Two-player Games

ZUI 2012/2013

Game-tree Search / Adversarial Search

- until now – only the searching player acts in the environment
- there could be others:
  - Nature – stochastic environment (MDP, POMDP, …)
  - other agents – rational opponents

- Game Theory
  - mathematical framework that describes optimal behavior of rational self-interested agents
  - Mag. OI → A4M36MAS (Multi-agent Systems)

Game-tree Search / Adversarial Search

- What are the basic games categories?
  - perfect / imperfect information
  - deterministic / stochastic
  - zero-sum / general-sum
  - finite / infinite
  - two-player / n-player
  - …

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- What is the goal?

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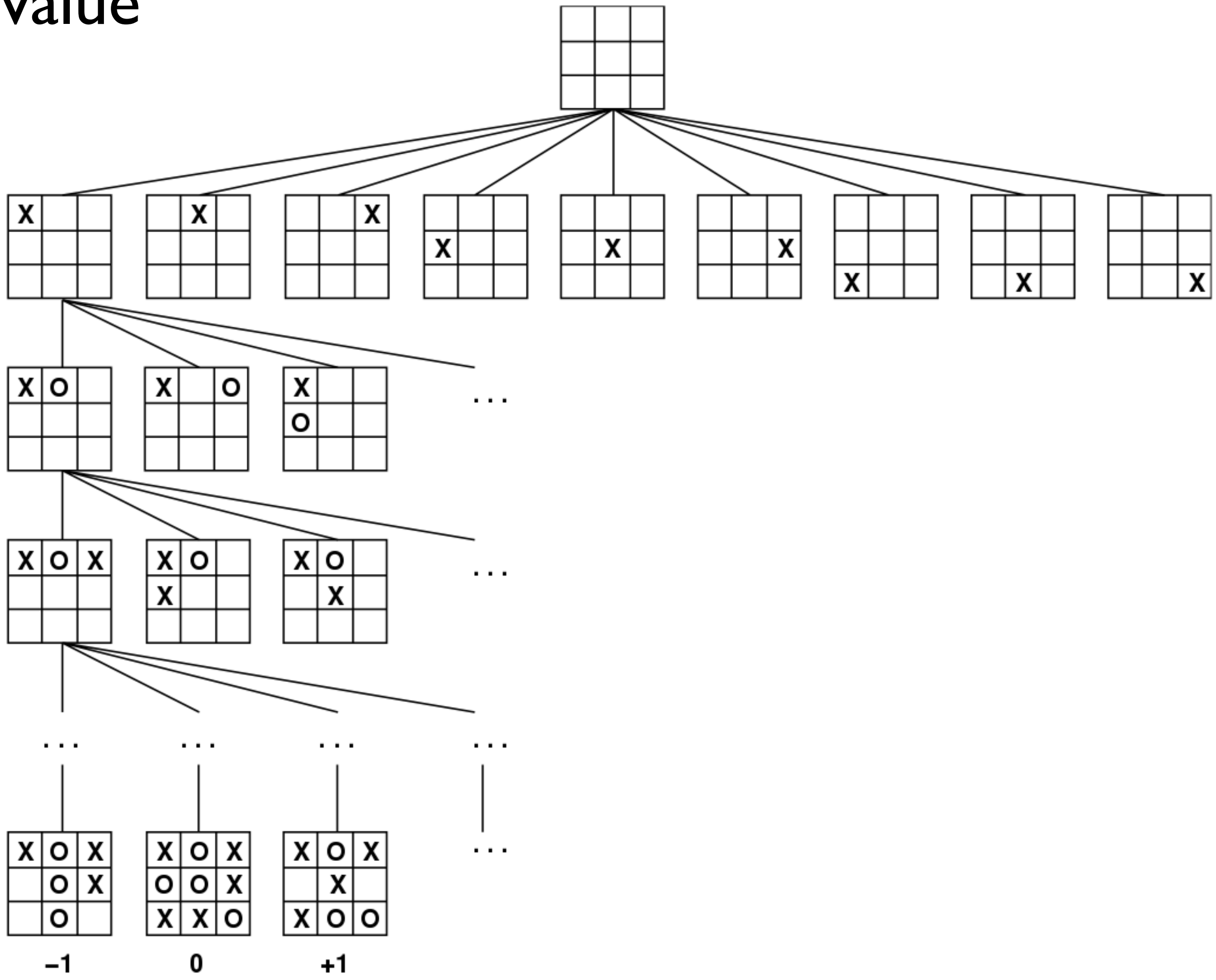
- What is the goal?
  - Finding an optimal **strategy** (i.e., what to play in which situation)

Game-tree Search / Adversarial Search

- Players are rational – each player wants to maximize her/his utility value



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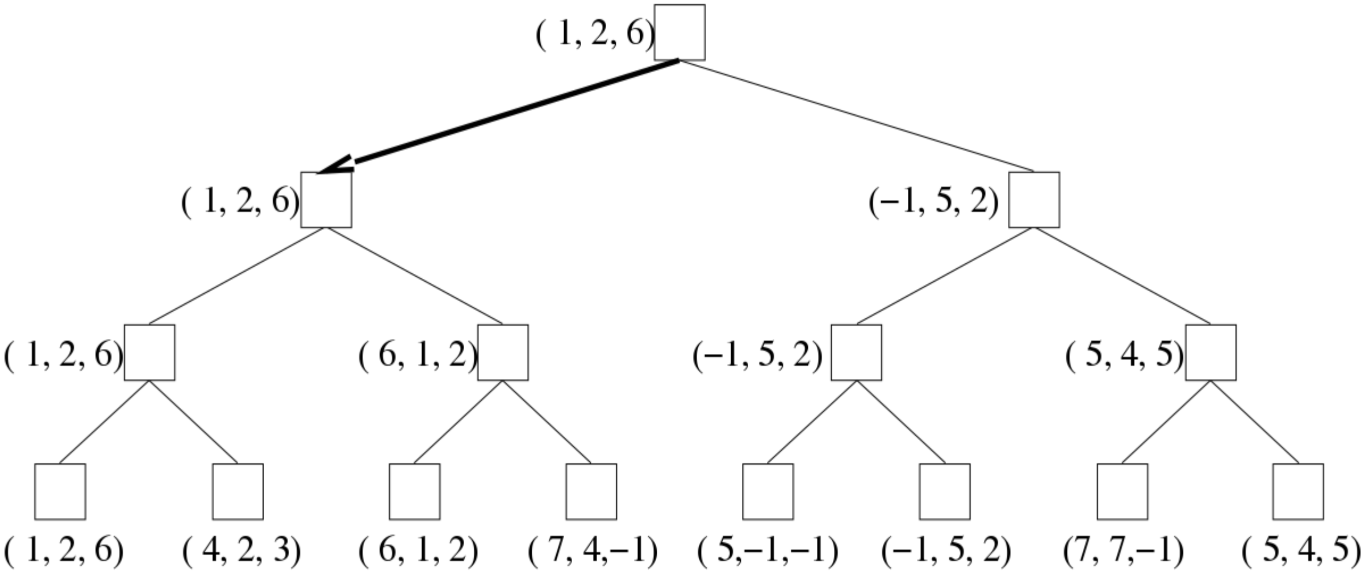

Minimax

```
function minimax(node, Player)
    if (node is a terminal node) return utility value of node
    if (Player = MaxPlayer)
        for each child of node
            $\alpha$ := max($\alpha$, minimax(child, switch(Player) ))

        return $\alpha$
    else
        for each child of node
            $\beta$ := min($\beta$, minimax(child, switch(Player) ))

        return $\beta$
```

Minimax in Real Games

- search space in games is typically **very large**
  - exponential in branching factor b^d
    - e.g., 35 in chess, up to 360 in Go, up to 45000 in Arimaa
- we have to limit the depth of the search
- we need an evaluation function

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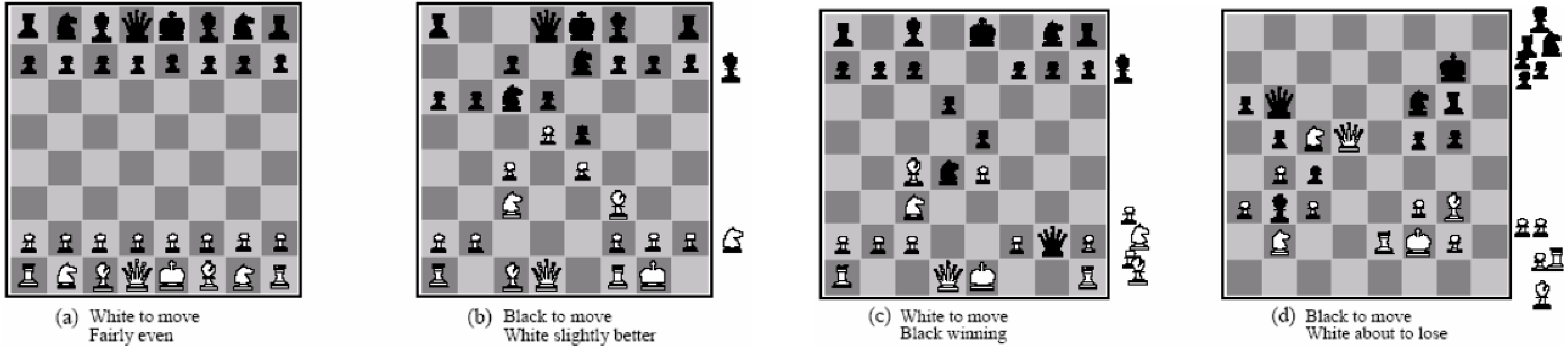


(a) White to move
Fairly even

(b) Black to move
White slightly better

(c) White to move
Black winning

(d) Black to move
White about to lose

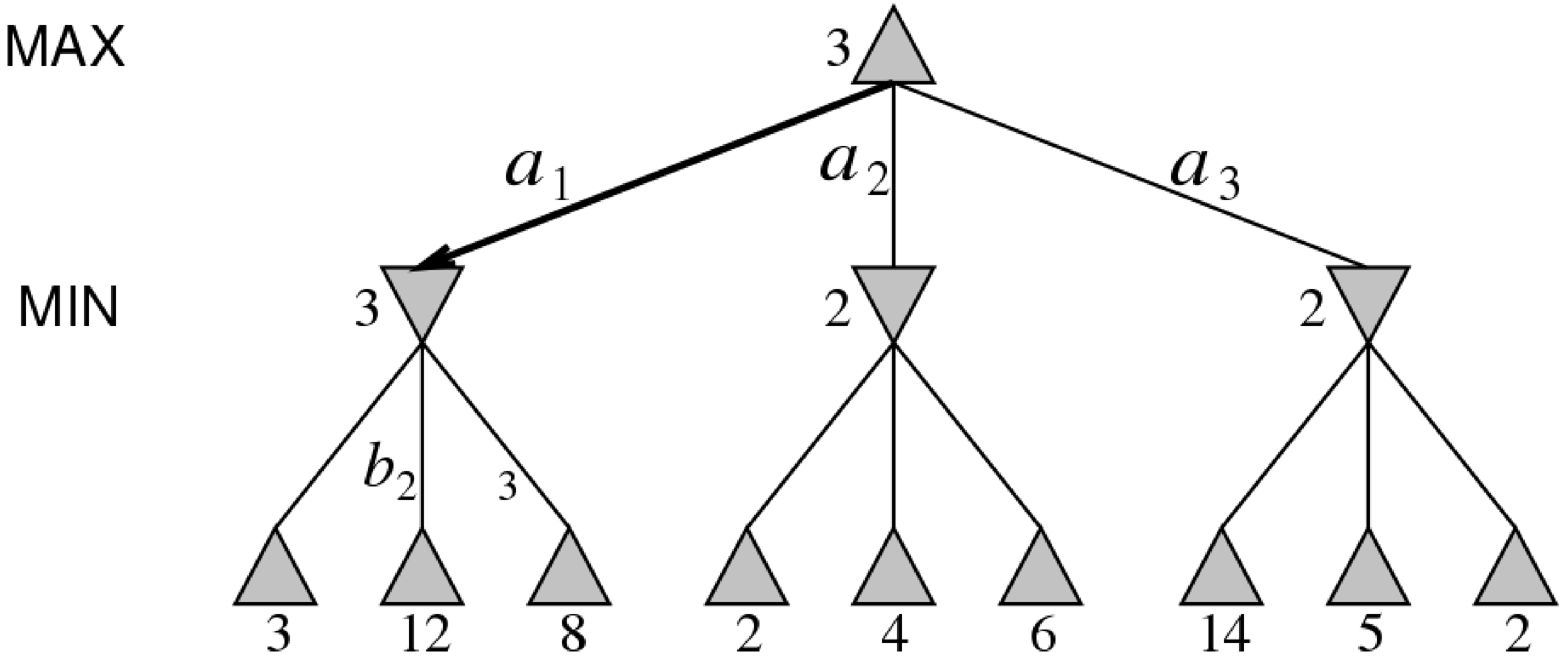
Minimax

```
function minimax(node, depth, Player)
    if (depth = 0 or node is a terminal node) return evaluation value of node
    if (Player = MaxPlayer)
        for each child of node
            α := max(α, minimax(child, depth-1, switch(Player) ))

        return α
    else
        for each child of node
            β := min(β, minimax(child, depth-1, switch(Player) ))

        return β
```

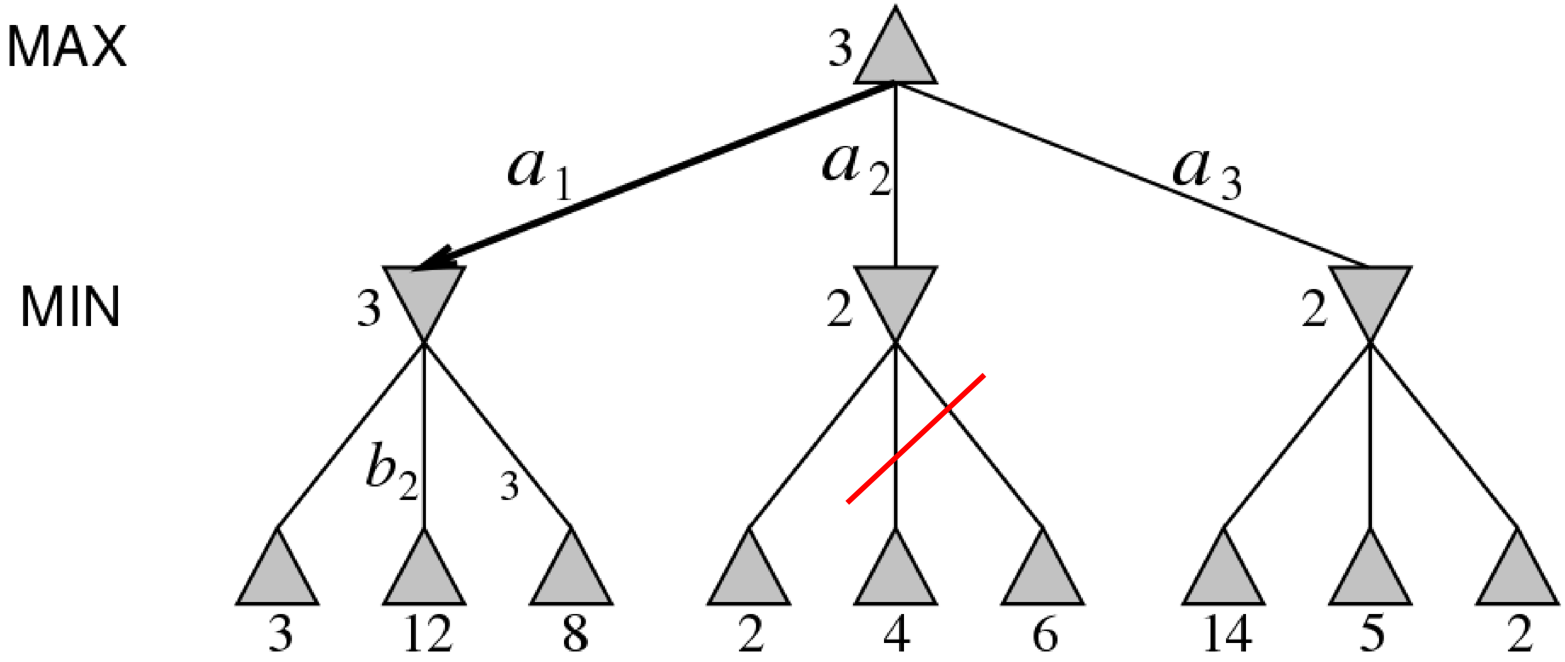
Minimax in Real Games - Problems

- good evaluation function
- depth?
  - horizon problem
  - iterative deepening
  - not always searching deeper improve the results
- caching the results (transposition tables)
- …

Alpha-Beta Pruning



Alpha-Beta Pruning



Alpha-Beta Pruning

```
function alphabeta(node, depth, α, β, Player)
    if (depth = 0 or node is a terminal node) return evaluation value of node
    if (Player = MaxPlayer)
        for each child of node
            α := max(α, alphabeta(child, depth-1, α, β, switch(Player) ))
            if (β≤α)  break
        return α
    else
        for each child of node
            β := min(β, alphabeta(child, depth-1, α, β, switch(Player) ))
            if (β≤α)   break
        return β
```

Negamax

```
function negamax(node, depth, $\alpha$, $\beta$, Player)
    if (depth = 0 or node is a terminal node) return the heuristic value of node
    if (Player = MaxPlayer)
        for each child of node
            $\alpha$ := max($\alpha$, -negamax(child, depth-1, -$\beta$, -$\alpha$, switch(Player) ))
            if ($\beta \leq \alpha$)  break
        return $\alpha$
    else
        for each child of node
            $\beta$ := min($\beta$, alphabeta(child, depth-1, $\alpha$, $\beta$, not(Player) ))
            if ($\beta \leq \alpha$)   break
        return $\beta$
```

Aspiration Search

- $[\alpha, \beta]$ interval – window
- alphabeta initialization $[-\infty, +\infty]$
- what if we use $[\alpha_0, \beta_0]$
  - x = alphabeta(node, depth, α_0, β_0,player)
  - $\alpha_0 \leq x \leq \beta_0$ - we found a solution
  - $x \leq \alpha_0$ - failing low (run again with $[-\infty, x]$)
  - $x \geq \beta_0$ - failing high (run again with $[x, +\infty]$)

Scout – Idea

- assume we are in a MAX node
- we are about to search a child 'c'
- we already have obtained a lower bound 'α'

- Is it worth searching the branch 'c'?

- we need to have some test ...

Scout –Test

- what we really need at that moment is a bound (not the precise value)
- Remember Aspiration Search?
  - $x \leq \alpha_0$ - failing low (we know, that solution is $\leq x$)
  - $x \geq \beta_0$ - failing high (we know, that solution is $\geq x$)
- What if we use a null-window $[\alpha, \alpha+1]$ (or $[\alpha,\alpha]$)?
  - we obtain a bound …

NegaScout

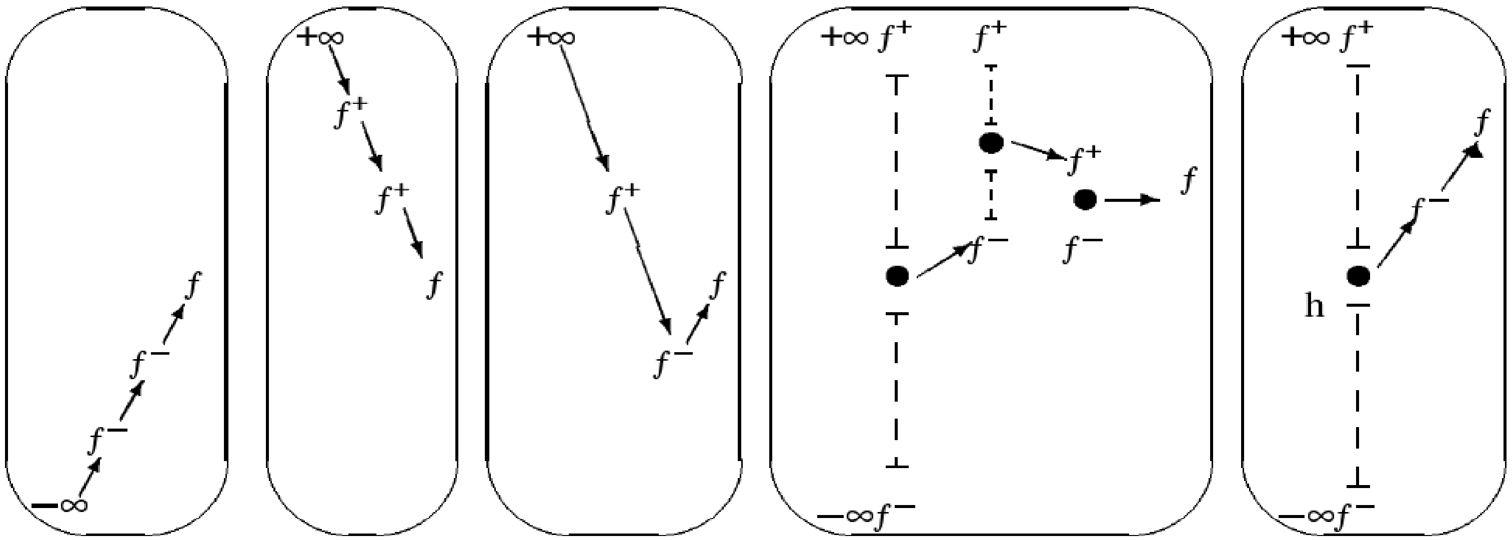
```
function negascout(node, depth, $\alpha$, $\beta$, Player)
    if ((depth = 0) or (node is a terminal node)) return eval(node)
    b := $\beta$
    for each child of node
            v := -negascout(child, depth-1, -b, -$\alpha$, switch(Player)))
        if (( $\alpha$ < v ) and (child is not the first child))
            v := -negascout(child, depth-1, -$\beta$, -$\alpha$, switch(Player)))
        $\alpha$ := max($\alpha$, v)
        if ($\beta \leq \alpha$)  break
        b := $\alpha$ + 1
    return $\alpha$
```

NegaScout

- also termed Principal Variation Search (PVS)
- dominates alpha-beta
  - never evaluates more different nodes than alpha-beta
  - can evaluate some nodes more than once
- depends on the move ordering
- can benefit from transposition tables
- generally 10-20% faster compared to alpha-beta

MTD

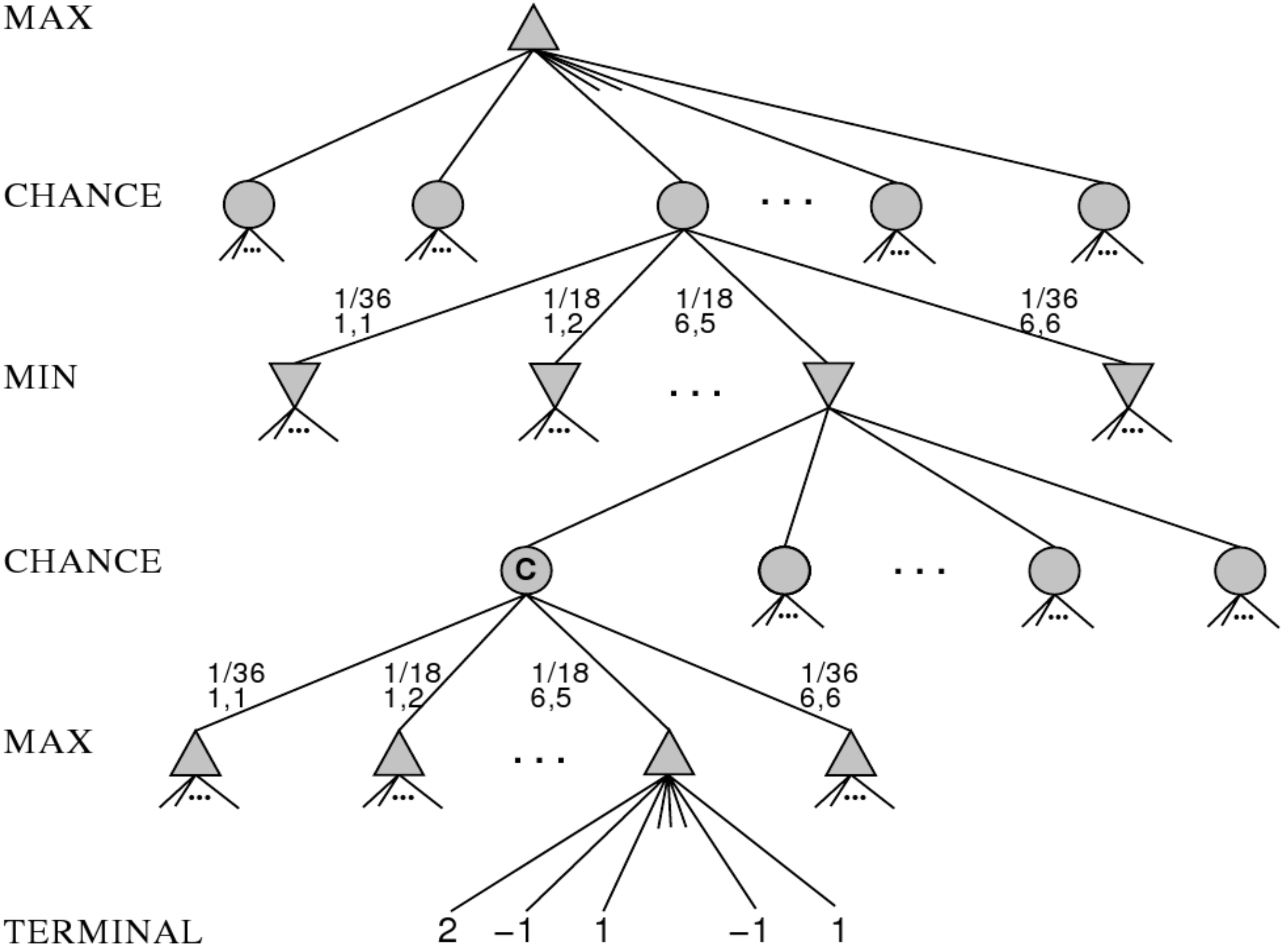
- Memory-enhanced Test Driver



Dual* SSS* MTD(step) MTD(bi) MTD(f)

- Best-first fixed-depth minimax algorithms. Plaat et. al. , In *Artificial Intelligence,* Volume 87, Issues 1-2, November 1996, Pages 255-293

Other Games - Chance nodes



Challenges?

- durative moves (asynchronous chess, Google AI Challenge, ...)
- General Game Playing
  - an algorithm receives rules of the game and has to play
- ARIMAA (created in 2002)
  - BF ≈ 17,000; no opening books; very few patterns
  - easy for people, very difficult for an algorithm
- using a 'real-AI-algorithms' in computer video-games
  - very few examples: F.E.A.R., World In Conflict, ...

Game Theory in ATG

- sequential games
  - with simultaneous moves
  - with imperfect information (Poker, Security Games)

- more general types of 'solutions'