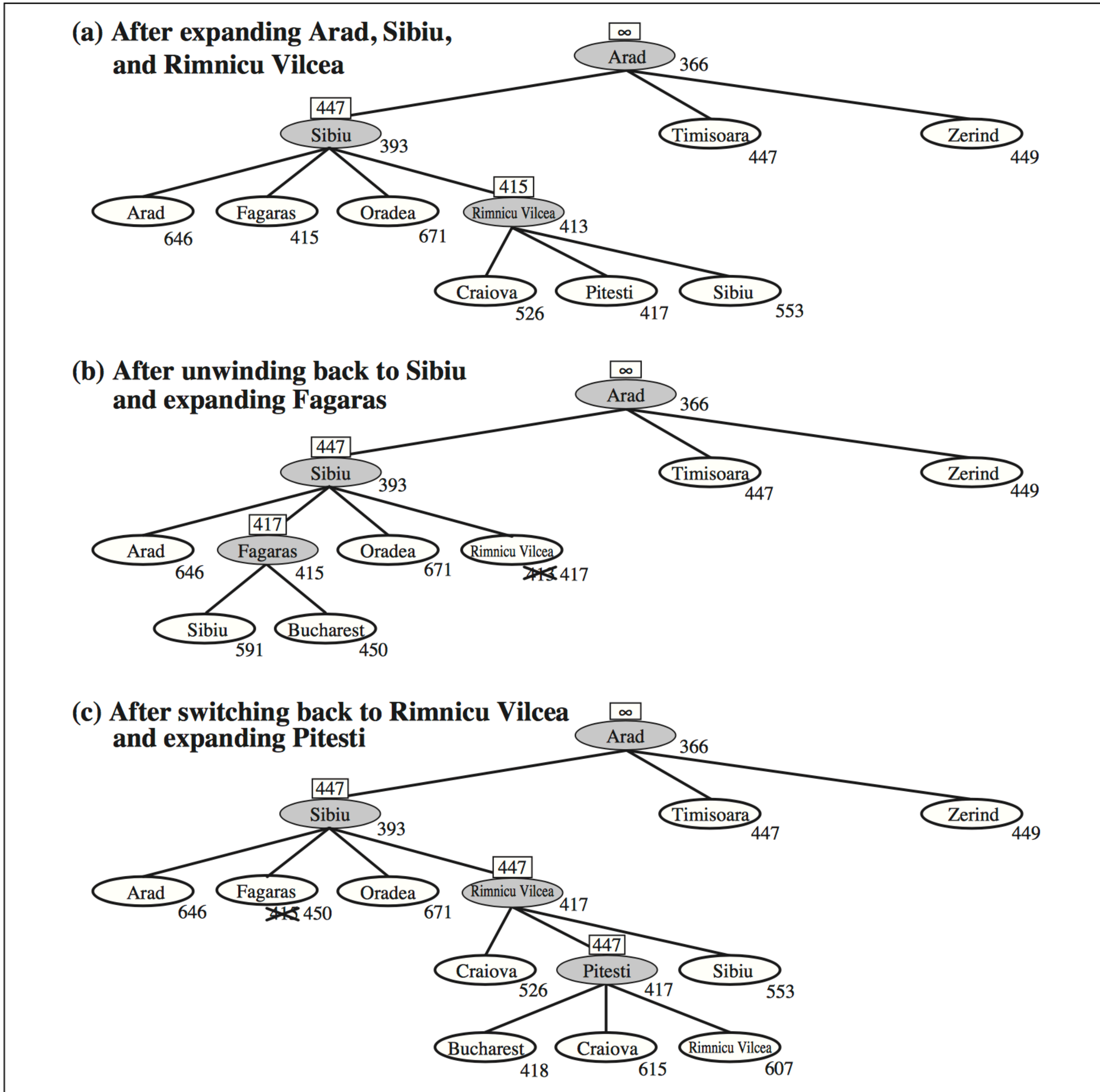


Figure 3.27 **FILES: figures/rbfs-progress.eps (Tue Nov 3 16:23:27 2009).** Stages in an RBFS search for the shortest route to Bucharest. The f-limit value for each recursive call is shown on top of each current node, and every node is labeled with its f-cost. (a) The path via Rimnicu Vilcea is followed until the current best leaf (Pitesti) has a value that is worse than the best alternative path (Fagaras). (b) The recursion unwinds and the best leaf value of the forgotten subtree (417) is backed up to Rimnicu Vilcea; then Fagaras is expanded, revealing a best leaf value of 450. (c) The recursion unwinds and the best leaf value of the forgotten subtree (450) is backed up to Fagaras; then Rimnicu Vilcea is expanded. This time, because the best alternative path (through Timisoara) costs at least 447, the expansion continues to Bucharest.


(a) After expanding Arad, Sibiu, and Rimnicu Vilcea
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
Fagaras 415
Oradea 671
415
Rimnicu Vilcea 413
Craiova 526
Pitesti 417
Sibiu 553
(b) After unwinding back to Sibiu and expanding Fagaras
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
417
Fagaras 415
Oradea 671
Rimnicu Vilcea 413 417
Sibiu 591
Bucharest 450
(c) After switching back to Rimnicu Vilcea and expanding Pitesti
∞
Arad 366
447
Sibiu 393
Timisoara 447
Zerind 449
Arad 646
Fagaras 415 450
Oradea 671
447
Rimnicu Vilcea 417
Craiova 526
447
Pitesti 417
Sibiu 553
Bucharest 418
Craiova 615
Rimnicu Vilcea 607

RBFS - Recursive Best-First Search

```
function RECURSIVE-BEST-FIRST-SEARCH(problem) returns a solution, or failure
    RBFS(MAKE-NODE(INITIAL-STATE[problem]), ∞)

function RBFS(problem, node, f-limit) returns a solution, or failure and a new f-cost limit
    if GOAL-TEST[problem](state) then return node
    successors ← EXPAND(node, problem)
    if successors is empty, then return failure, ∞
    for each s in successors do f[s] ← max(g(s) + h(s), f[node])
    repeat
        best ← the lowest f-value node in successors
        if f[best] > f-limit then return failure, f[best]
        alternative ← the second-lowest f-value among successors
        result, f[best] ← RBFS(problem, best, min(f-limit, alternative))
        if result ∓ failure then return result
    end
```

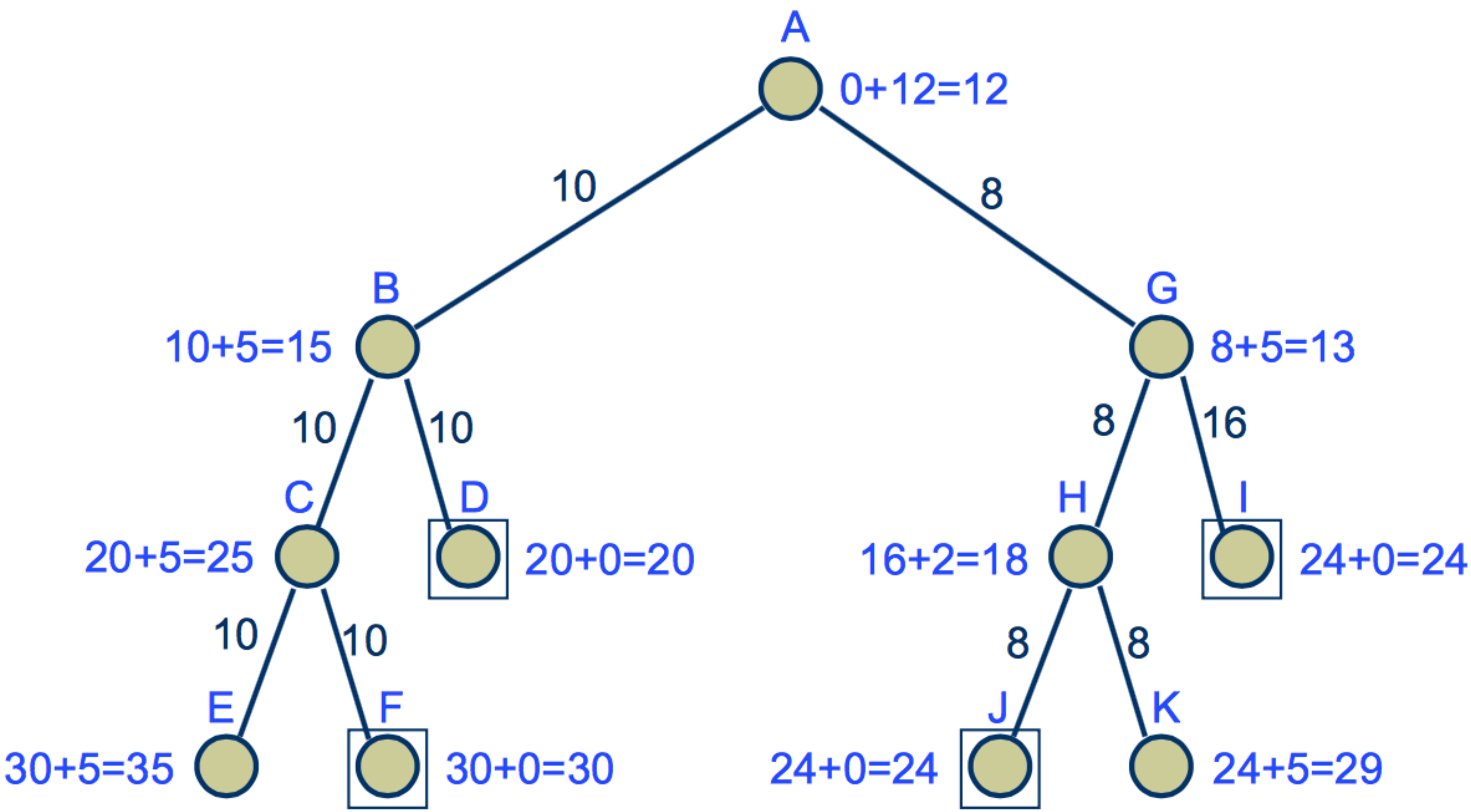
RBFS - Recursive Best-First Search

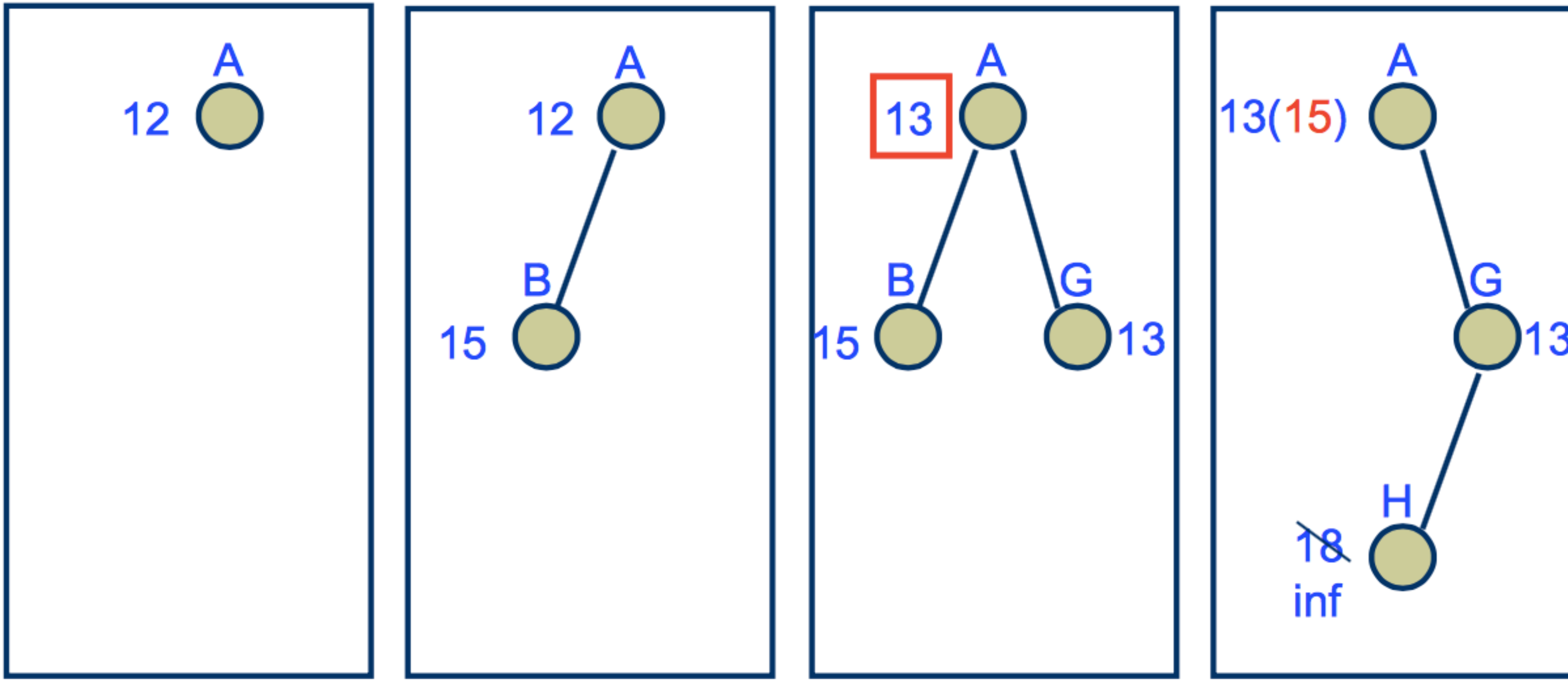
- More efficient than IDA* and still optimal
  - Best-first Search based on next best f-contour; fewer regeneration of nodes
  - Exploit results of search at a specific f-contour by saving next f- countour associated with a node who successors have been explored.
- Like IDA* still suffers from excessive node regeneration IDA* and RBFS not good for graphs
- Can't check for repeated states other than those on current path Both are hard to characterize in terms of expected time complexity

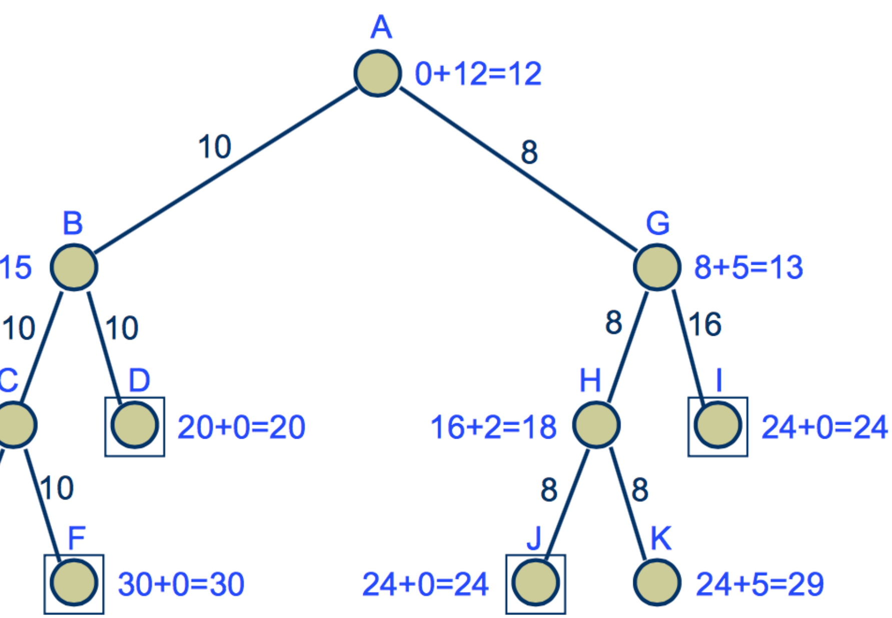
SMA* Simplified Memory Bounded A*

- The implementation of SMA* is very similar to the one of A*, the only difference is that when there isn't any space left, nodes with the highest f are pruned away.
- Because those nodes are deleted, the SMA* also has to remember the f of the best forgotten child with the parent node.
- When it seems that all explored paths are worse than such a forgotten path, the path is re-generated.

SMA* Simplified Memory Bounded A*

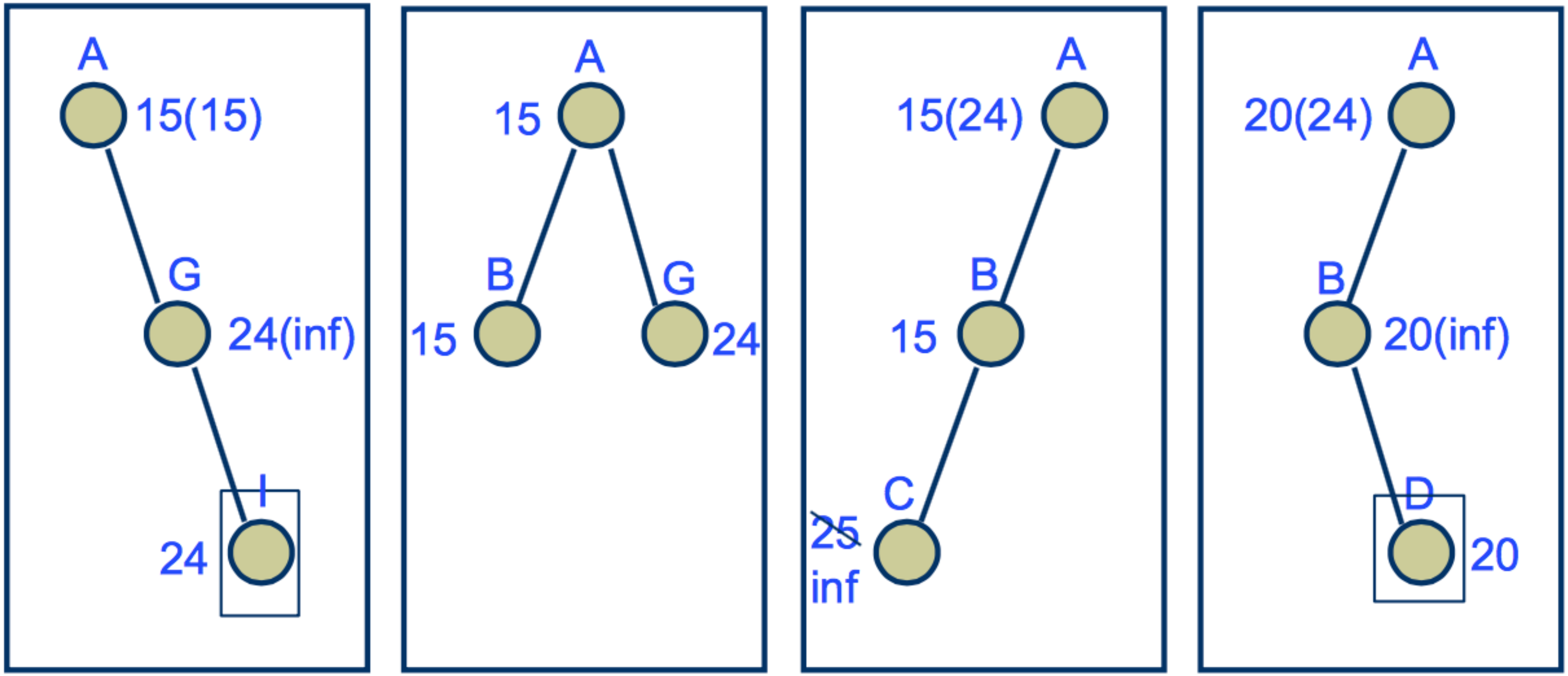






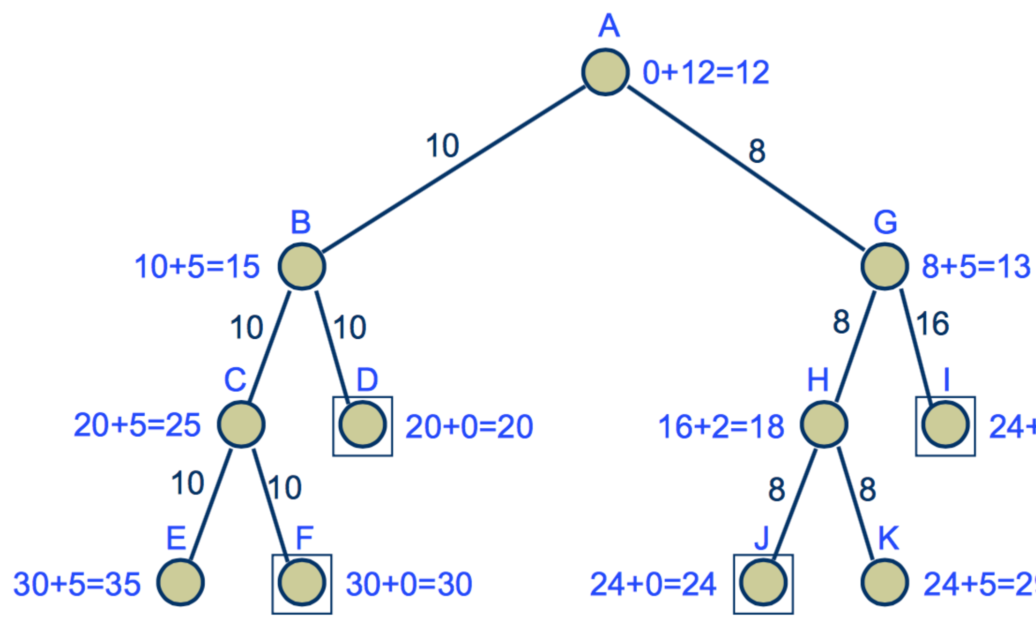
Update A based on lowest cost f successor?

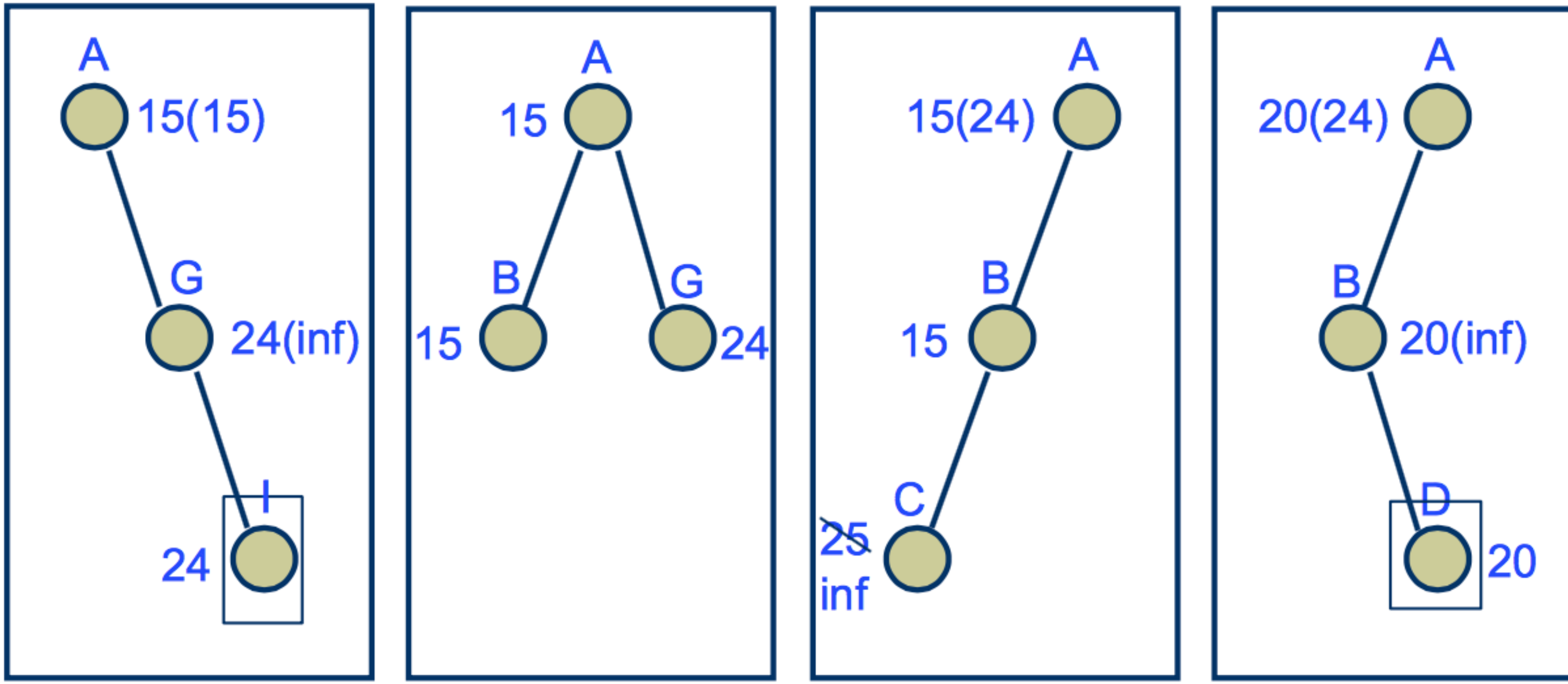
Remember next lowest cost f node B that is removed

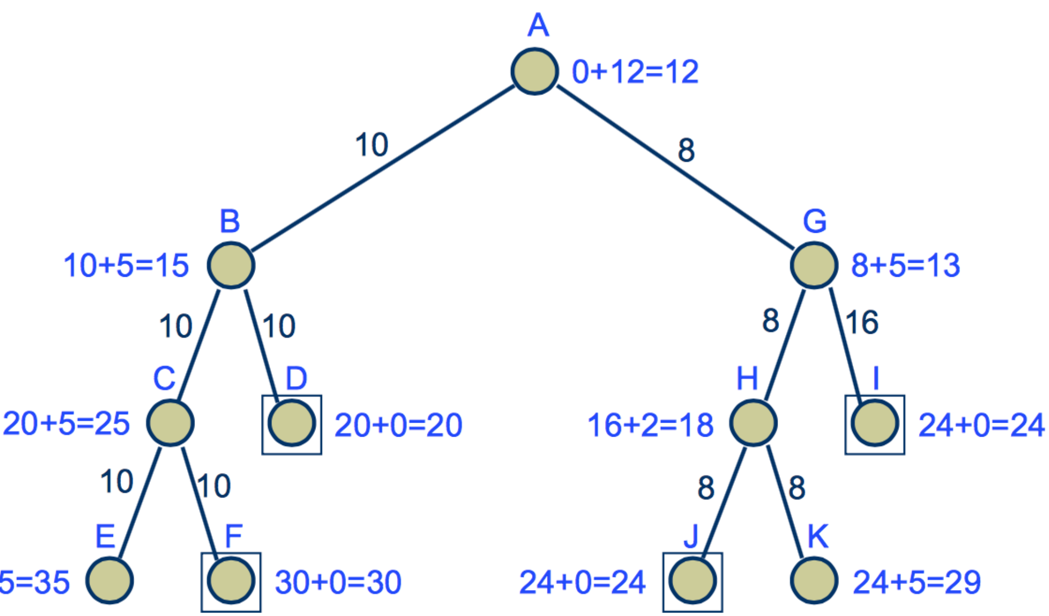


Reach goal node I but it is not the cheapest so continue search

Regenerate node B, remember that there was node with f =15 that had been removed, remember successor of G has f=24







C is not goal node and it is at max depth

Why don't we need to search anymore after finding D.

SMA* Simplified Memory Bounded A*

- It is complete, provided the available memory is sufficient to store the shallowest solution path.
- It is optimal, if enough memory is available to store the shallowest optimal solution path. Otherwise, it returns the best solution (if any) that can be reached with the available memory.
- Can keep switching back and forth between a set of candidate solution paths, only a few of which can fit in memory (thrashing)
- Memory limitations can make a problem intractable wrt time
- With enough memory for the entire tree, same as A*

Memory-bounded heuristic search

- **IDA* - Iterative-deepening A***
  - Use f-cost as cutoff - at each iteration, the cutoff value is the smallest f-cost of any node that exceeded the cutoff on the previous iteration
- **Recursive best-first search (RBFS)**
  - Best-first search with only linear space
  - Keep track of the f-value of the best alternative
  - As the recursion unwinds, it forgets the sub-tree and back-up the f-value of the best leaf as its parent's f-value.
- **SMA***
  - Expanding the best leaf until memory is full
  - Drop the worst leaf, and back-up the value of this node to its parent.
  - Complete IF there is any reachable solution.
  - Optimal IF any optimal solution is reachable.

Steepest Descent

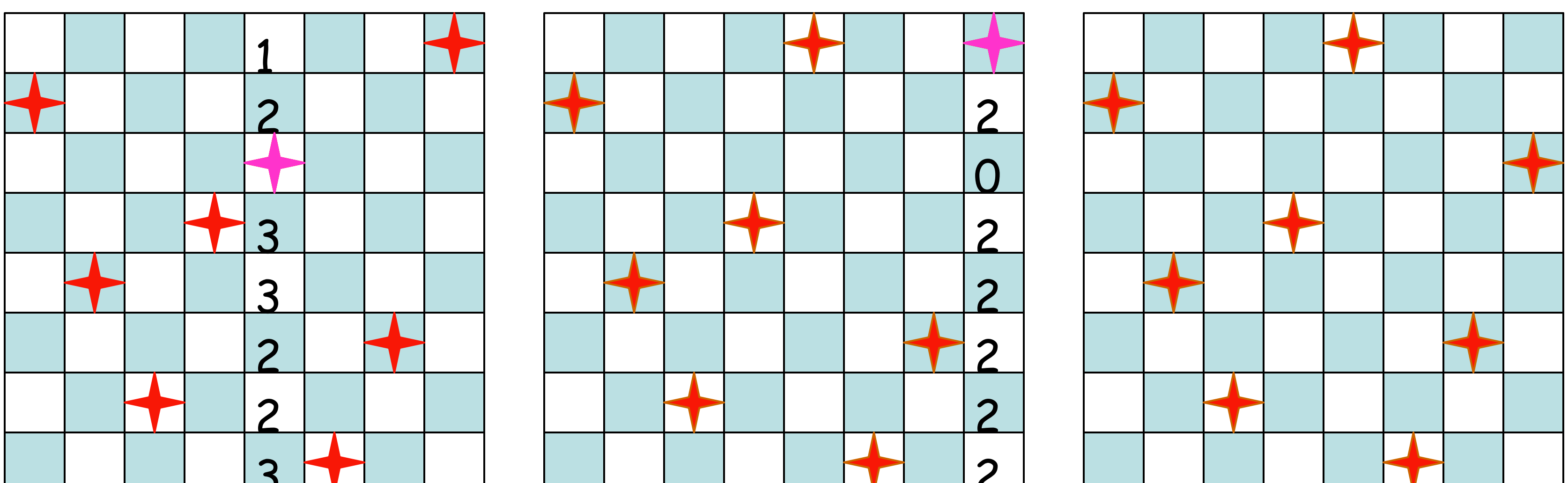
1) $S \leftarrow$ initial state
2) Repeat:
   a) $S' \leftarrow \arg\min_{S' \in SUCCESSORS(S)}\{h(S')\}$
   b) if GOAL?(S') return S'
   c) if $h(S') < h(S)$ then $S \leftarrow S'$ else return failure

Similar to:

- hill climbing with –h
- gradient descent over continuous space

Application: 8-Queen

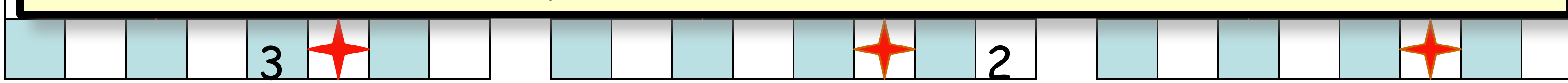
1) Pick an initial state S at random with one queen in each column
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) Pick an attacked queen Q at random
   c) Move Q in its column to minimize the number of attacking queens → new S [min-conflicts heuristic]
3) Return failure



Application: 8-Queen

Why does it work ???

1) There are **many** goal states that are well-distributed over the state space
2) If no solution has been found after a few steps, it's better to start it all over again. Building a search tree would be much less efficient because of the high branching factor
3) Running time almost independent of the number of queens



Steepest Descent

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   c) if $h(S') < h(S)$ then $S \leftarrow S'$ else return failure

may easily get stuck in local minima

à Random restart (as in n-queen example)

à Monte Carlo descent

Monte Carlo Descent

1) S ← initial state
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) S' ← successor of S picked at random
   c) if h(S') ≤ h(S) then S ← S'
   d) else
      - Δh = h(S')-h(S)
      - with probability ~ exp(−Δh/T), where T is called the "temperature", do: S ← S' [Metropolis criterion]
3) Return failure

Simulated annealing lowers T over the k iterations.

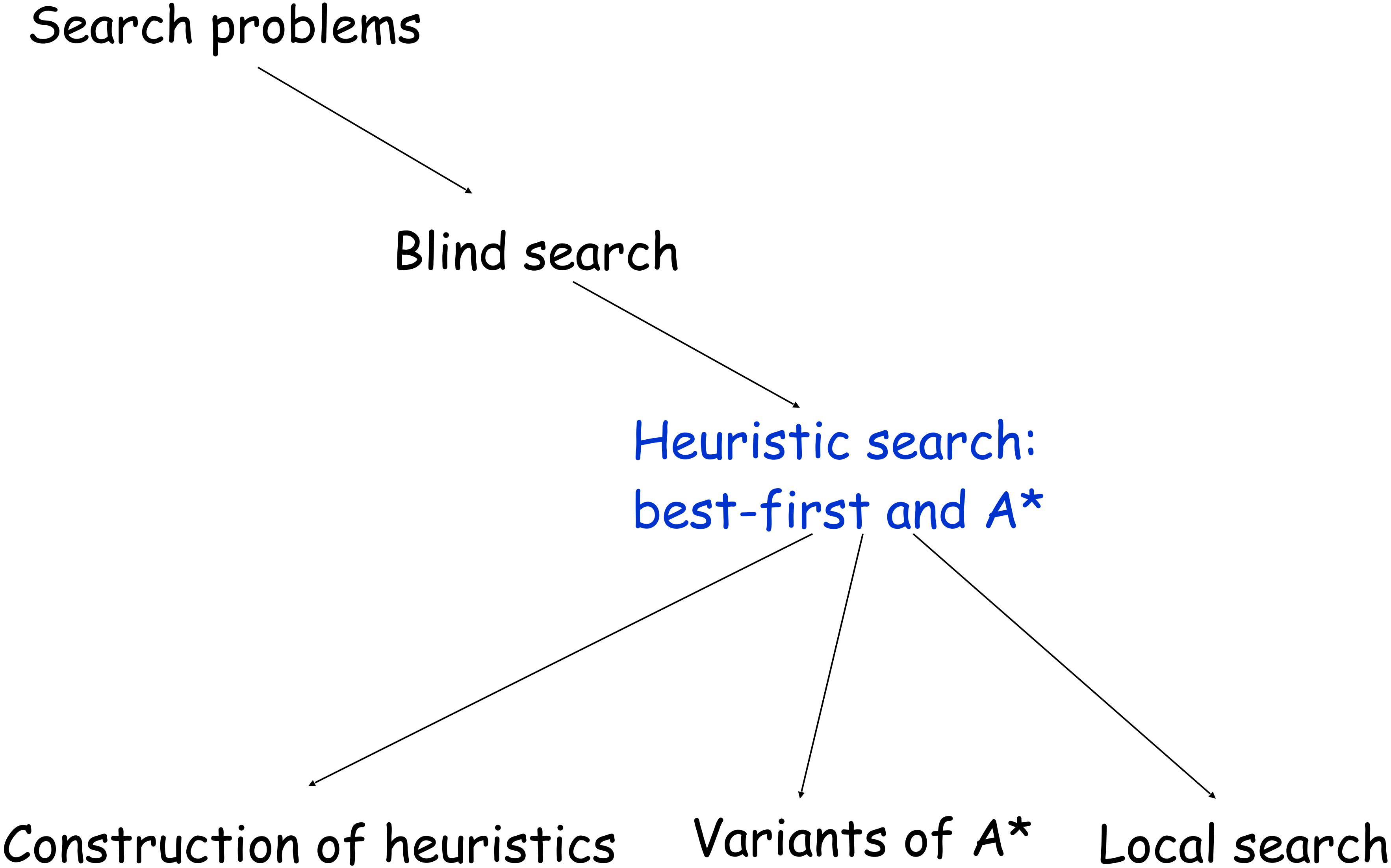
It starts with a large T and slowly decreases T

"Parallel" Local Search Techniques

They perform several local searches concurrently, but not independently:

- Beam search
- Genetic algorithms

See R&N, pages 115-119


Search problems
Blind search
Heuristic search:
best-first and A*
Construction of heuristics
Variants of A*
Local search

When to Use Search Techniques?

1) The search space is small, and
   - No other technique is available, or
   - Developing a more efficient technique is not worth the effort

2) The search space is large, and
   - No other available technique is available, and
   - There exist “good” heuristics