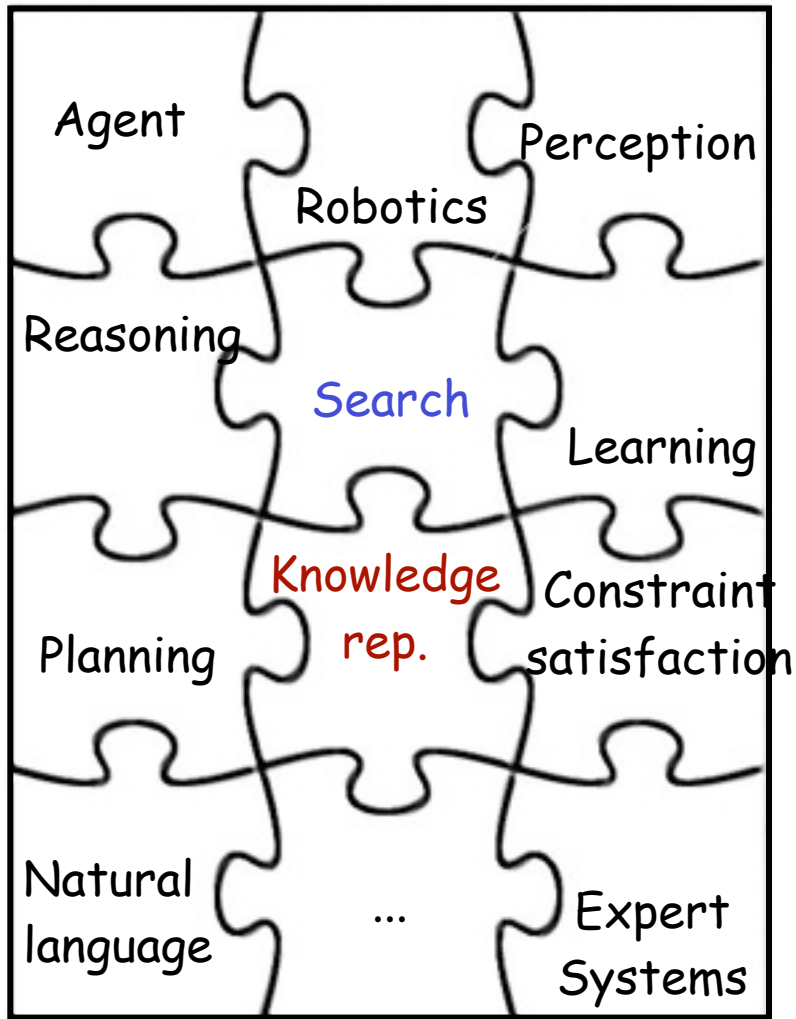


Search Problems

(Where reasoning consists of exploring alternatives)



- Declarative knowledge creates alternatives:
 - Which pieces of knowledge to use?
 - How to use them?
- Search is a about exploring alternatives.
It is a major approach to exploit knowledge

Example: 8-Puzzle

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

8-Puzzle: Successor Function

8	2	7
3	4	
5	1	6

SUCC(state) → subset of states

The **successor function** is knowledge about the 8-puzzle game, but it does not tell us which outcome to use, nor to which state of the board to apply it.

8	2	
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8	2	7
3	4	6
5	1	

8	2	7
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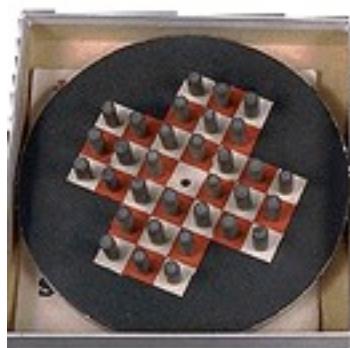
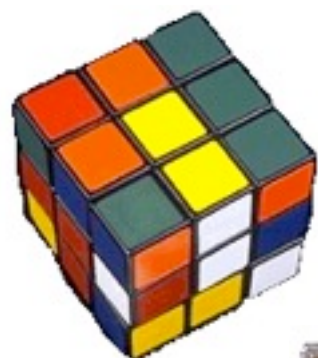
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Search is about the
exploration of alternatives

Across history, puzzles and games requiring the exploration of alternatives have been considered a challenge for human intelligence:

- **Chess** originated in Persia and India about 4000 years ago
- **Checkers** appear in 3600-year-old Egyptian paintings
- **Go** originated in China over 3000 years ago

So, it's not surprising that AI uses games to design and test algorithms



$(n^2 - 1)$ -puzzle

algorithm scalability

8	2	
3	4	7
5	1	6

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

■ ■ ■ ■

15-Puzzle

Introduced (?) in 1878 by Sam Loyd, who dubbed himself “America’s greatest puzzle-expert”

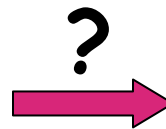


SAM LOYD,
Journalist and Advertising Expert,
ORIGINAL
Games, Novelties, Supplements, Souvenirs,
Etc., for Newspapers.
Unique Sketches, Novelties, Puzzles, &c.,
FOR ADVERTISING PURPOSES.
Author of the famous
"Get Off The Earth Mystery," "Trick Donkeys,"
"15 Block Puzzle," "Pigs In Clover,"
"Parcheesi," Etc., Etc..
P. O. BOX 876.
New York, *April 15* 1903

15-Puzzle

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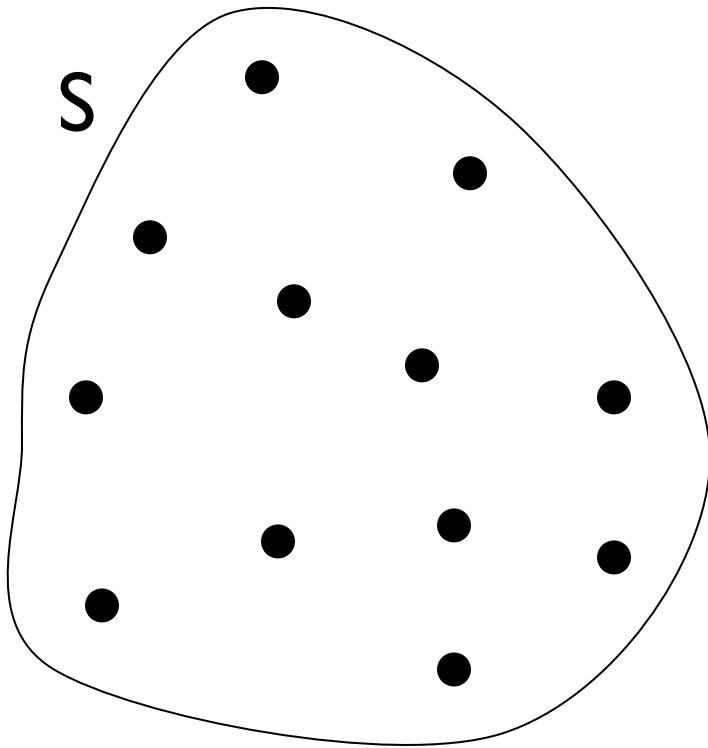
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THE 14-15 PUZZLE IN PUZZLELAND

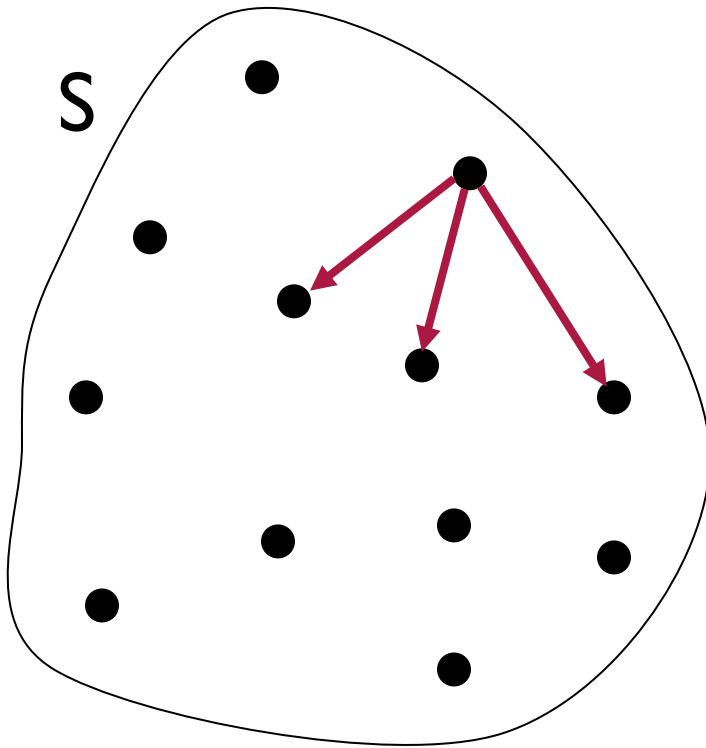


But no one ever won the prize !!

Stating a Problem as a Search Problem

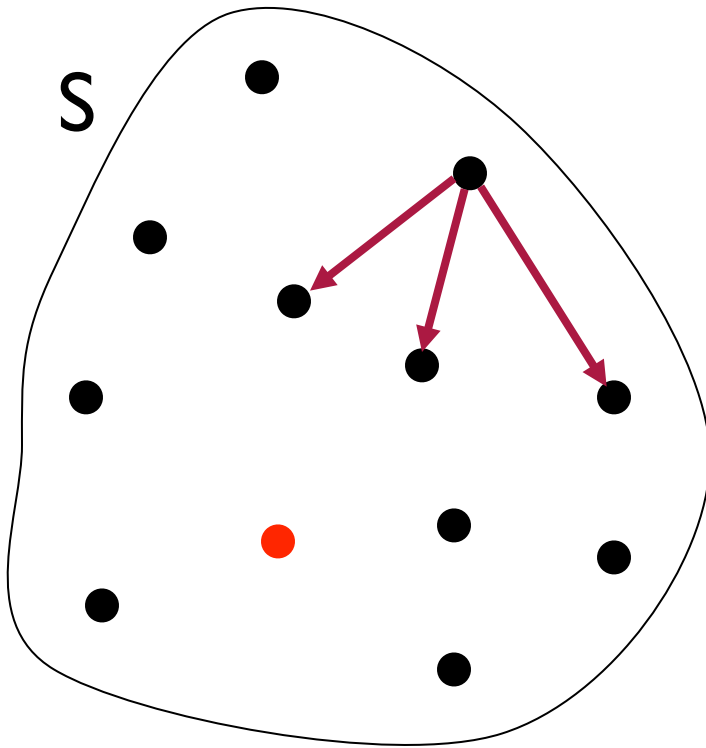


Stating a Problem as a Search Problem



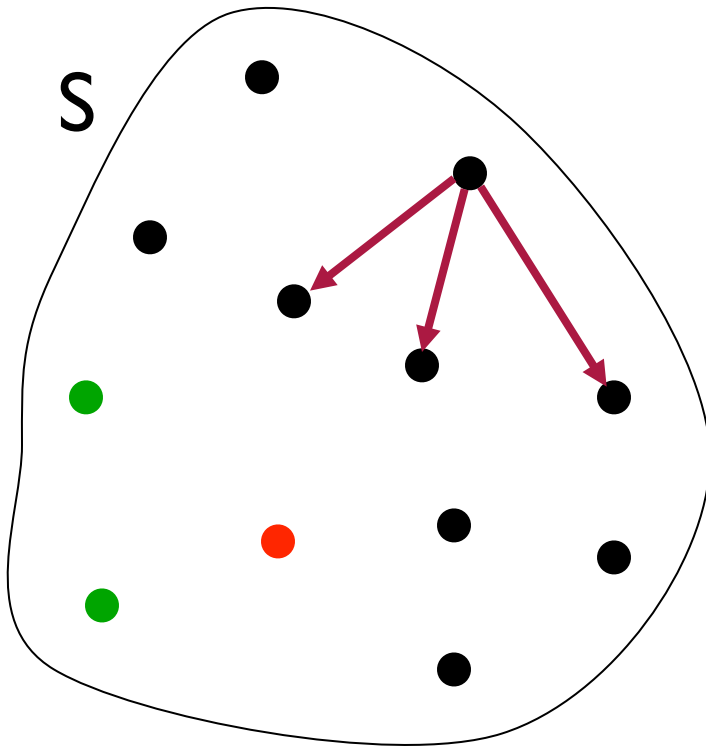
- State space S

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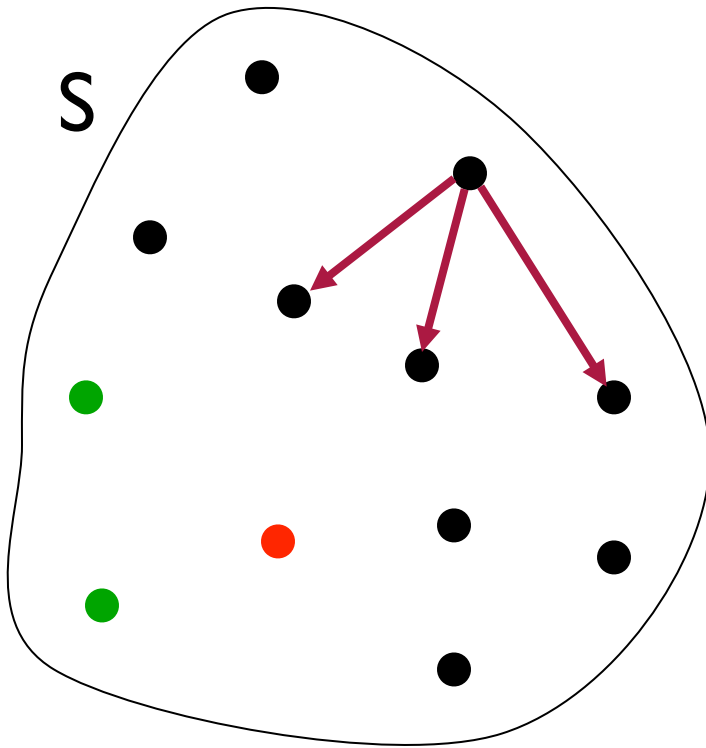
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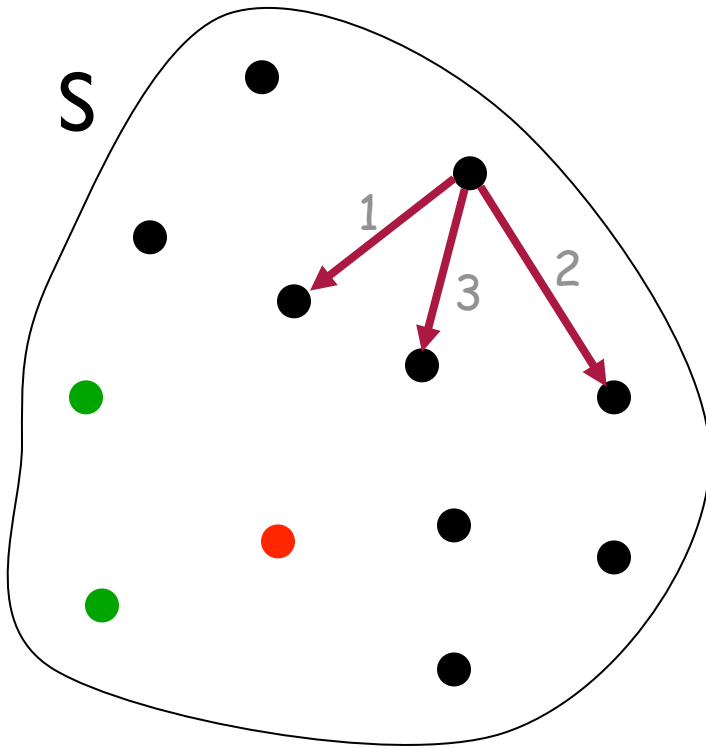
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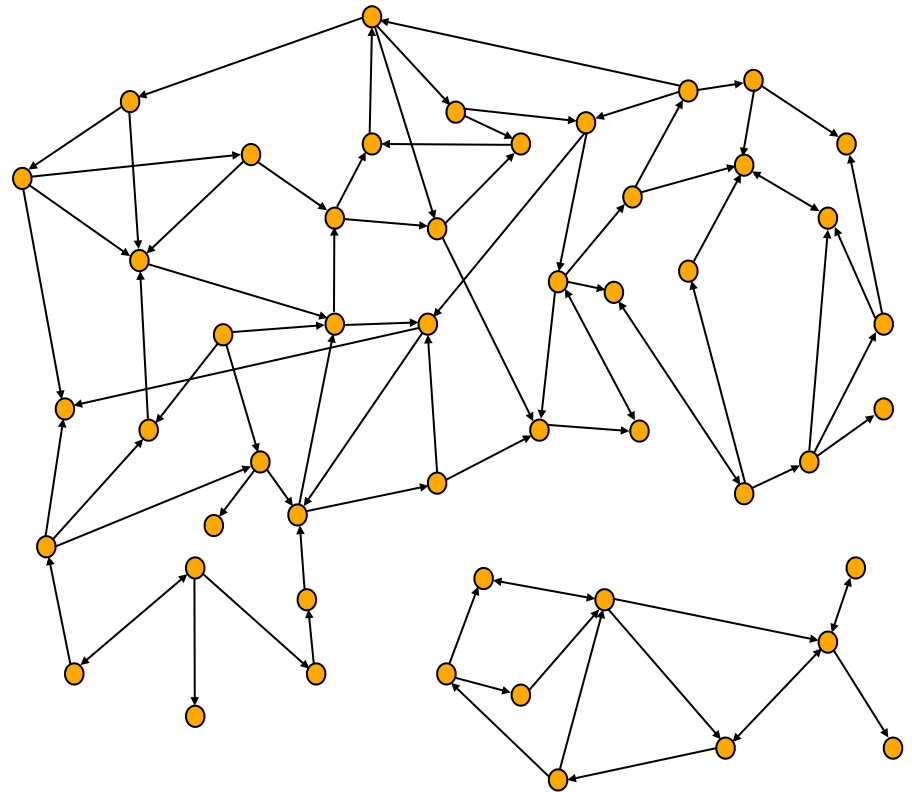
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- Arc cost

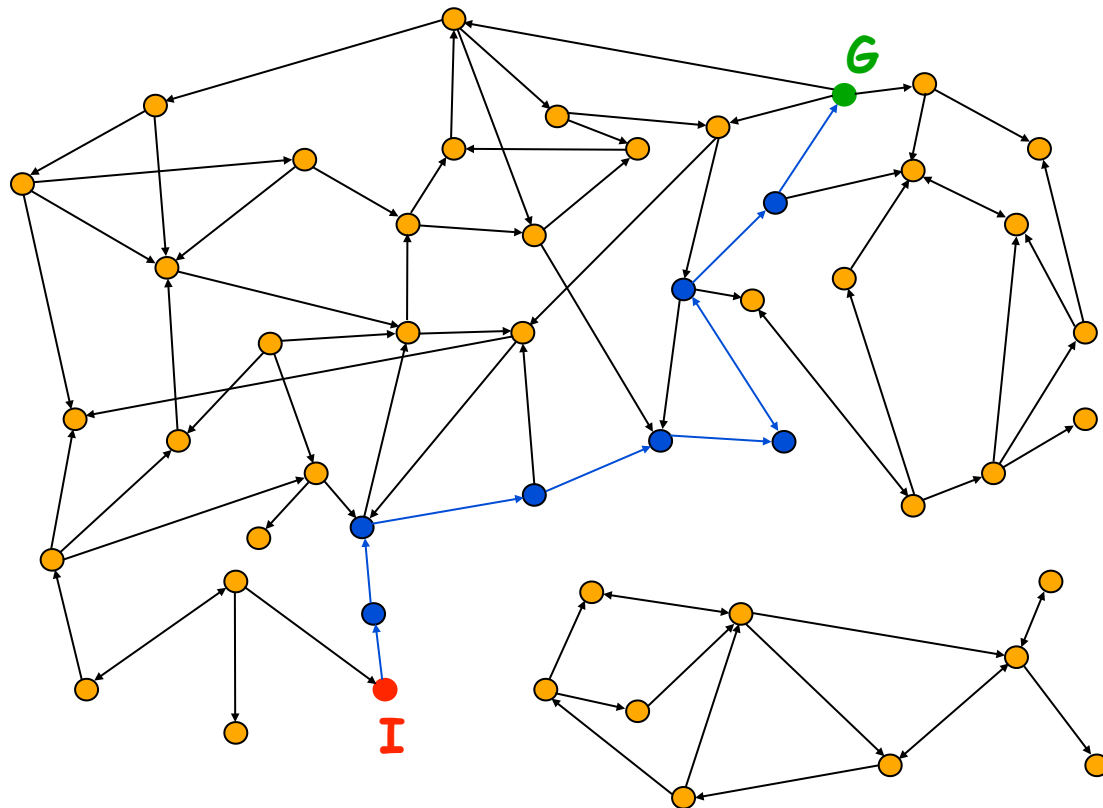
State Graph

- Each state is represented by a distinct node
- An arc (or edge) connects a node s to a node s' if $s' \in \text{SUCCESSORS}(s)$
- The state graph may contain more than one connected component



Solution to the Search

- A **solution** is a path connecting the initial node to a goal node (any one)



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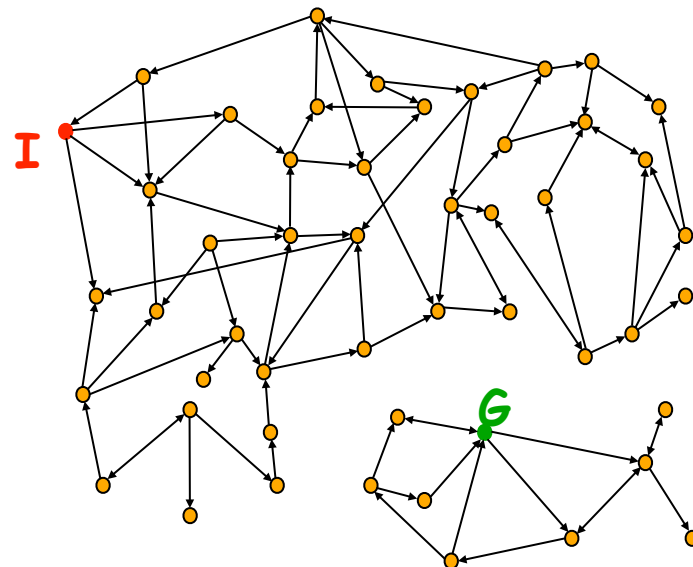
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- A **solution** is a path connecting the initial node to a goal node (any one)
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- An **optimal** solution is a solution path of minimum cost
- There might be no solution !



How big is the state space of the $(n^2 - 1)$ -puzzle?

- 8-puzzle \rightarrow ?? states

**How big is the state space of the
($n^2 - 1$)-puzzle?**

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But only half of these states are reachable
from any given state
(but you may not know that in advance)

Permutation Inversions

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Permutation Inversions

- Let the goal be:

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$$\begin{array}{lll}
 n_2 = 0 & n_3 = 0 & n_4 = 0 \\
 n_5 = 0 & n_6 = 0 & n_7 = 1 \\
 n_8 = 1 & n_9 = 1 & n_{10} = 4 \\
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- Let the goal be:

1	2	3	4
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- A tile j **appears after** a tile i if either j appears on the same row as i to the right of i , or on another row below the row of i .

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- For every $i = 1, 2, \dots, 15$, let n_i be the number of tiles $j < i$ that appear after tile i (permutation inversions)

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- For every $i = 1, 2, \dots, 15$, let n_i be the number of tiles $j < i$ that appear after tile i (permutation inversions)
- $N = n_2 + n_3 + \dots + n_{15} + \text{row number of empty tile}$

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$$\rightarrow N = 7 + 4$$

- Proposition: $(N \bmod 2)$ is invariant under any legal move of the empty tile
- Proof:
 - Any horizontal move of the empty tile leaves N unchanged
 - A vertical move of the empty tile changes N by an even increment $(\pm 1 \pm 1 \pm 1 \pm 1)$

$s =$

1	2	3	4
5	6		7
9	10	11	8
13	14	15	12

$s' =$

1	2	3	4
5	6	11	7
9	10		8
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$$N(s') = N(s) + 3 + 1$$

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- It can be shown that this is also a sufficient condition
- \rightarrow The state graph consists of two connected components of equal size

15-Puzzle

Sam Loyd offered \$1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
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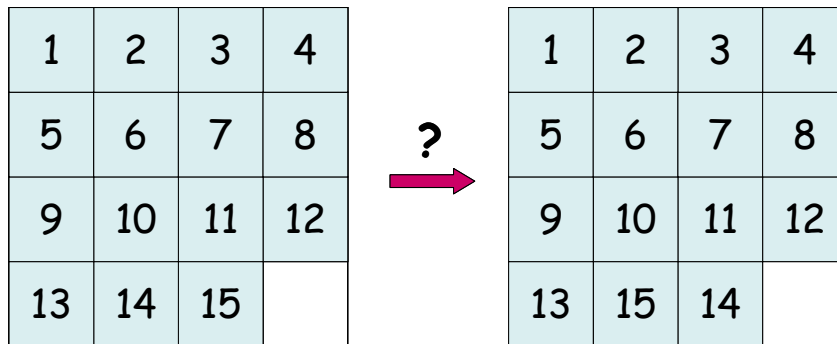
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N = 4

N = 5

15-Puzzle

Sam Loyd offered \$1,000 of his own money to the first person who would solve the following problem:



N = 4

N = 5

So, the second state is not reachable from the first, and Sam Loyd took no risk with his money ...

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- a) The set of all states?
[e.g., a set of $16!$ states for the 15-puzzle]

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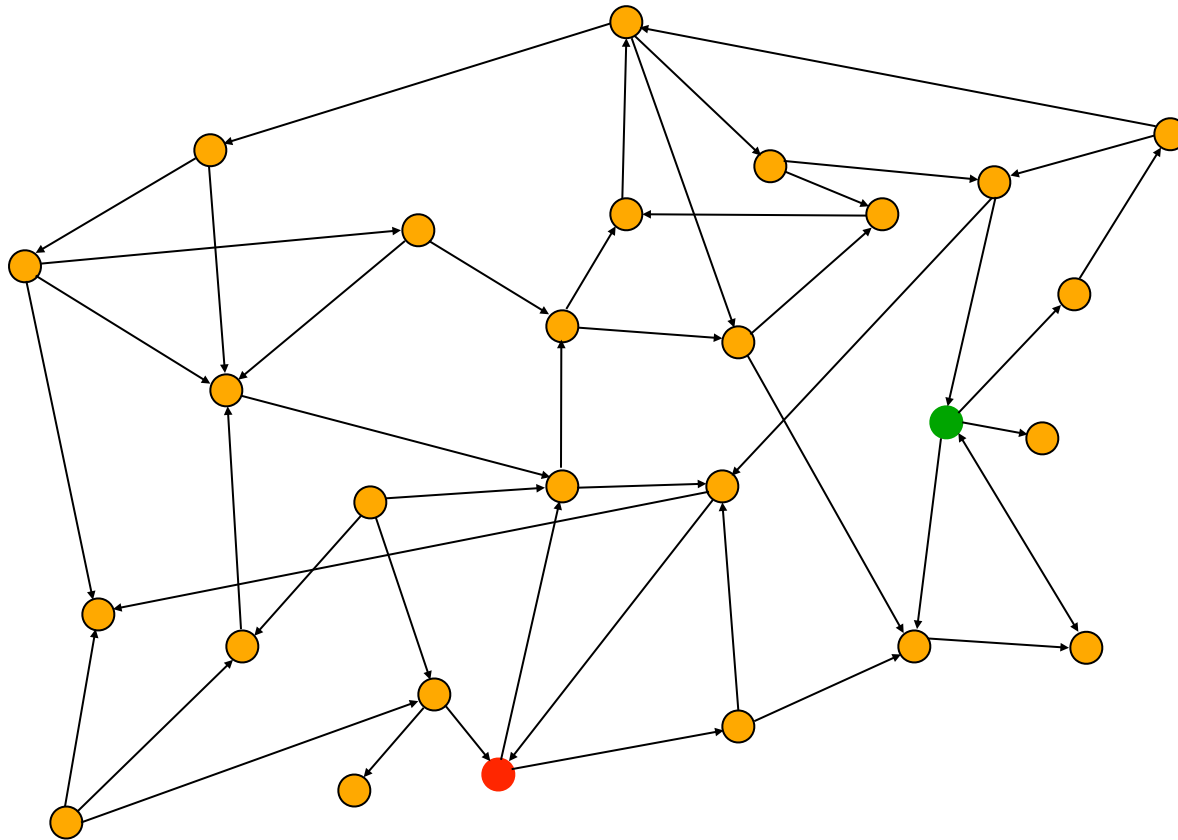
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In general, the answer is a)
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But a fast test determining whether a state is reachable from another is very useful, as search techniques are often **inefficient** when a problem has no solution

Searching the State Space



- It is often not feasible (or too expensive) to build a complete representation of the state graph

8-, 15-, 24-Puzzles

100 millions states/sec

8-, 15-, 24-Puzzles

8-puzzle \rightarrow 362,880 states

0.036 sec

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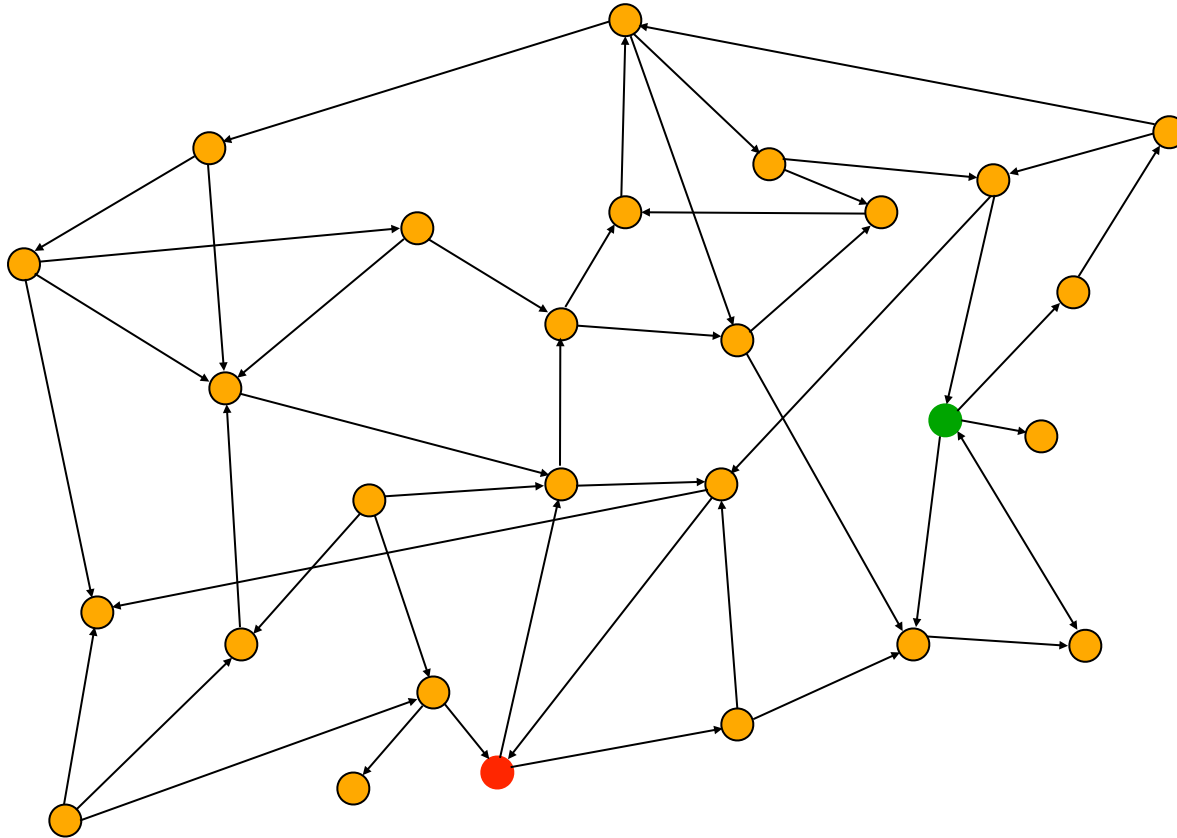
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24-puzzle $\rightarrow 10^{25}$ states,
 $> 10^9$ years

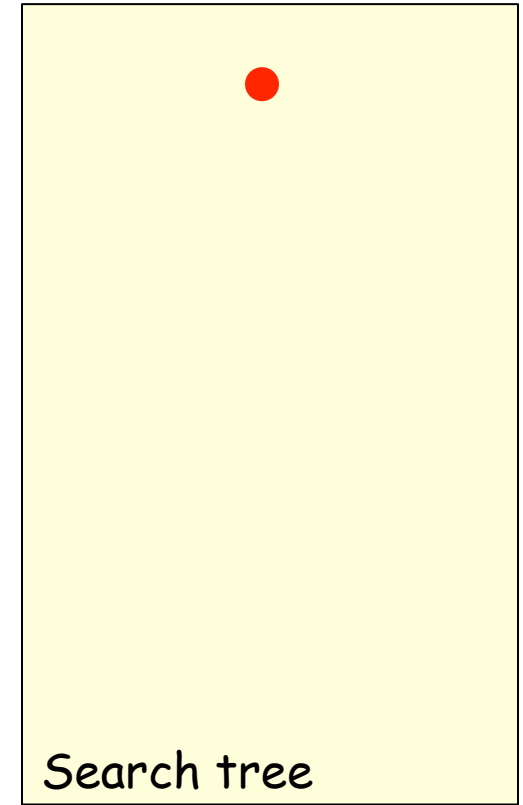
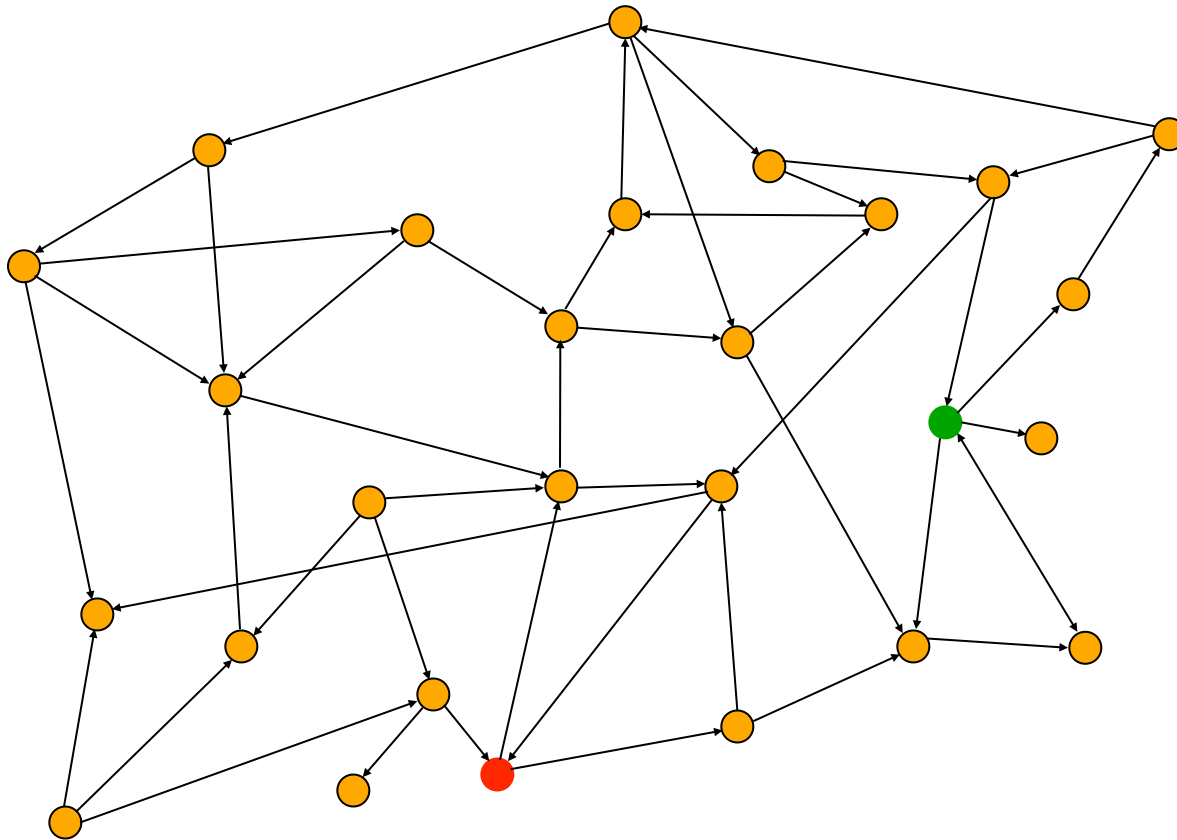
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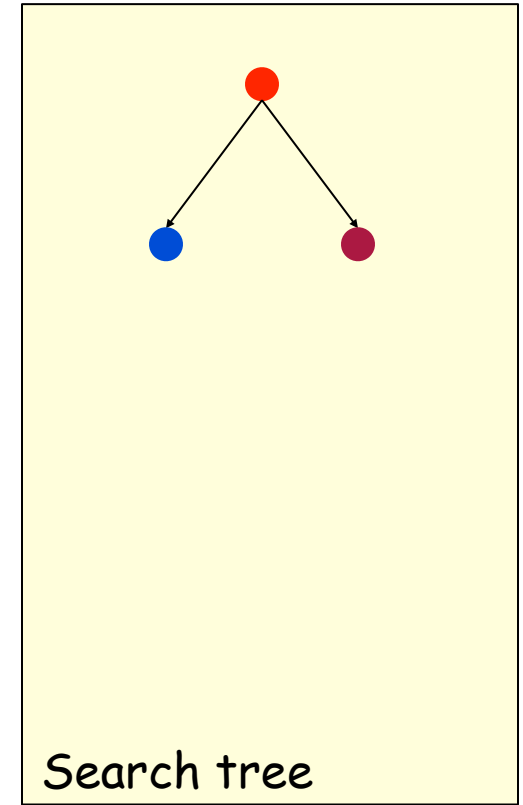
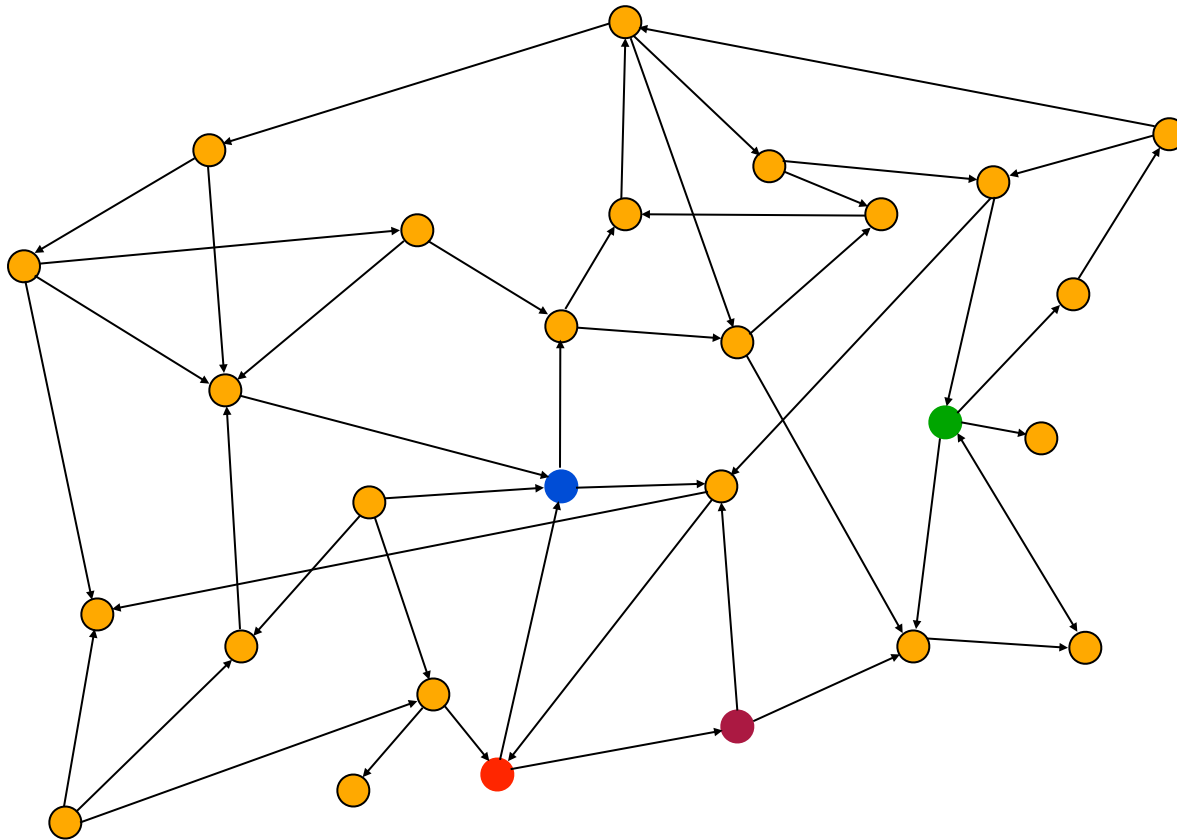


- Often it is not feasible (or too expensive) to build a complete representation of the state graph
- A problem solver must construct a solution by exploring a small portion of the graph

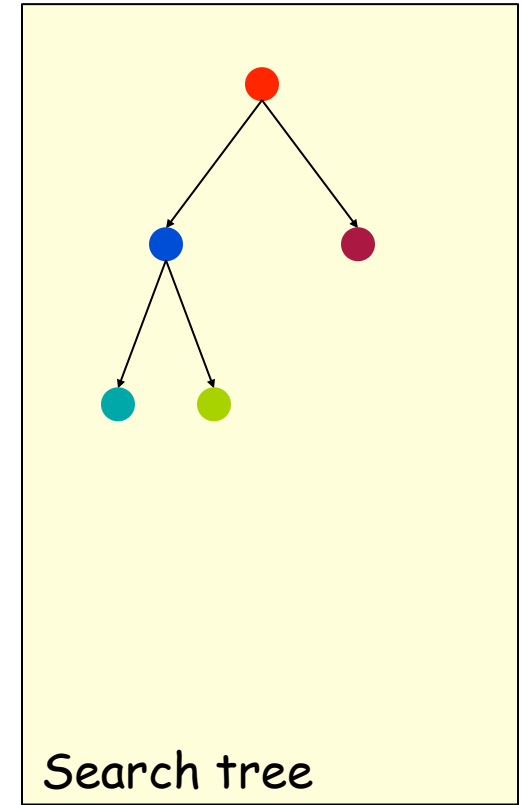
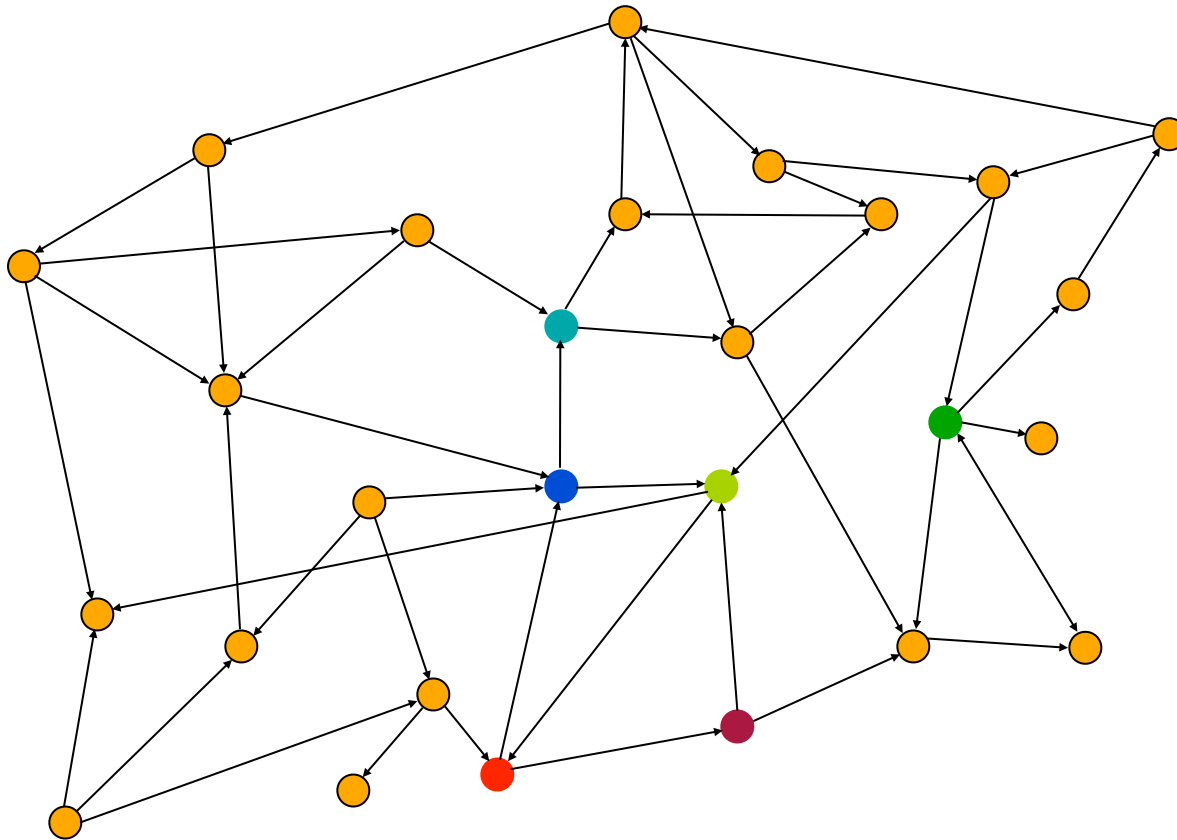
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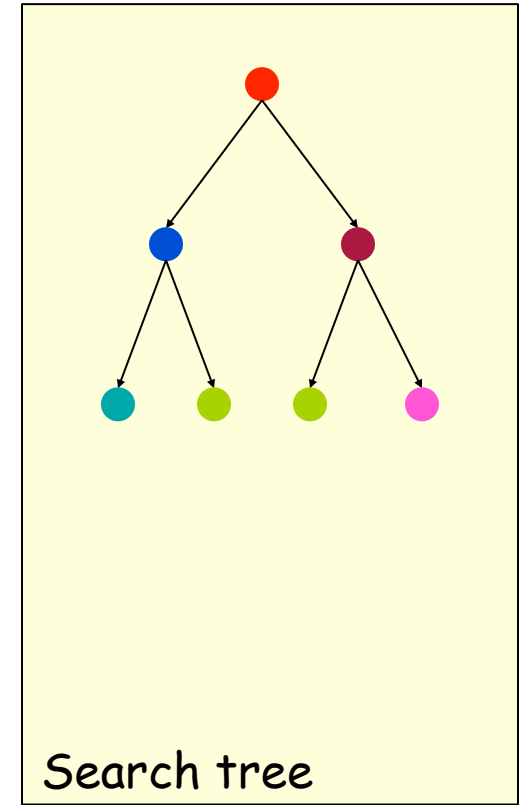
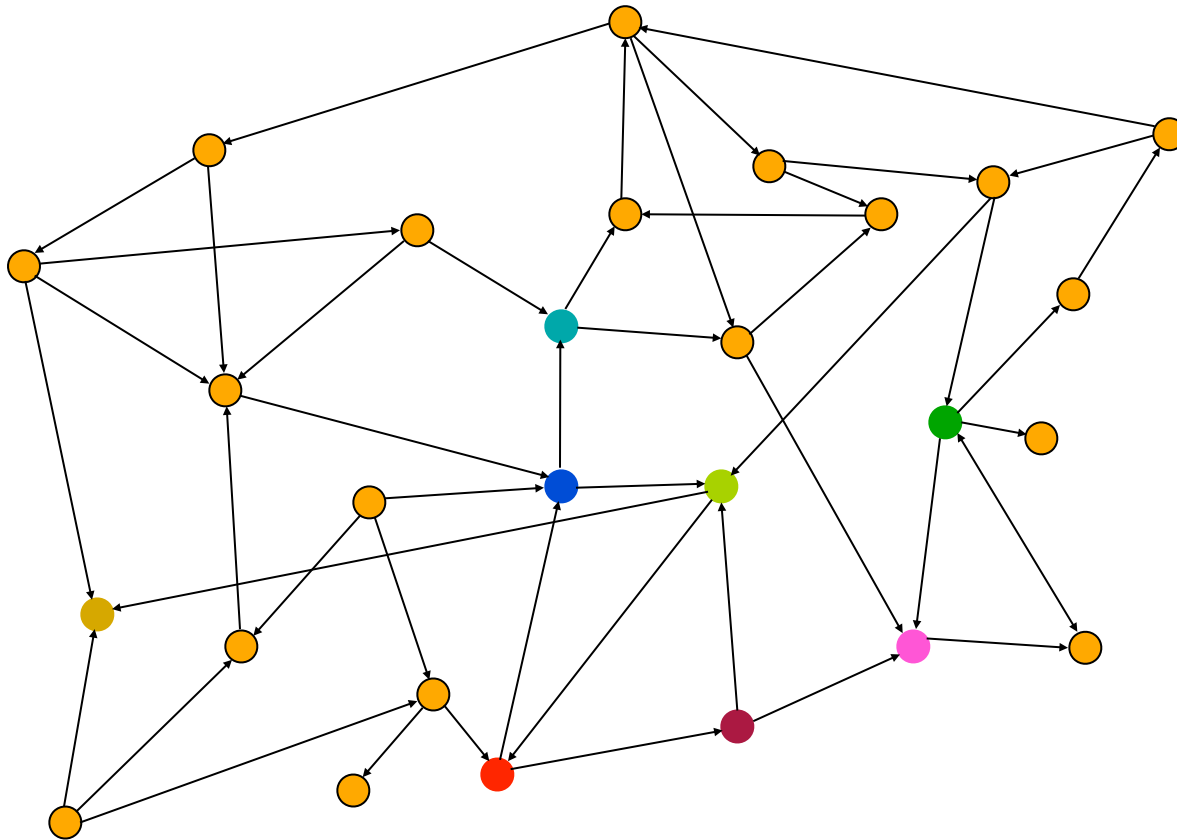
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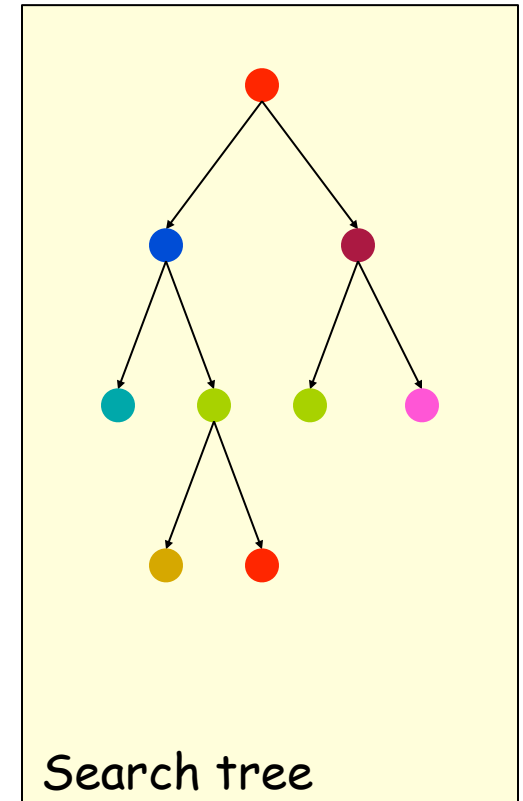
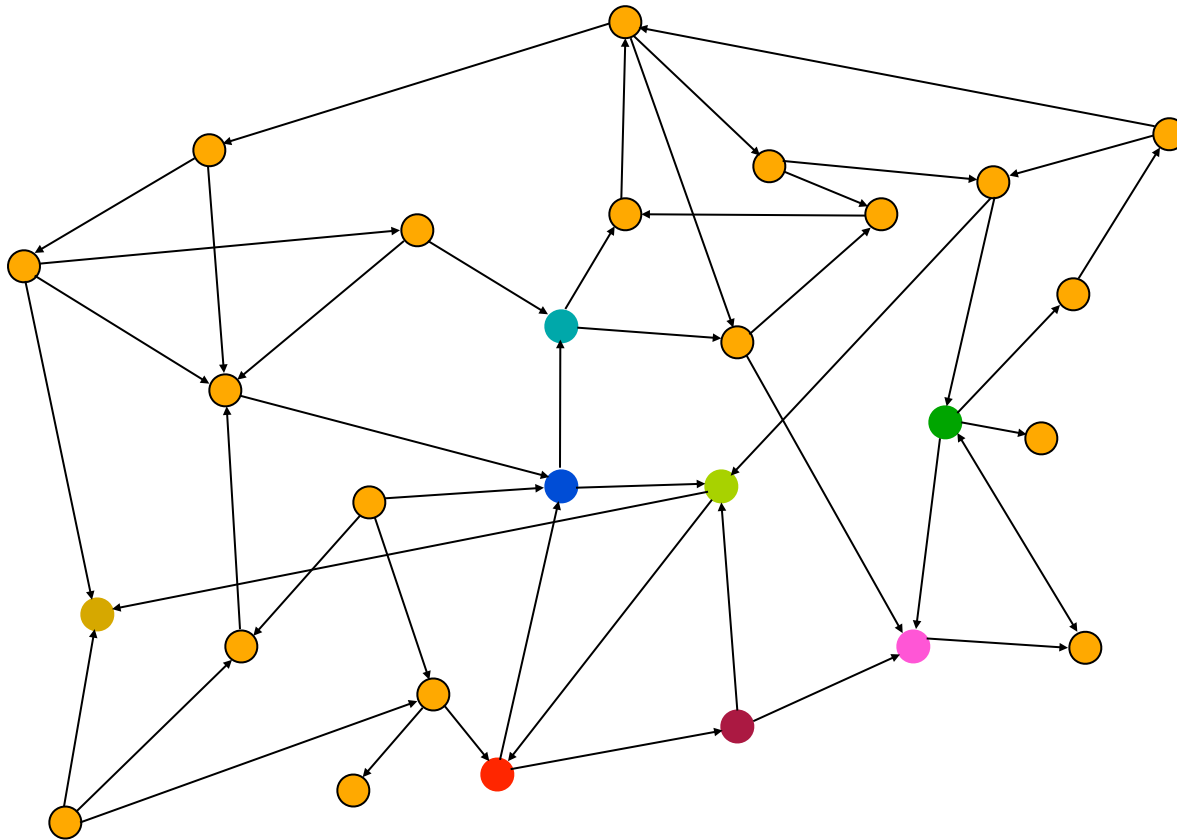
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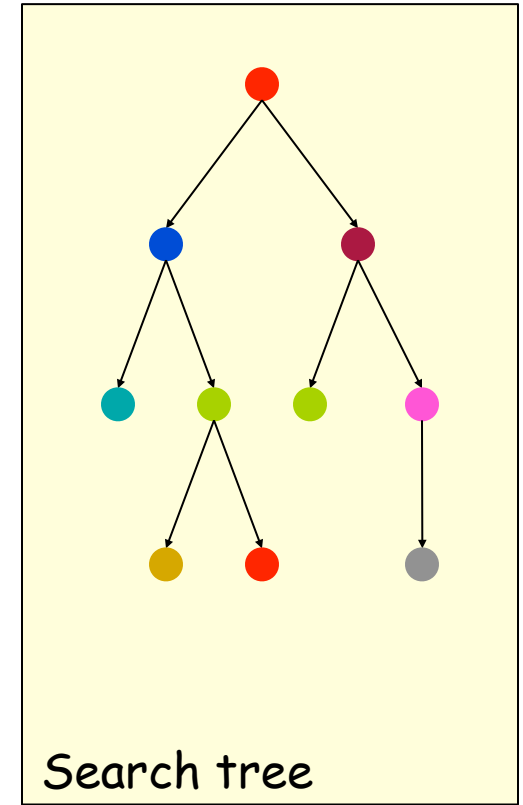
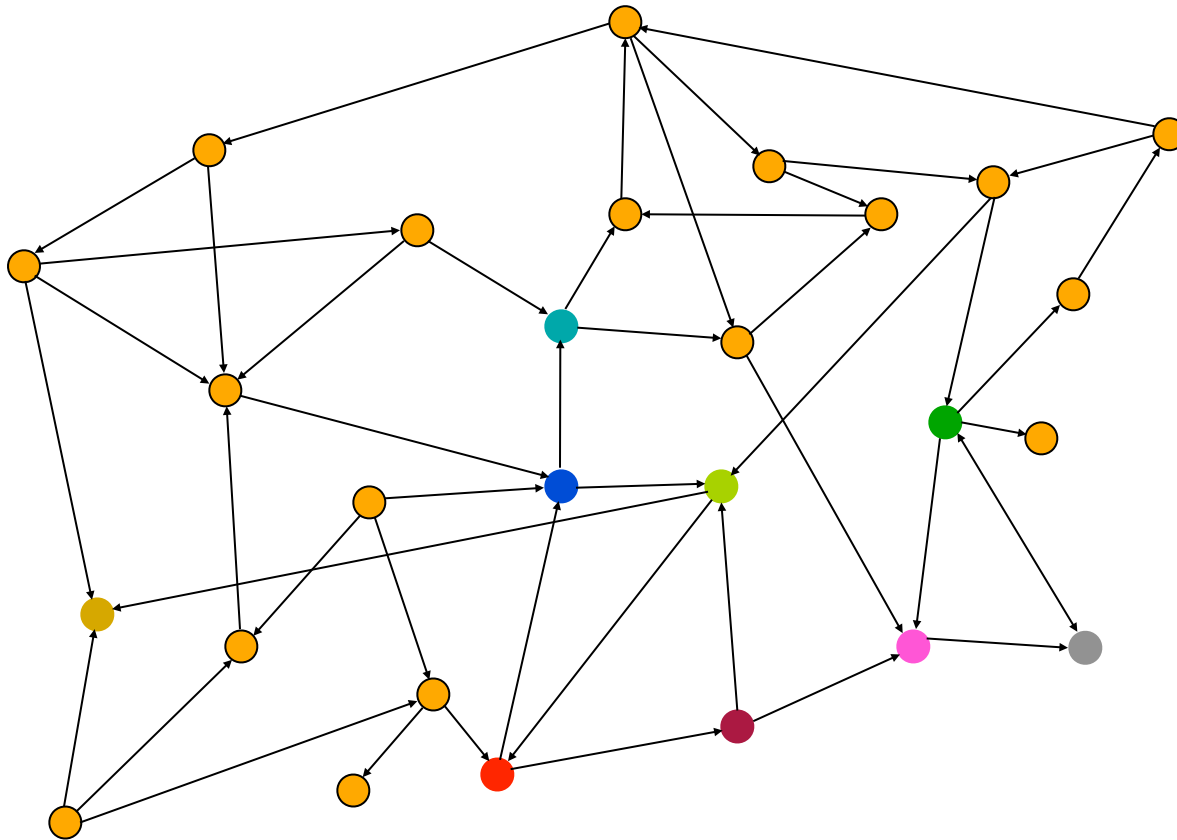
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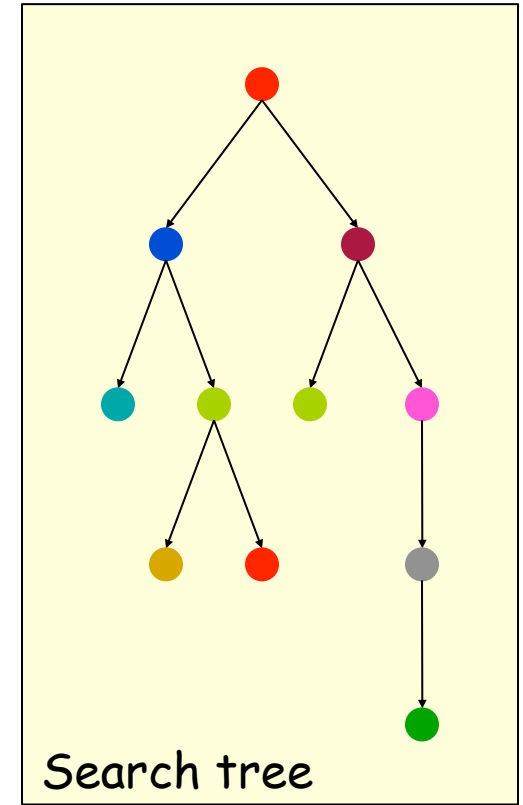
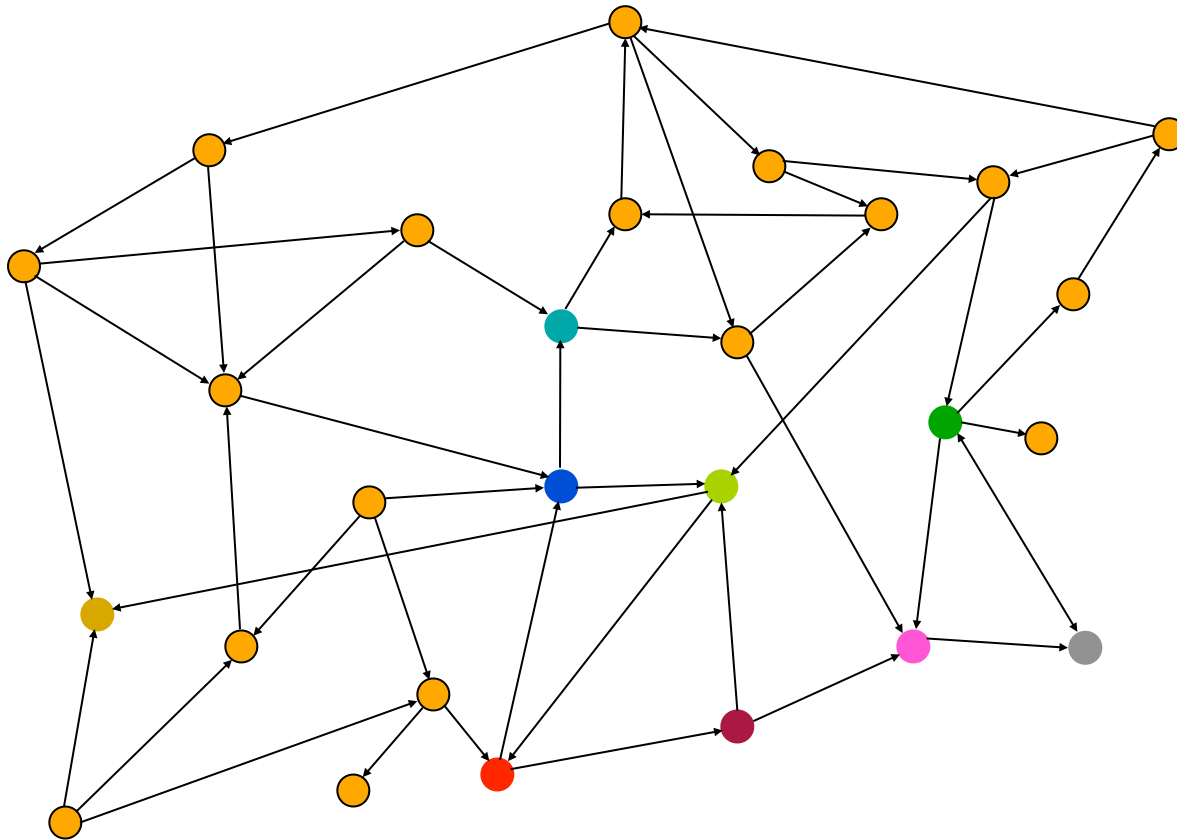
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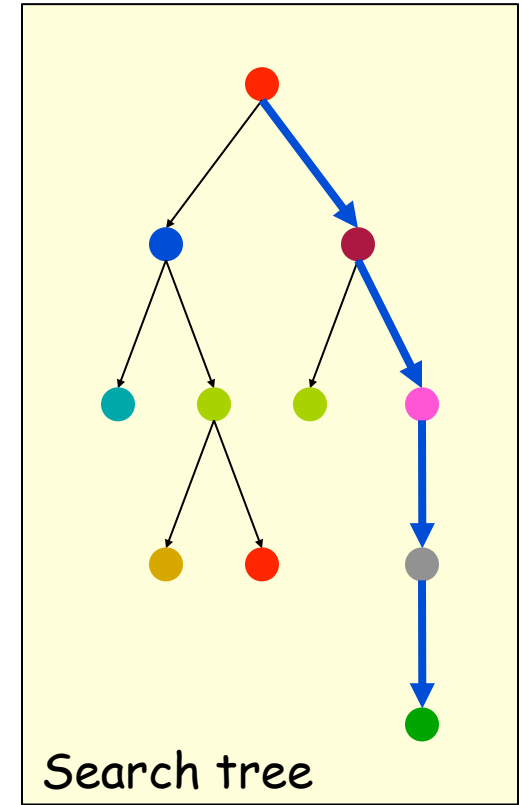
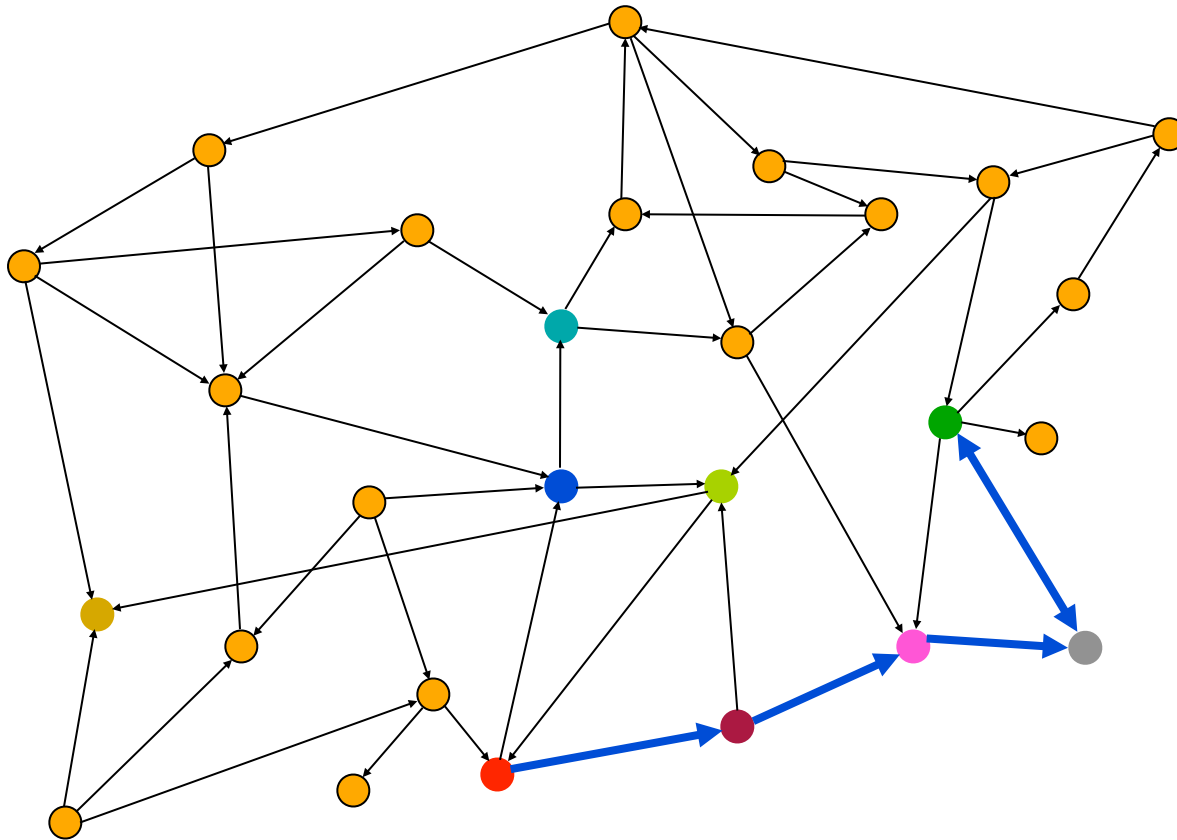
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Searching the State Space



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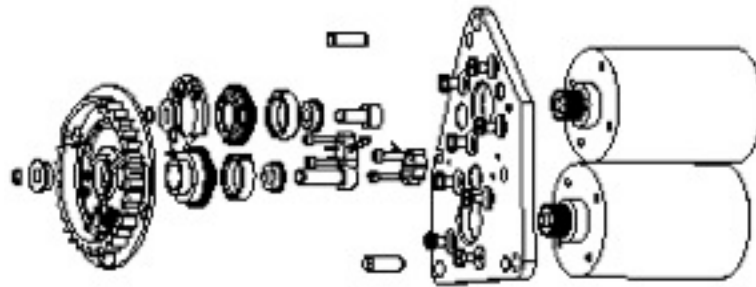
Simple Problem-Solving-Agent Algorithm

1. $I \leftarrow$ sense/read initial state
2. $GOAL? \leftarrow$ select/read goal test
3. $Succ \leftarrow$ select/read successor function
4. solution \leftarrow **search**(I , $GOAL?$, $Succ$)
5. perform(solution)

State Space

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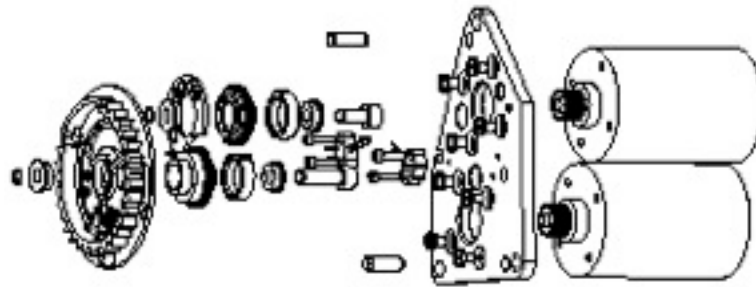
- Each state is an **abstract** representation of a collection of possible worlds sharing some crucial properties and differing on non-important details only



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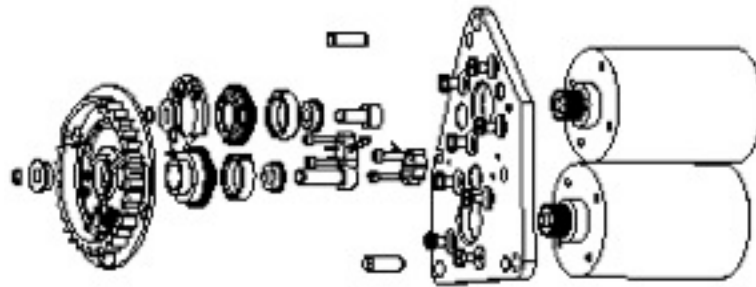
E.g.: In assembly planning, a state does not define exactly the absolute position of each part



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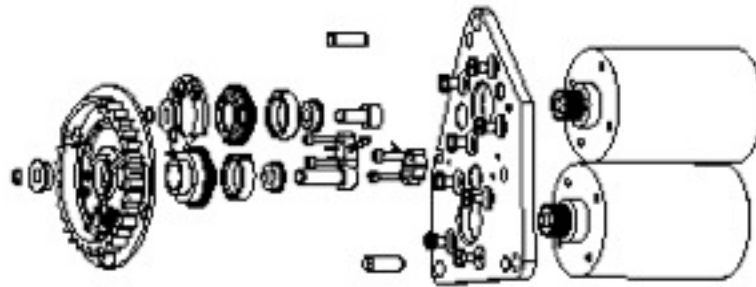
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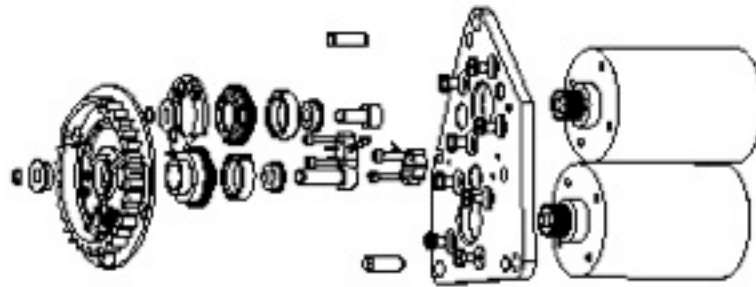
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State Space

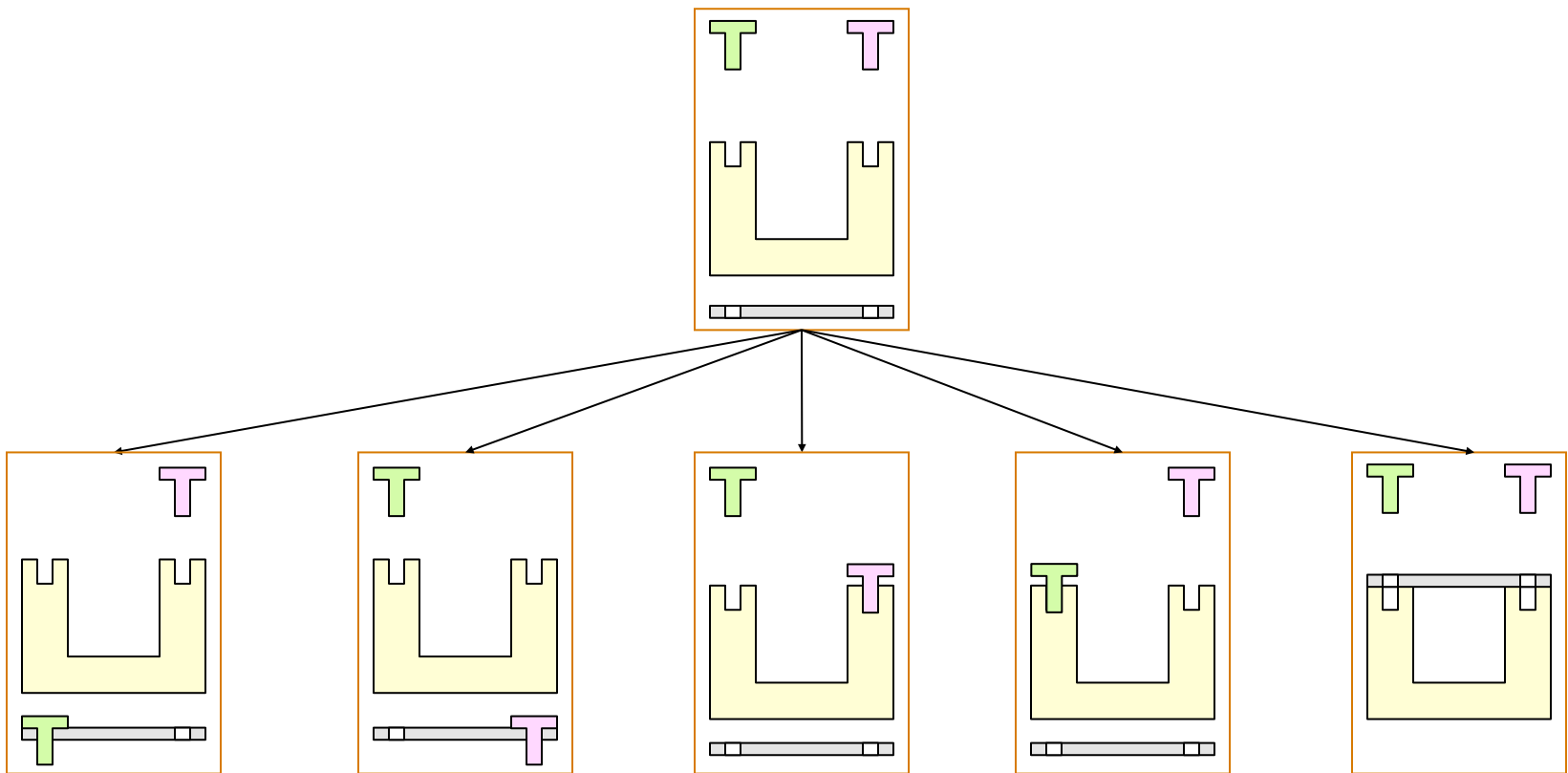
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E.g.: In assembly planning, a state does not define exactly the absolute position of each part



Successor Function

- It implicitly represents all the actions that are feasible in each state



Successor Function

- It implicitly represents all the actions that are feasible in each state
- Only the results of the actions (the successor states) and their costs are returned by the function
- The successor function is a “black box”: its content is unknown
E.g., in assembly planning, the successor function may be quite complex (collision, stability, grasping, ...)

Path Cost

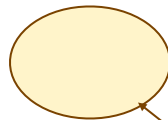
Path Cost

- An arc cost is a positive number measuring the “cost” of performing the action corresponding to the arc, e.g.:
 - 1 in the 8-puzzle example
 - expected time to merge two sub-assemblies

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- We will assume that for any given problem the cost c of an arc always verifies: $c \geq \varepsilon > 0$, where ε is a constant

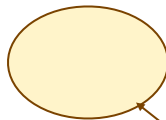
Path Cost



Why is this needed?

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[This condition guarantees that, if path becomes arbitrarily long, its cost also becomes arbitrarily large]

Why is this needed?

Goal State

1	2	3
4	5	6
7	8	

Goal State

- It may be explicitly described:

1	a	a
a	5	a
a	8	a

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4	5	6
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("a" stands for "any" other than 1, 5, and 8)

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- or partially described:

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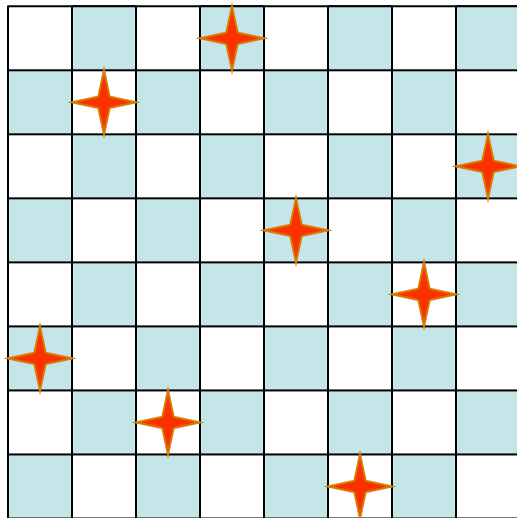
- or defined by a condition,
e.g., the sum of every row, of every column, and of every diagonal equals 30

15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	

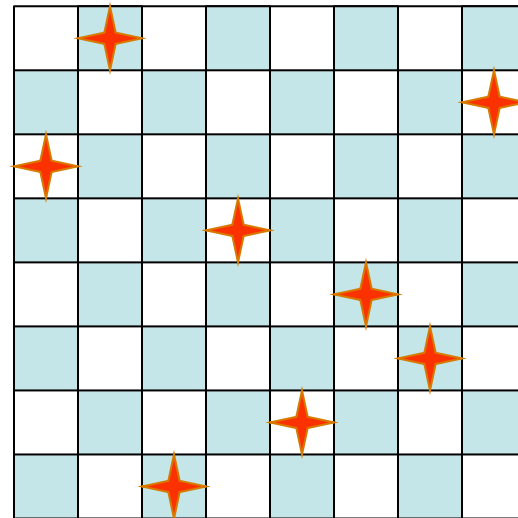
Other examples

8-Queens Problem

Place 8 queens in a chessboard so that no two queens are in the same row, column, or diagonal.

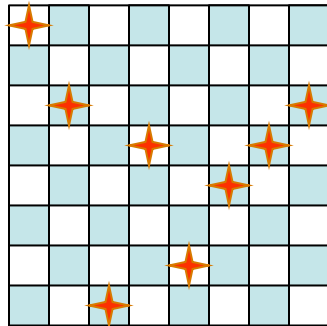
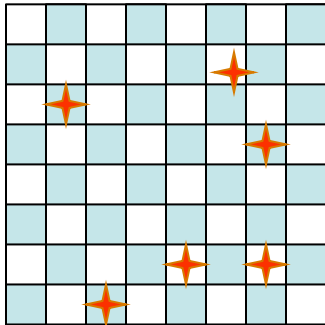
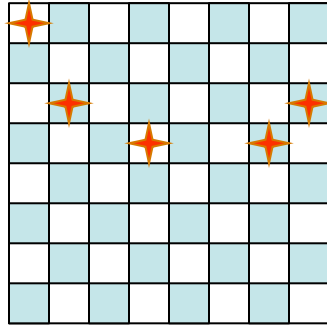
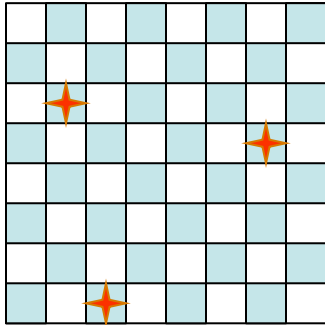


A solution



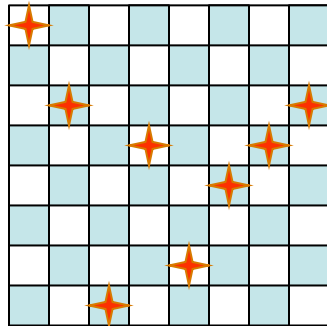
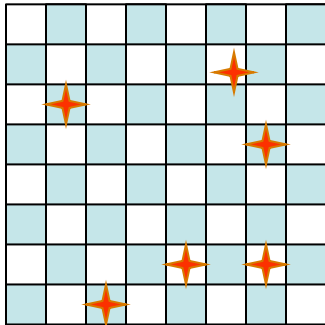
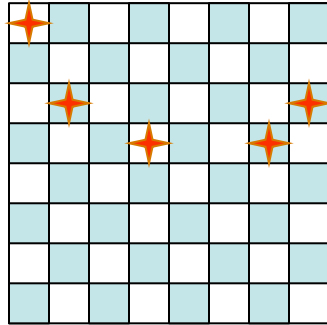
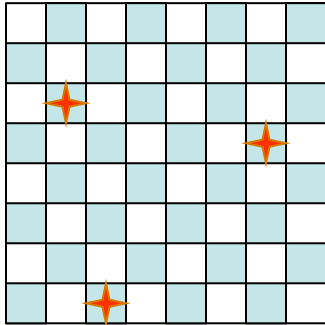
Not a solution

Formulation #1



- **States:** all arrangements of 0, 1, 2, ..., 8 queens on the board
- **Initial state:** 0 queens on the board
- **Successor function:** each of the successors is obtained by adding one queen in an empty square
- **Arc cost:** irrelevant
- **Goal test:** 8 queens are on the board, with no queens attacking each other

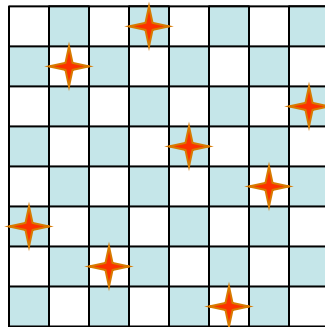
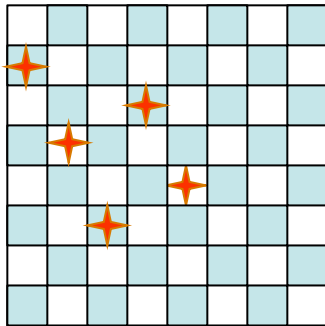
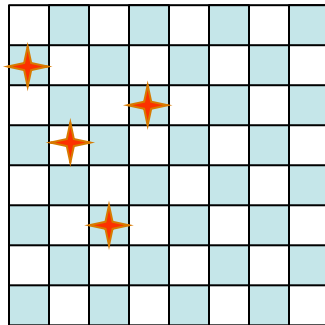
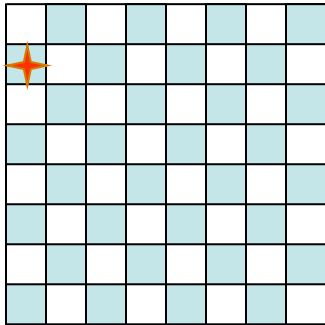
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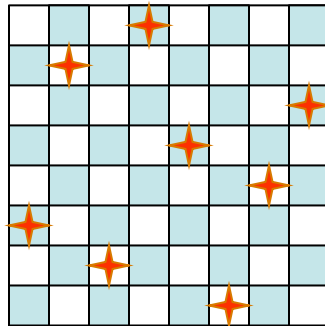
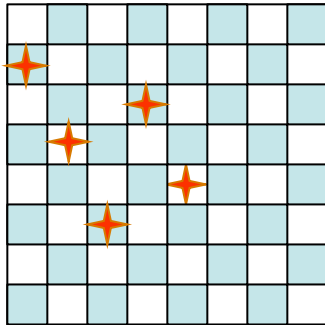
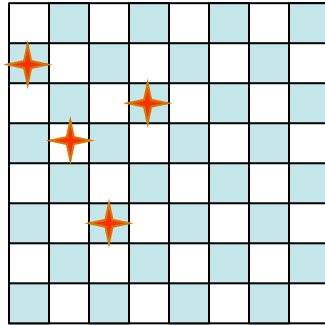
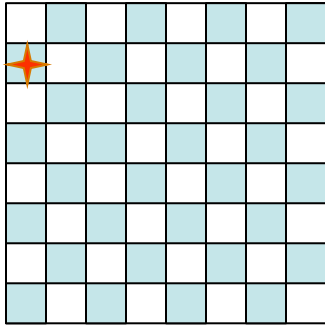
→ $\sim 64 \times 63 \times \dots \times 57 \sim 3 \times 10^{14}$ states

Formulation #2



- **States:** all arrangements of $k = 0, 1, 2, \dots, 8$ queens in the k leftmost columns with no two queens attacking each other
- **Initial state:** 0 queens on the board
- **Successor function:** each successor is obtained by adding one queen in any square that is not attacked by any queen already in the board, in the leftmost empty column
- **Arc cost:** irrelevant

Formulation #2



- **States:** all arrangements of $k = 0, 1, 2, \dots, 8$ queens in the k leftmost columns with no two queens attacking each other
- **Initial state:** 0 queens on the board
- **Successor function:** each successor is obtained by adding one queen in any square that is not attacked by any queen already in the board, in the leftmost empty column
- **Arc cost:** irrelevant

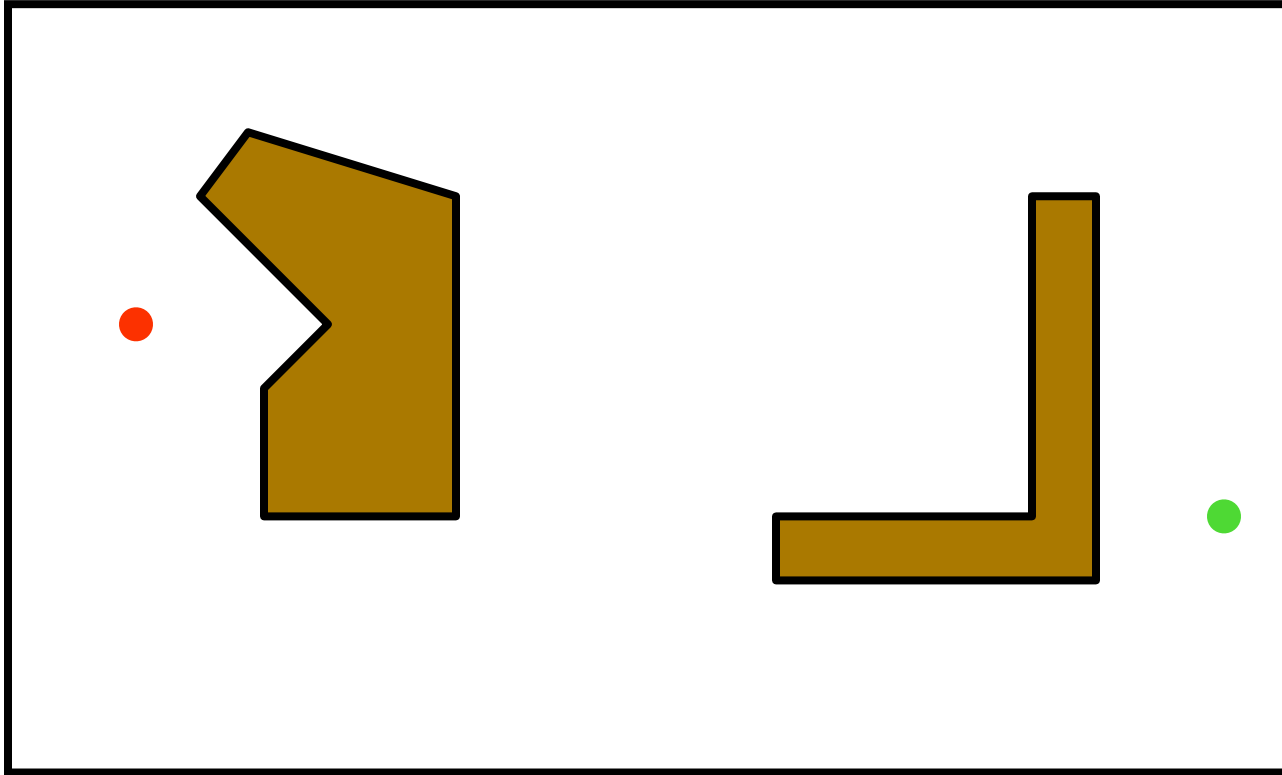
→ 2,057 states

n-Queens Problem

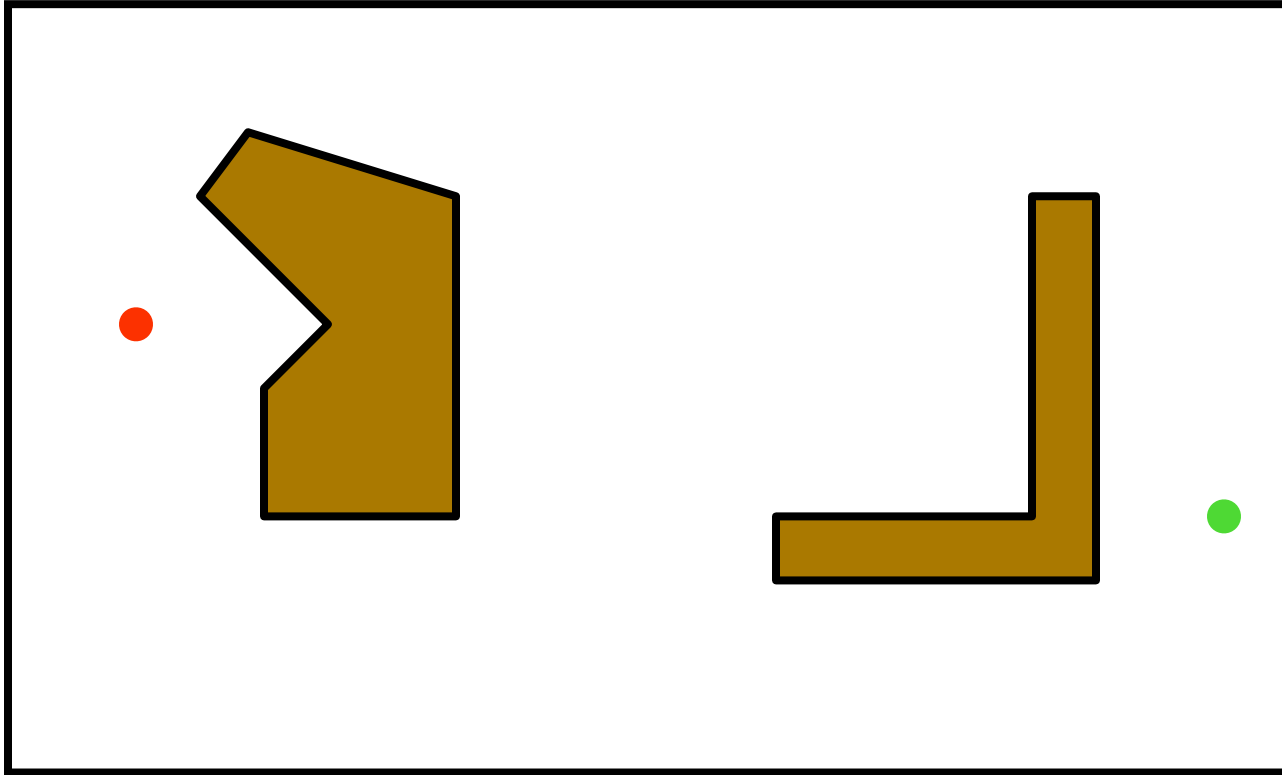
- A solution is a **goal node**, not a path to this node (typical of design problem)
- Number of states in state space:
 - 8-queens \rightarrow 2,057
 - **100-queens \rightarrow 10^{52}**
- But techniques exist to solve n-queens problems efficiently for large values of n

They exploit the fact that there are many solutions well distributed in the state space

Path Planning

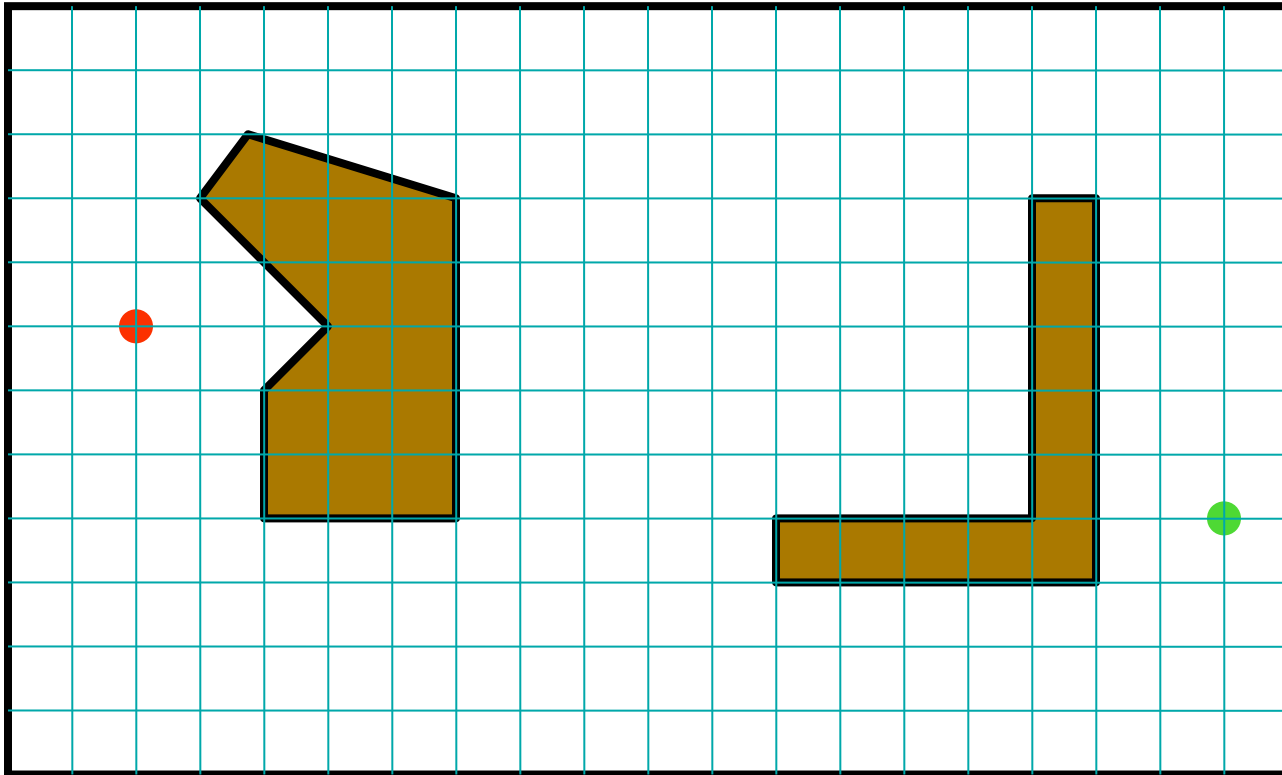


Path Planning

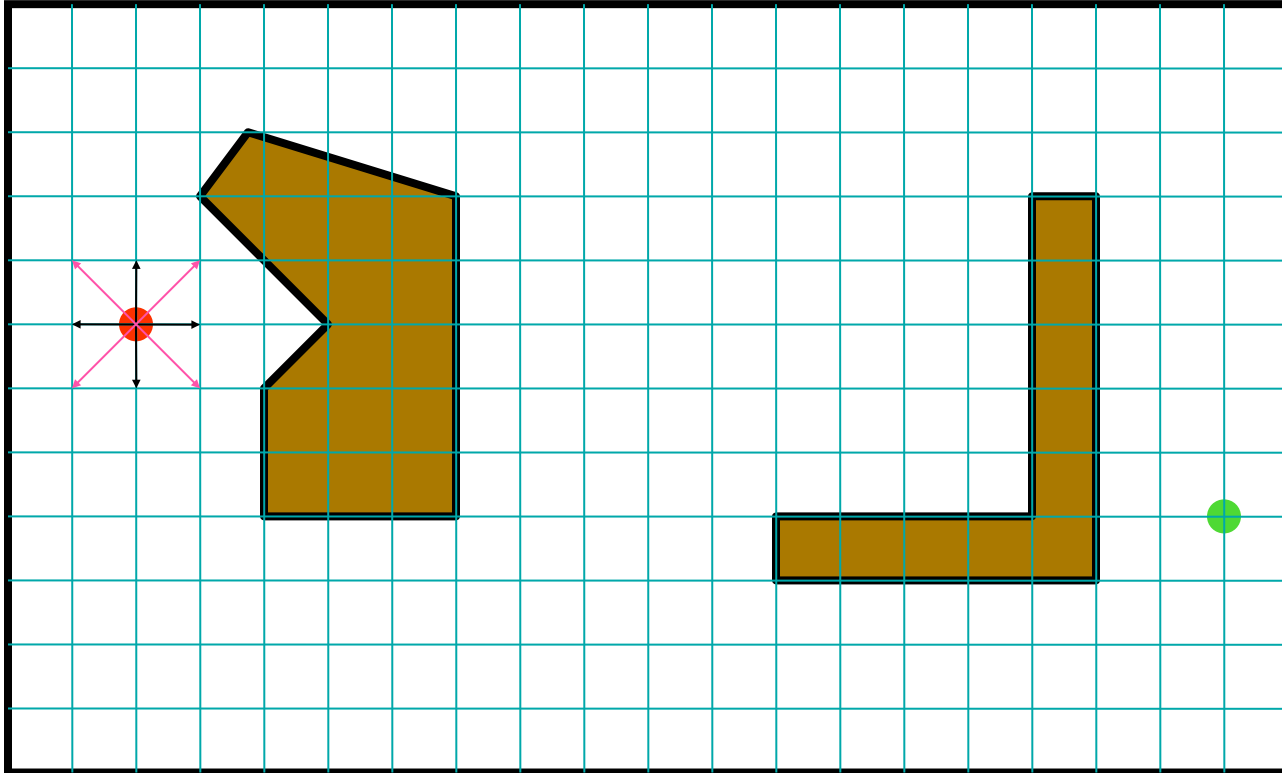


What is the state space?

Formulation #1



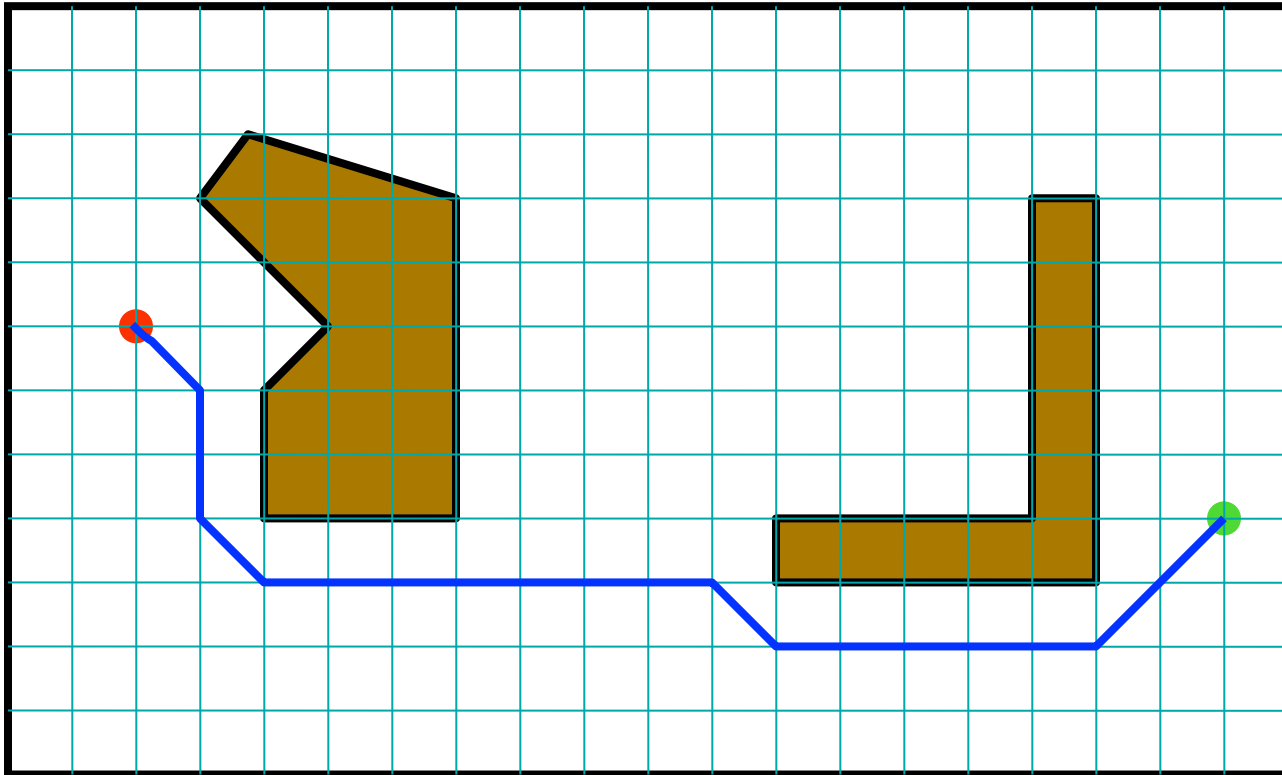
Formulation #1



Cost of one horizontal/vertical step = 1

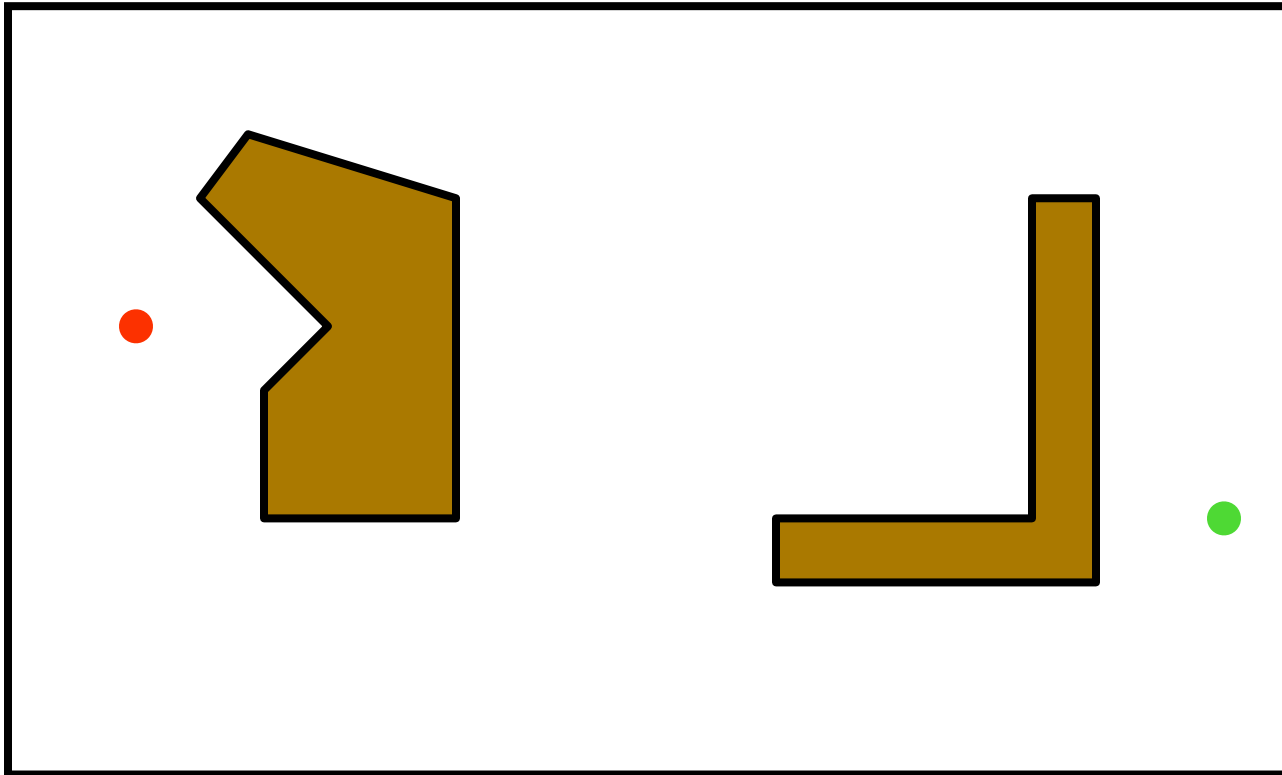
Cost of one diagonal step = $\sqrt{2}$

Optimal Solution



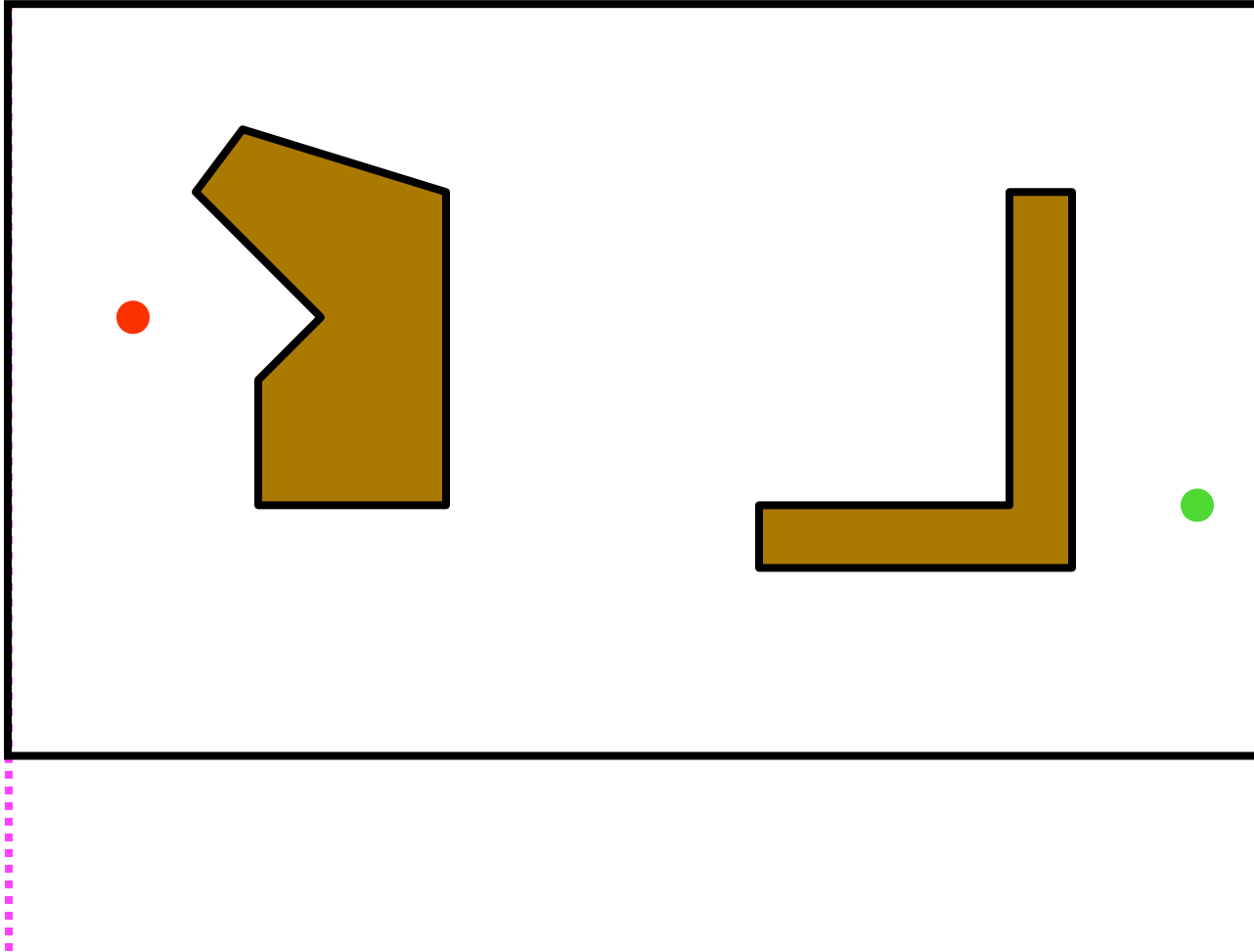
This path is the shortest in the discretized state space, but not in the original continuous space

Formulation #2

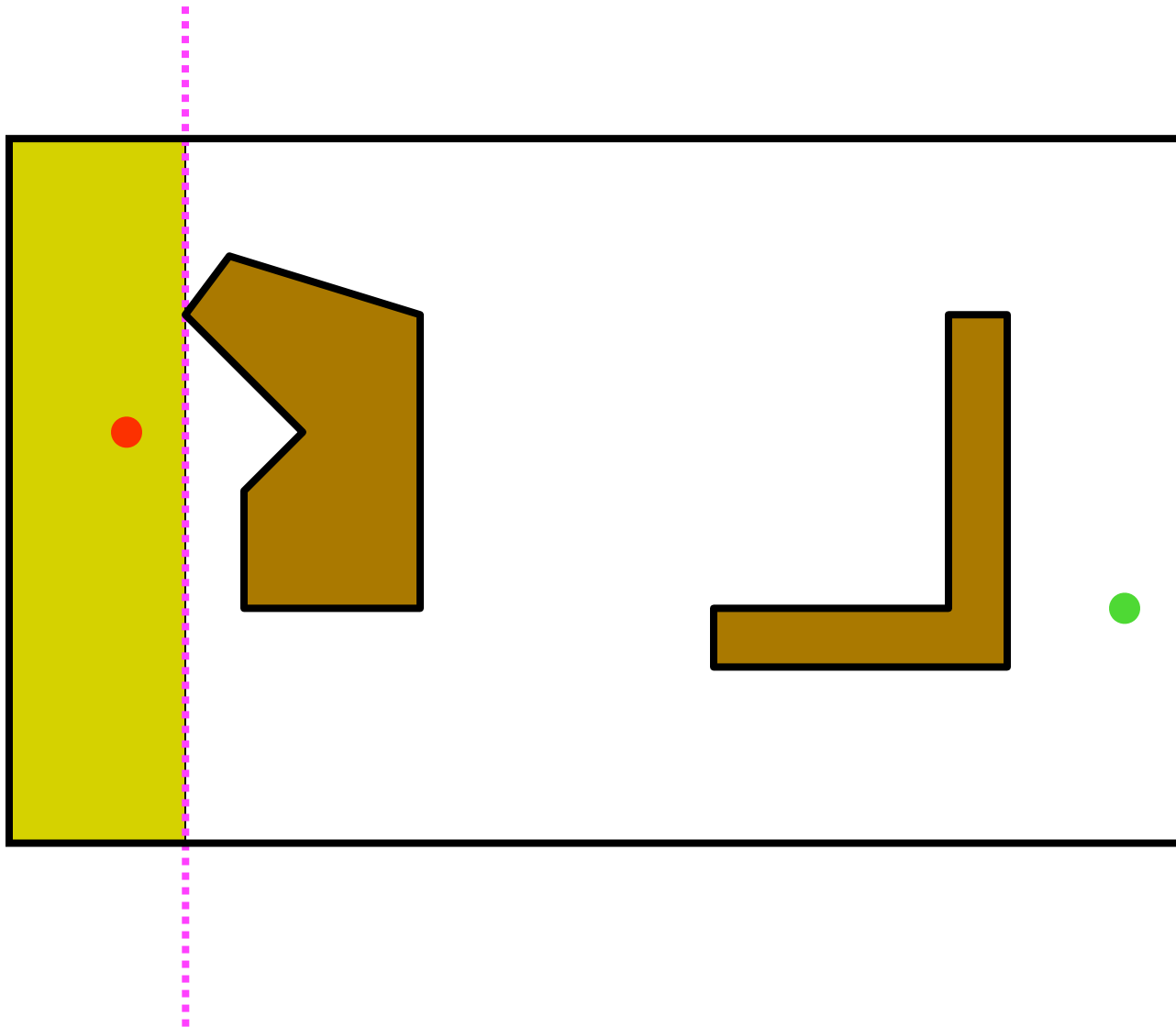


Formulation #2

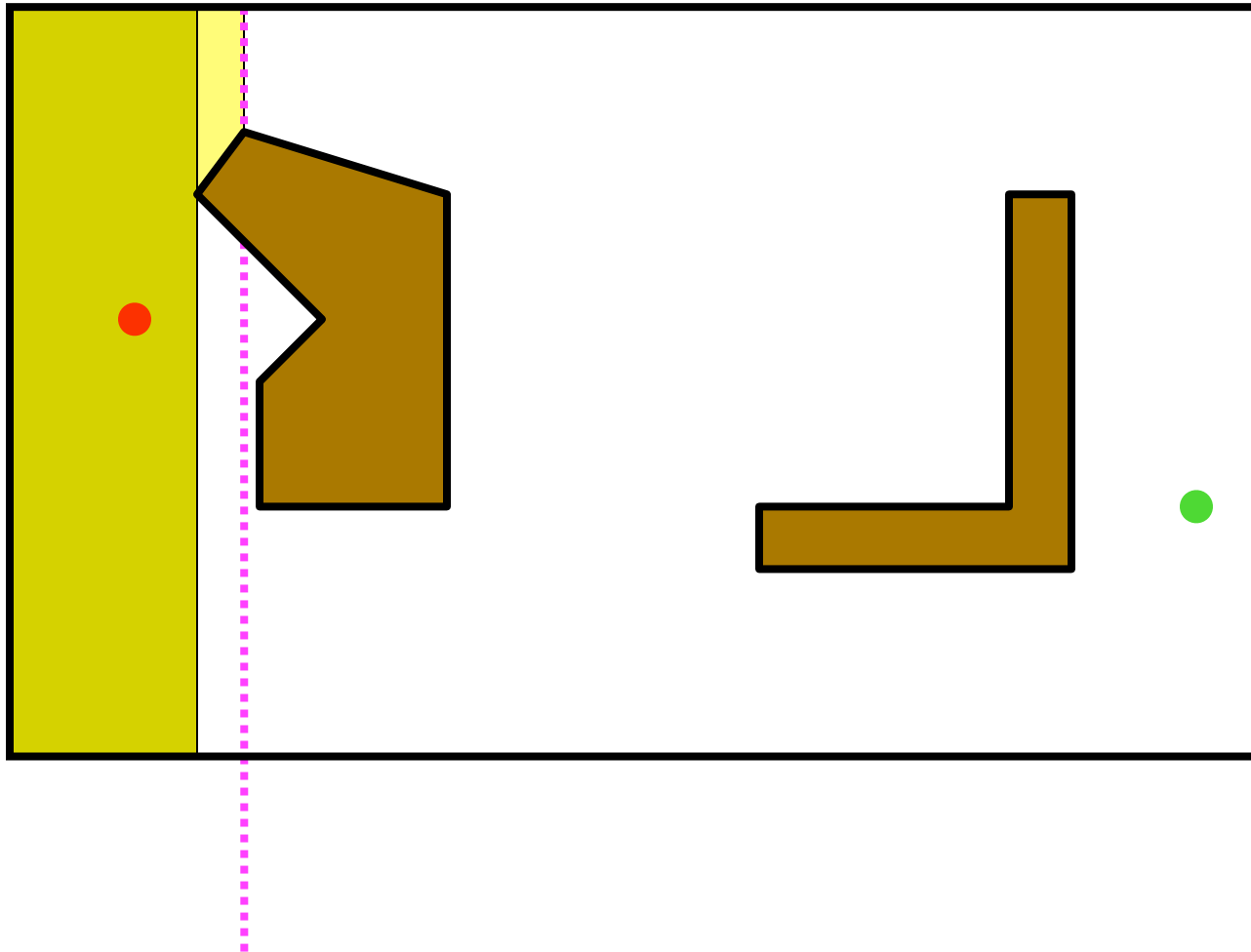
sweep-line



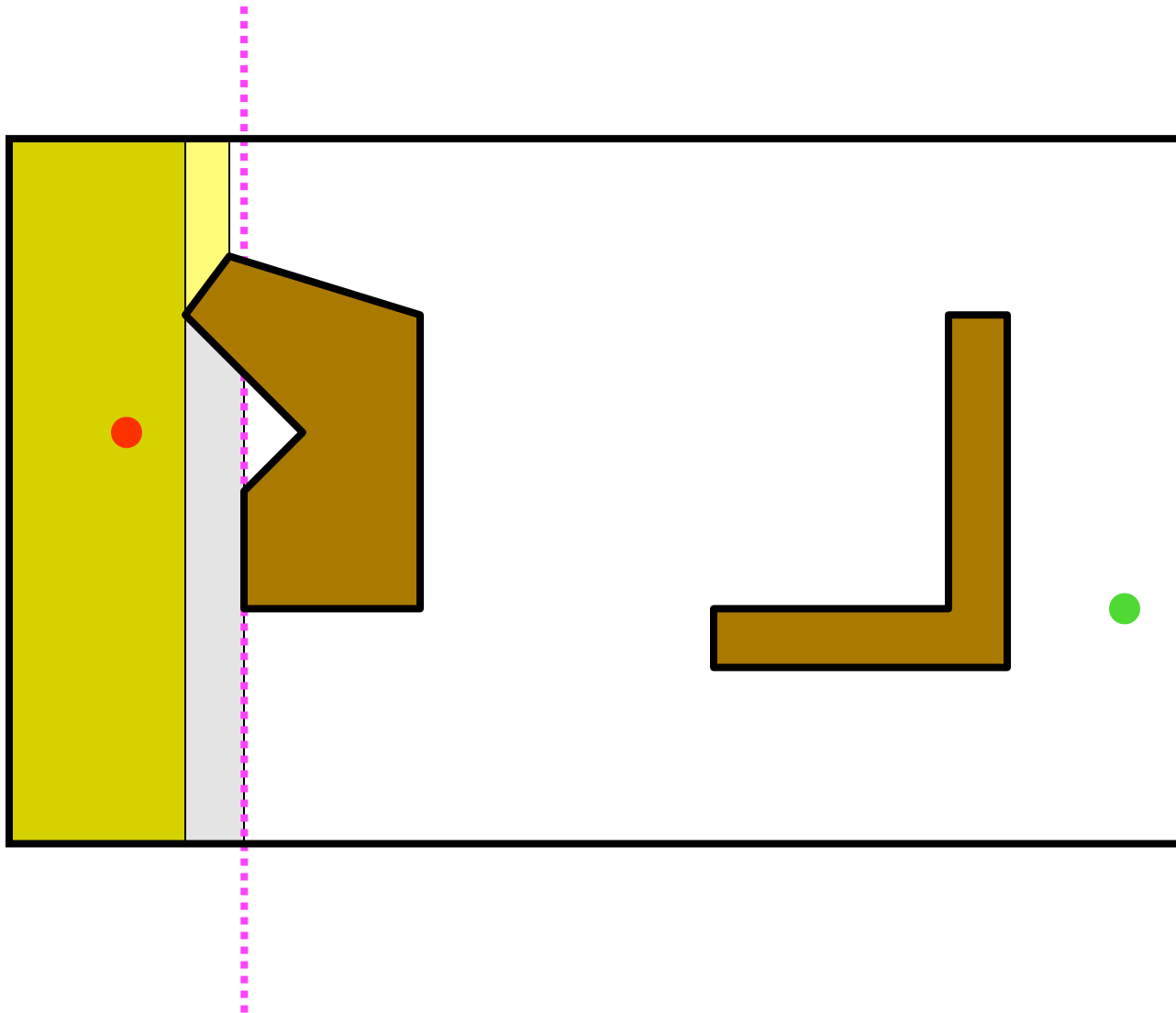
Formulation #2



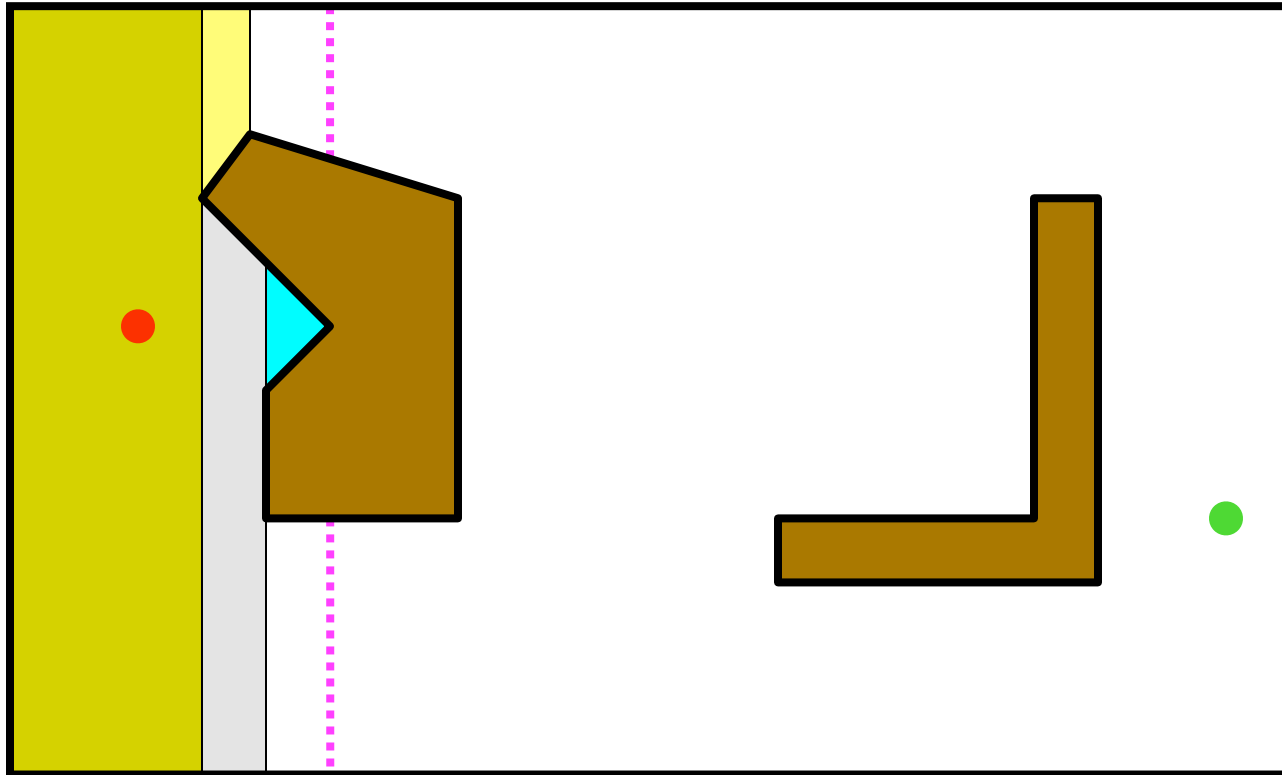
Formulation #2



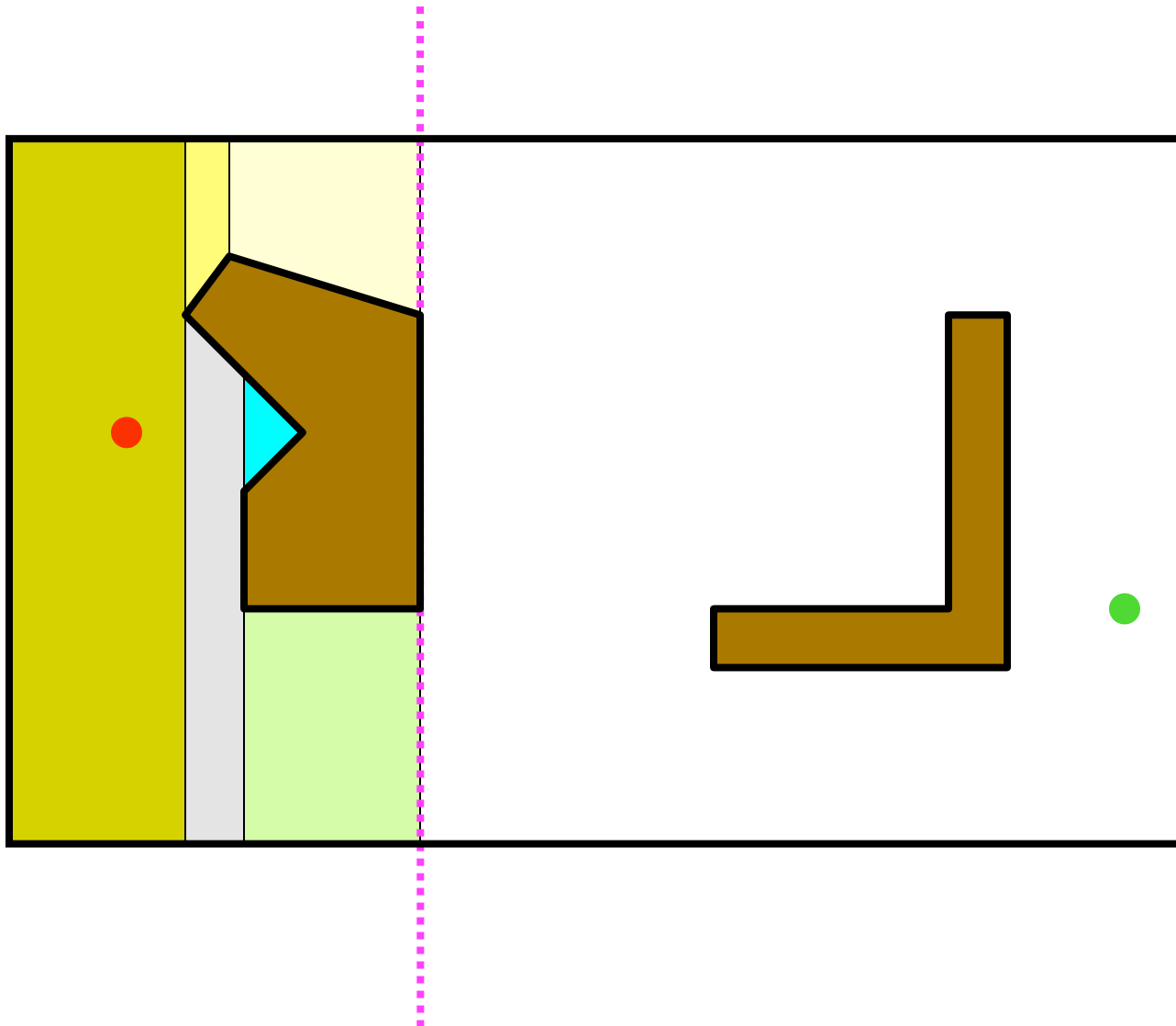
Formulation #2



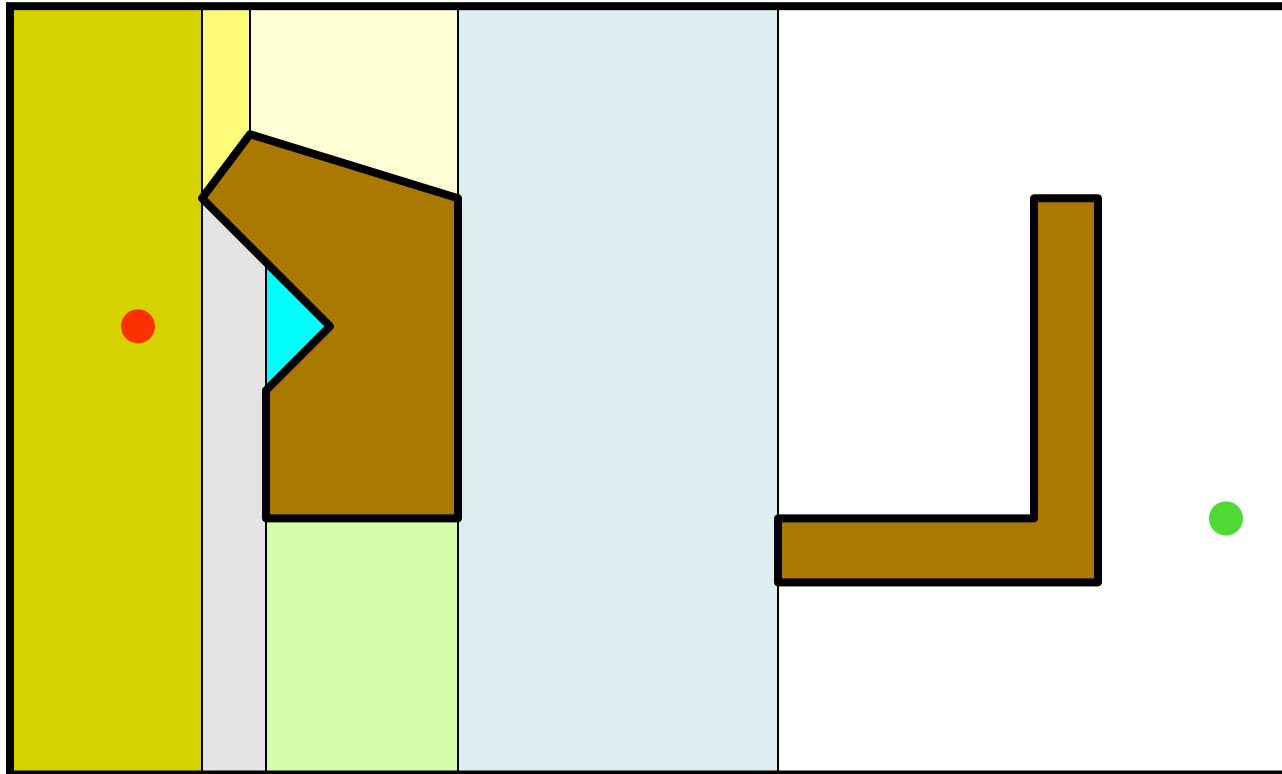
Formulation #2



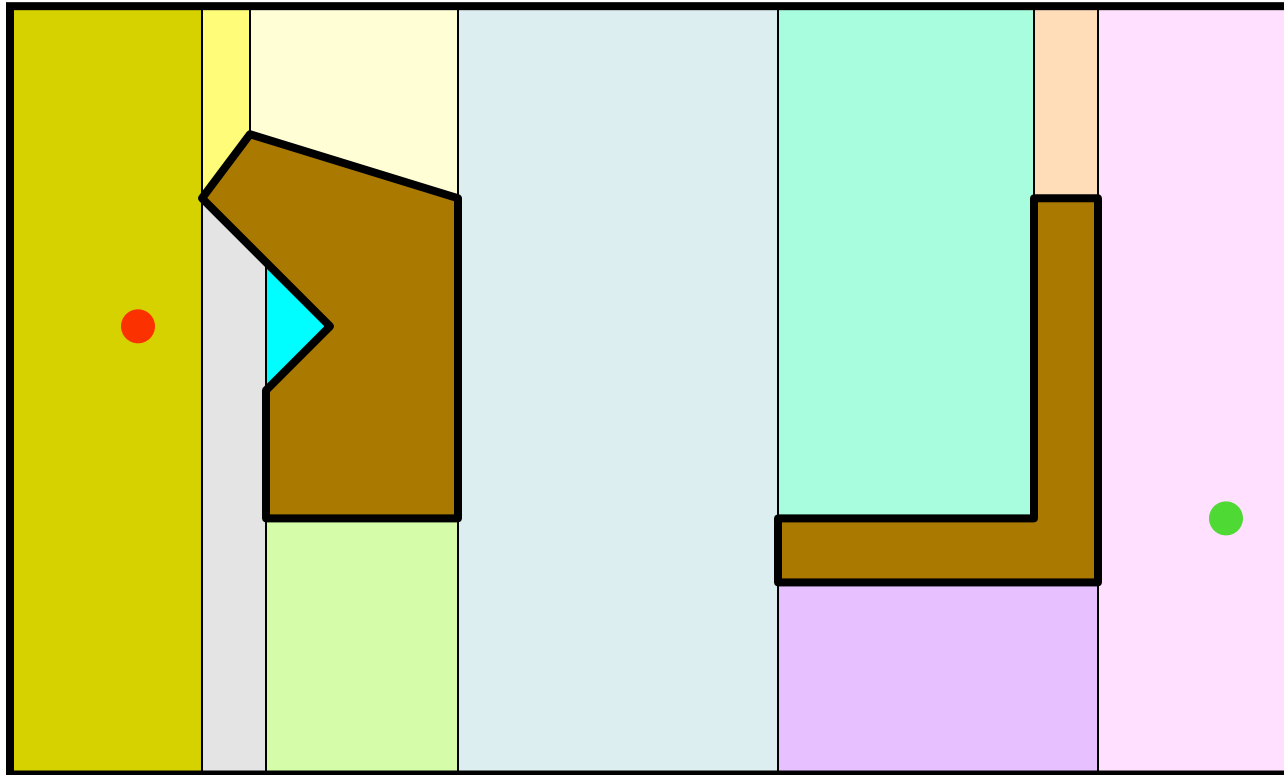
Formulation #2



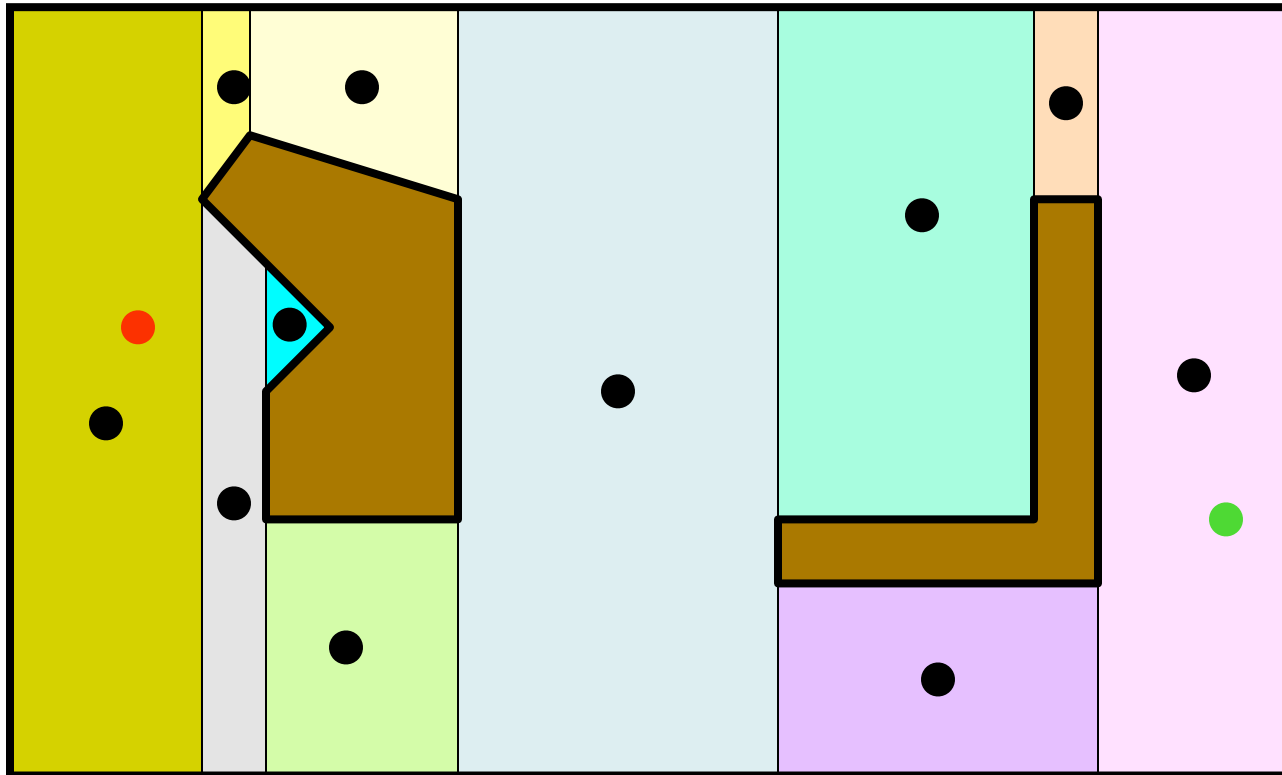
Formulation #2



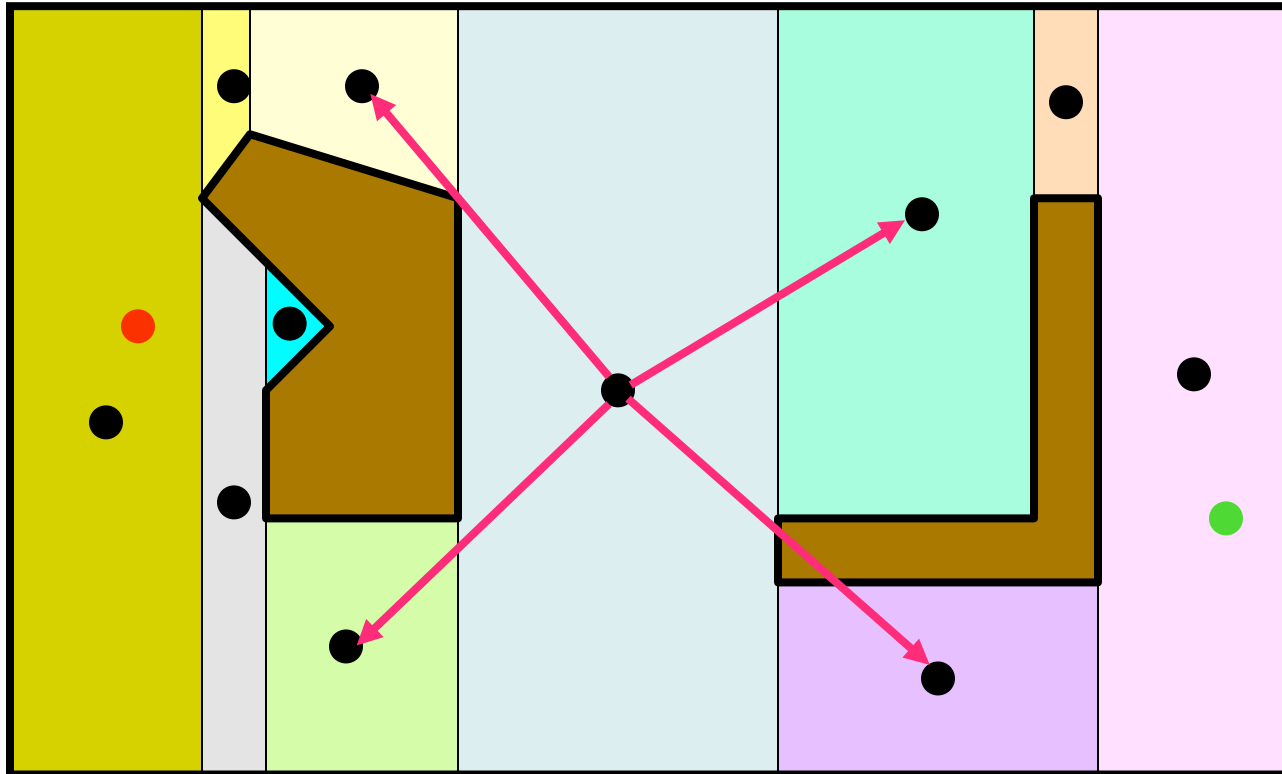
Formulation #2



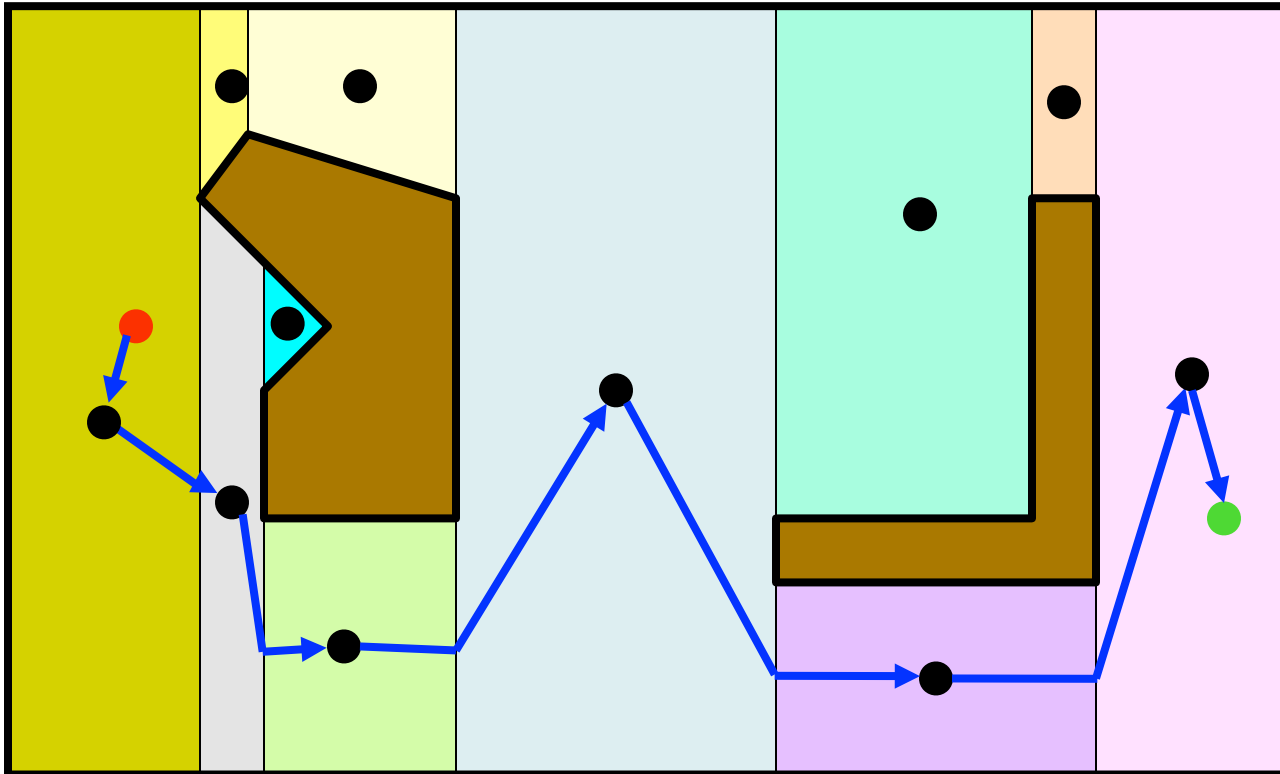
States



Successor Function

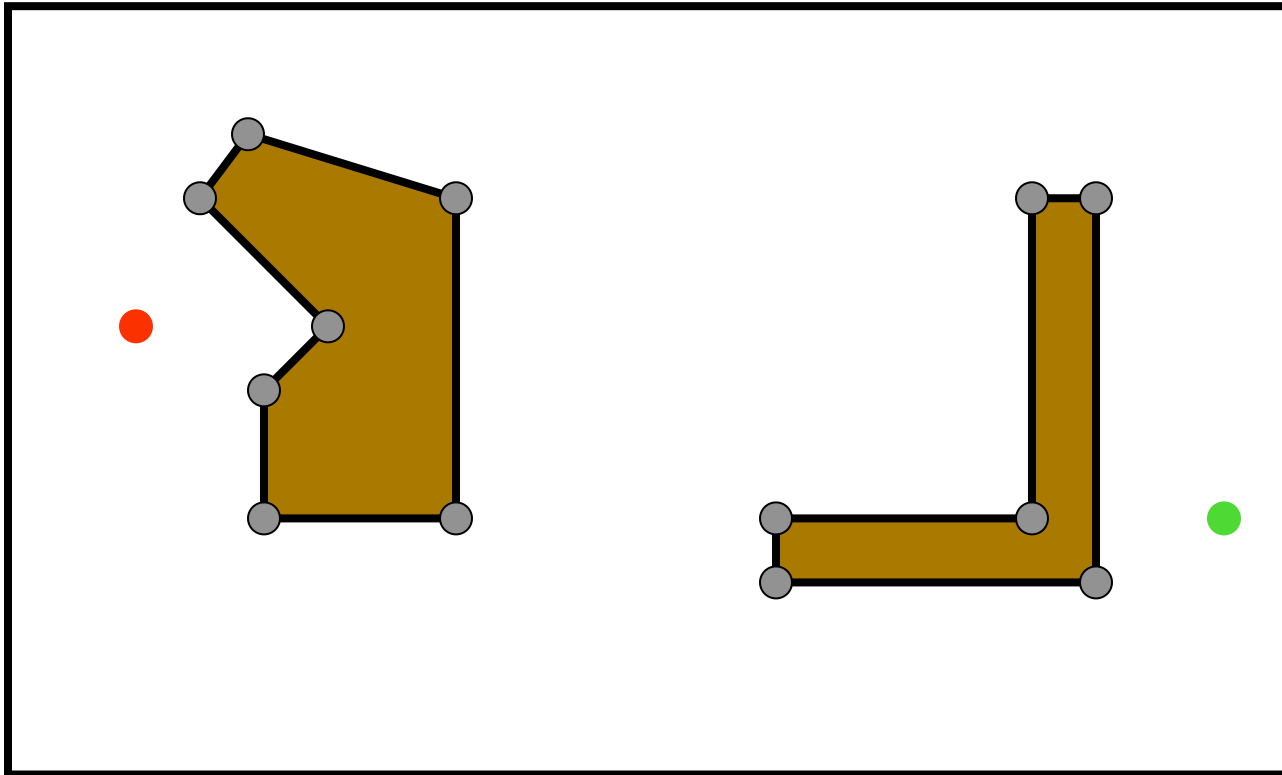


Solution Path

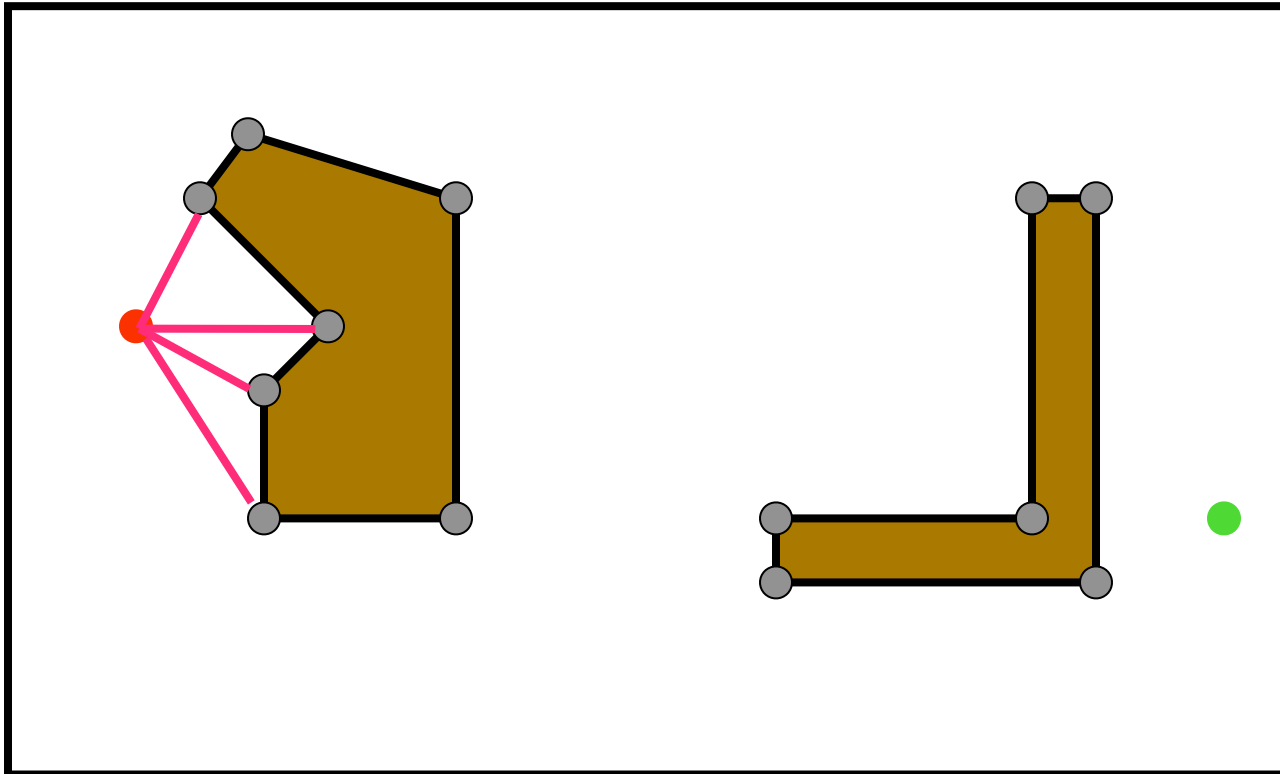


A path-smoothing post-processing step is usually needed to shorten the path further

Formulation #3

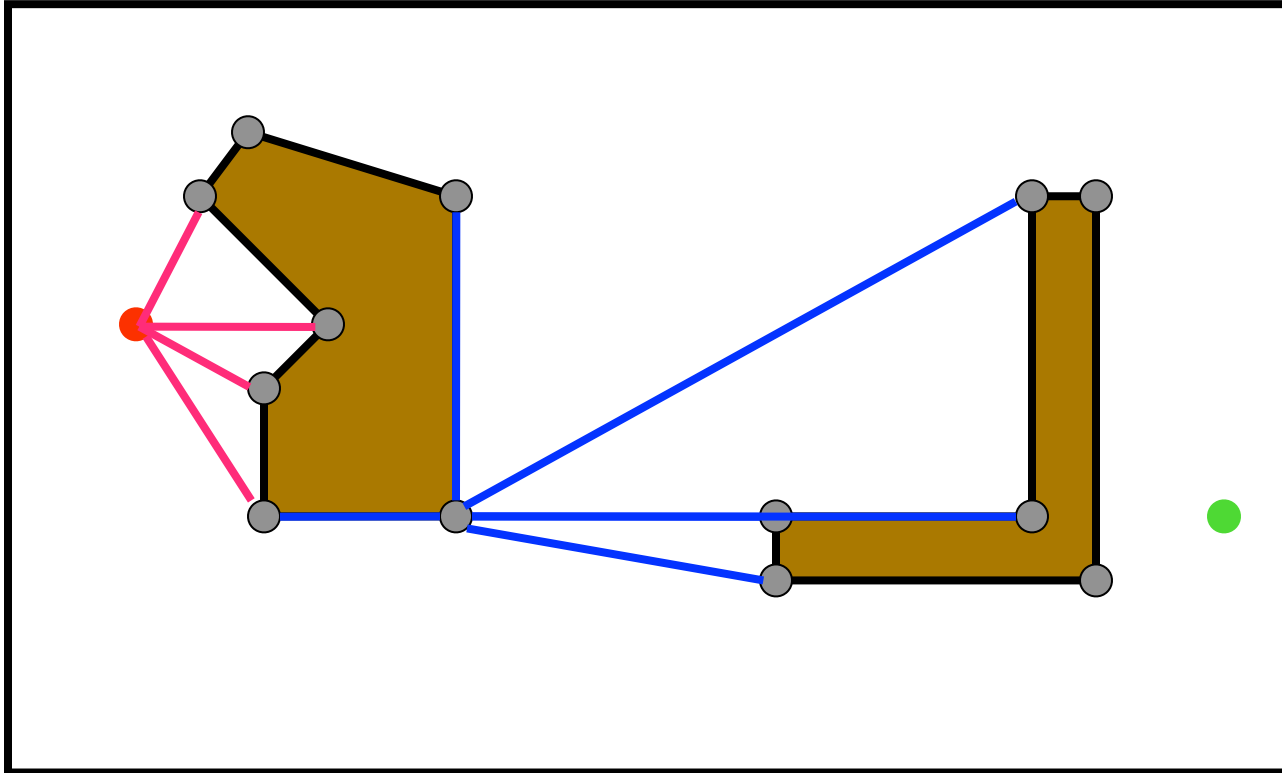


Formulation #3



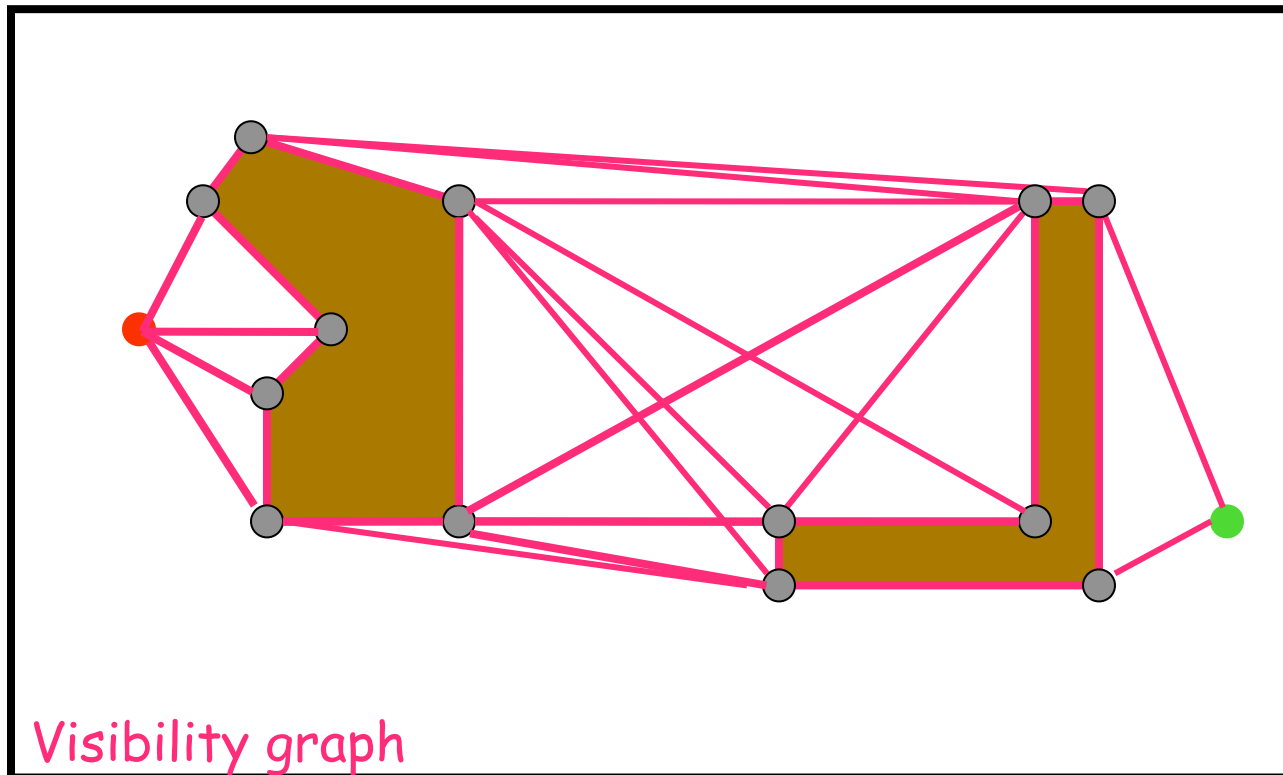
Cost of one step: length of segment

Formulation #3



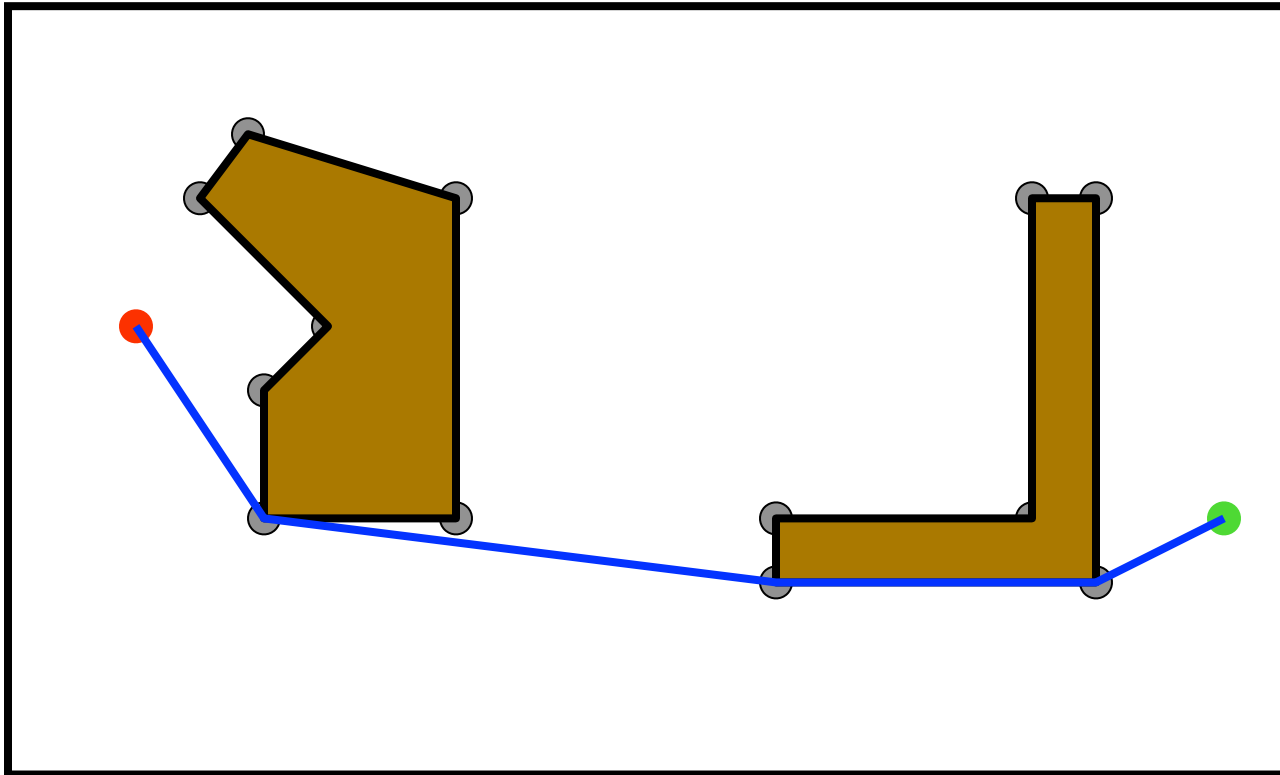
Cost of one step: length of segment

Formulation #3



Cost of one step: length of segment

Solution Path



The shortest path in this state space is also the shortest in the original continuous space

Assumptions in Basic Search

- The world is static
- The world is discretizable
- The world is observable
- The actions are deterministic

But many of these assumptions can be removed, and search still remains an important problem-solving tool

Search and AI

- Search methods are **ubiquitous** in AI systems. They often are the backbones of both core and peripheral modules
- An **autonomous robot** uses search methods:
 - to decide which actions to take and which sensing operations to perform,
 - to quickly anticipate collision,
 - to plan trajectories,
 - to interpret large numerical datasets provided by sensors into compact symbolic representations,
 - to diagnose why something did not happen as expected,
 - etc...
- Many searches may occur concurrently and sequentially

Applications

Search plays a key role in many applications, e.g.:

- Route finding: airline travel, networks
- Package/mail distribution
- Pipe routing, VLSI routing
- Comparison and classification of protein folds
- Pharmaceutical drug design
- Design of protein-like molecules
- Video games