Sequential decisions under uncertainty
Policy iteration

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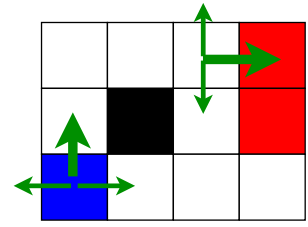
Notes

Unreliable actions in observable grid world

- Walls block movement – agent/robot stays in place.
- Actions do not always go as planned.
- Agent receives rewards each time step:
  - Small "living" reward/penalty.
  - Big rewards/penalties at the end.
- Goal: maximize sum of (discounted) rewards



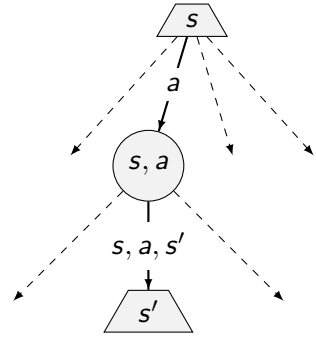
Notes

This is a recap slide, we already know this from the last lecture.

MDPs recap

Markov decision processes (MDPs):

- Set of states $\mathcal{S}$
- Set of actions $\mathcal{A}$
- Transitions $p(s'|s,a)$ or $T(s,a,s')$
- Rewards $r(s,a,s')$; and discount γ



MDP quantities:

- Policy $\pi(s) : \mathcal{S} \to \mathcal{A}$
- Utility – sum of (discounted) rewards.
- Values – expected future utility from a state (max-node), $v(s)$
- Q-Values – expected future utility from a q-state (chance-node), $q(s,a)$

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Q-values – like values but given that I have commited to do action a from state s.

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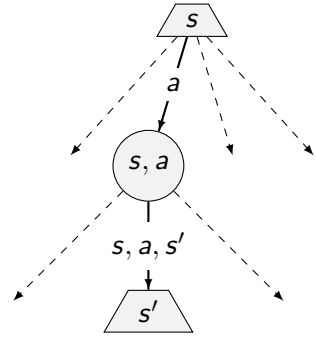

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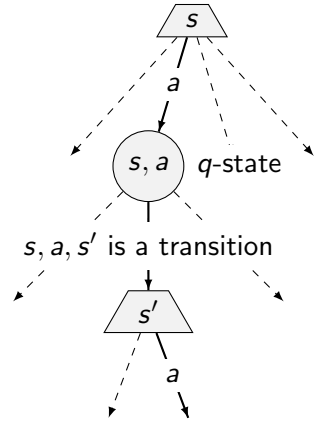
Q-values – like values but given that I have commited to do action a from state s.

Optimal quantities

- The optimal policy: $\pi^*(s)$ – optimal action from state s
- Expected utility/return of a policy.

$$U^{\pi}(S_t) = \mathrm{E}^{\pi}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

Best policy π^* maximizes above.



- The value of a state s: $v^*(s)$ – expected utility starting in s and acting optimally.
- The value of a q-state (s, a): $q^*(s, a)$ - expected utility having taken a from state s and acting optimally thereafter.

Notes

Remember: Discounted return G_t
Returns are successive steps related to each other

$$\begin{aligned} G_t &= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \cdots \\ &= R_{t+1} + \gamma(R_{t+2} + \gamma^1 R_{t+3} + \gamma^2 R_{t+4} + \cdots) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned}$$

$G_t \doteq \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ including the possibility that $T = \infty$ or $\gamma = 1$, but not both.

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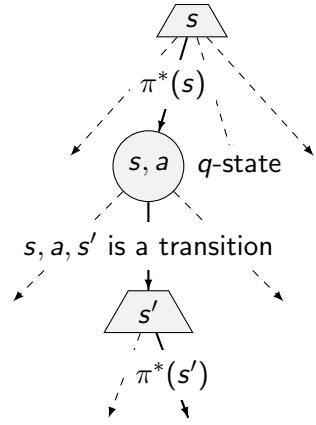

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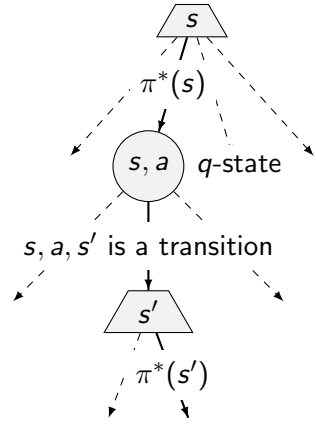

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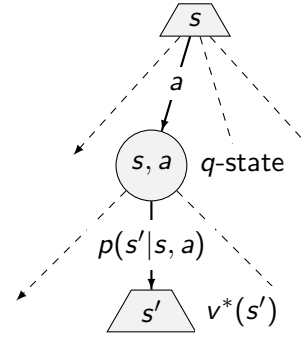
v^* and q^*

The value of a q-state (s, a):

$$q^*(s,a) = \sum_{s'} p(s'|a,s) \left[r(s,a,s') + \gamma \, v^*(s') \right]$$



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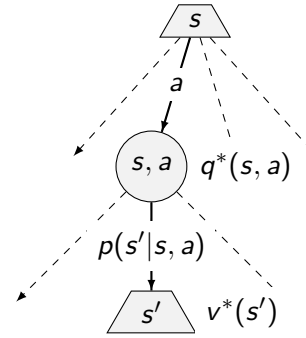

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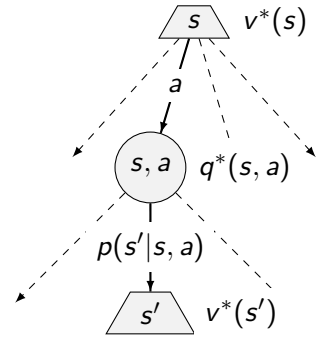
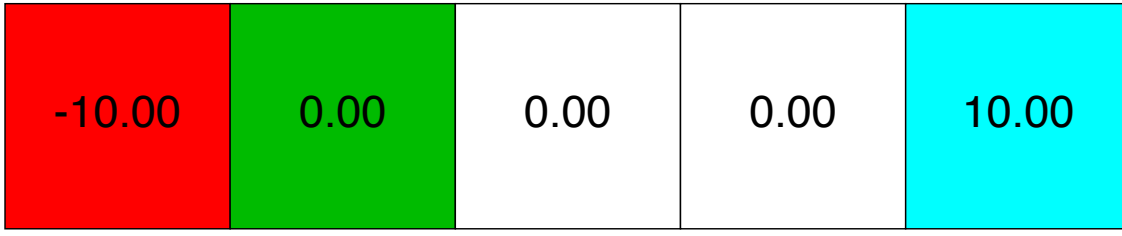


Notes

Maze: $V_0 = [0, 0, 0]^\top$, $r(s) = -1$, deterministic robot, $\mathcal{A} = \{\leftarrow, \uparrow, \downarrow, \rightarrow\}$

0 1 2 3 4



$$
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q^*(s,a) &= \sum_{s'} p(s'|a,s) \left[r(s,a,s') + \gamma \, v^*(s') \right] \\
v^*(s) &= \max_a q^*(s,a)
\end{aligned}
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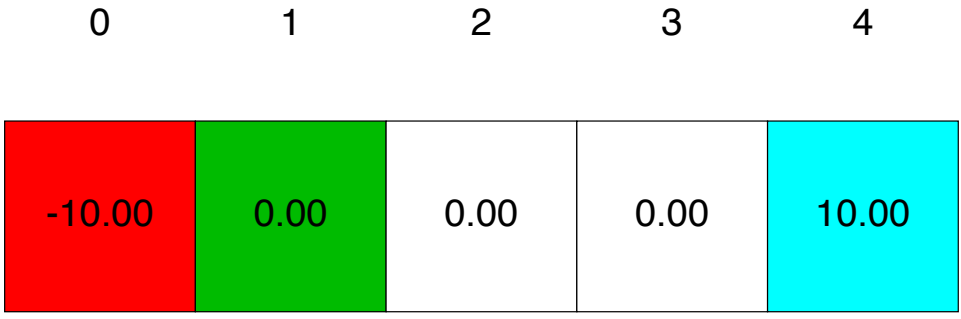
Notes

What will be V^* after first sweep? $V_1^* = [v_1^*(1), v_1^*(2), v_1^*(3)]^\top$?



Sweep is meant as the Bellmann update for all states: $V_1^* = BV_0^*$. $r(s) = -1$. Assume sync version of the algorithm.

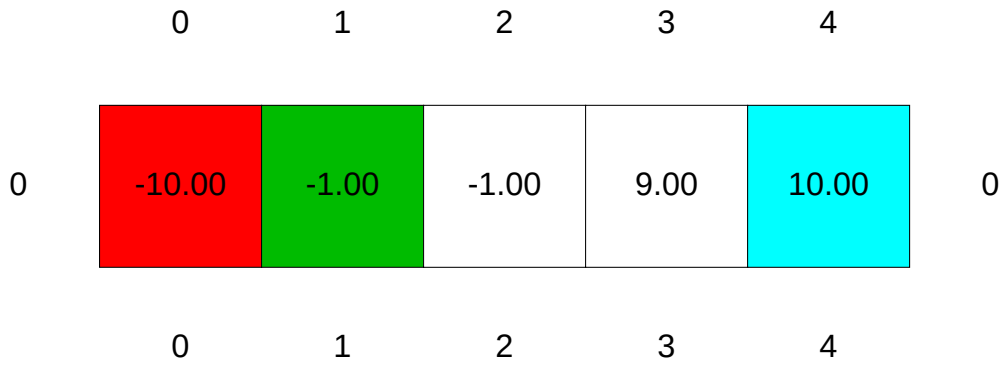
A: $V_1^* = [-1, -1, 9]^\top$

B: $V_1^* = [0, 8, 9]^\top$

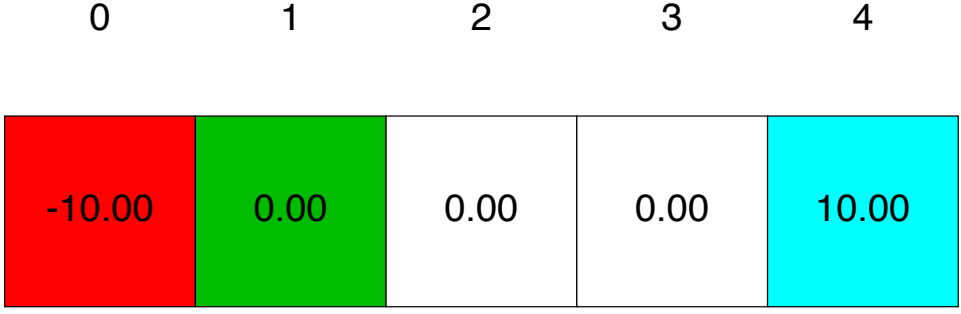
C: $V_1^* = [-1, 0, 0]^\top$

D: $V_1^* = [-11, 8, 9]^\top$

Notes



What will be V^* after second sweep? $V_2^* = [v_2^*(1), v_2^*(2), v_2^*(3)]^\top$?



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A: $V_2^* = [-1, -1, 9]^\top$
B: $V_2^* = [-1, 8, 9]^\top$
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D: $V_2^* = [7, 8, 9]^\top$

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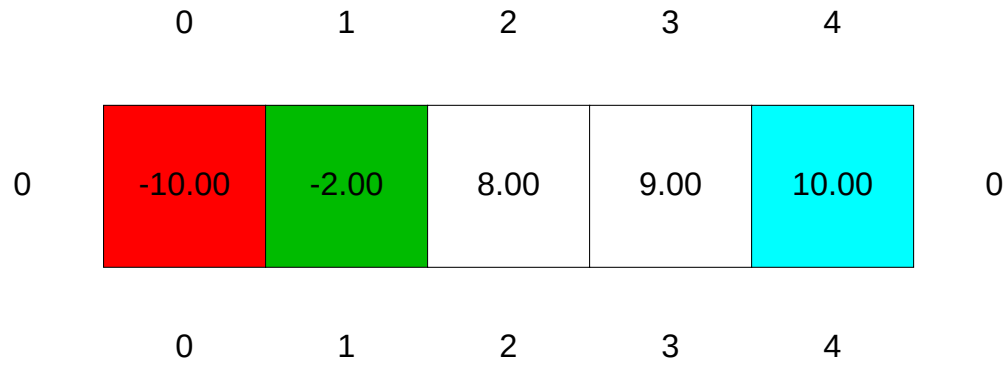
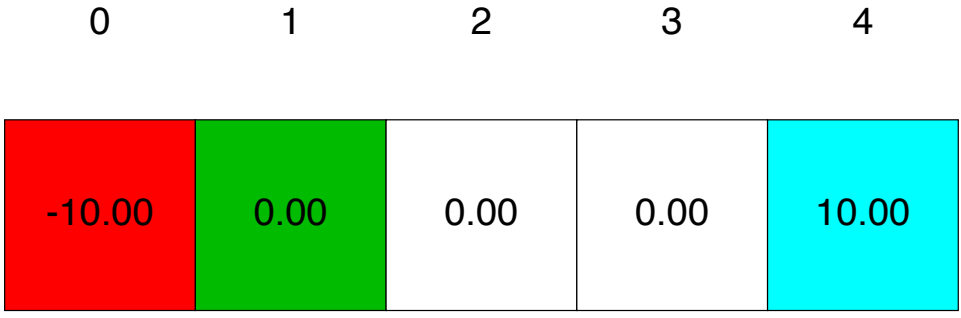


What will be the $q^*(s, a)$ values after first value-iter sweep?



Pick the option which is *wrong*:

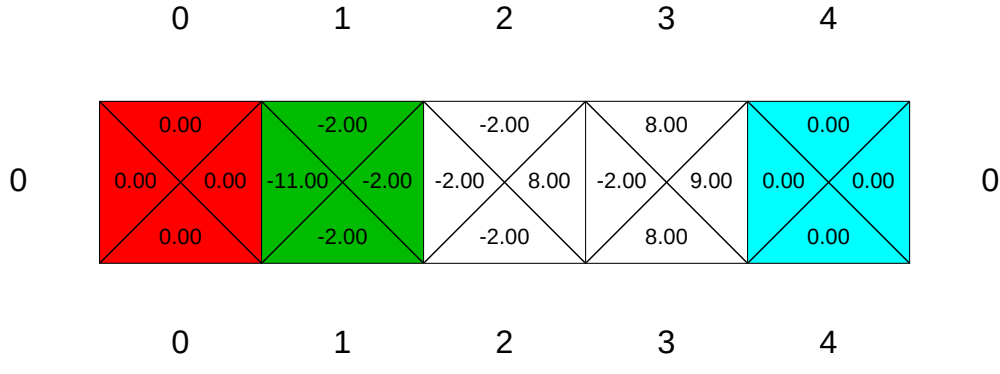
A: $q^*(1, \uparrow) = -2$

B: $q^*(1, \rightarrow) = -2$

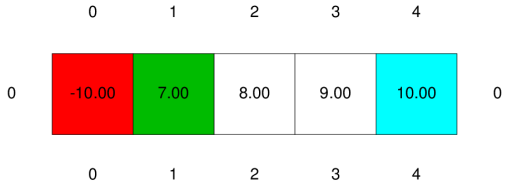
C: $q^*(2, \rightarrow) = -2$

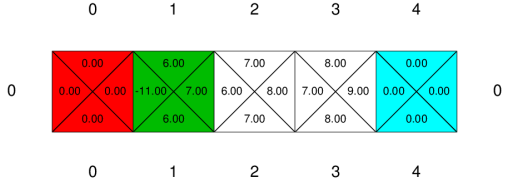
D: $q^*(3, \leftarrow) = -2$

Notes



Maze: v^* vs. q^*, deterministic robot, $\mathcal{A} = \{\leftarrow, \uparrow, \downarrow, \rightarrow\}$





$$q^*(s,a) = \sum_{s'} p(s'|a,s)\left[r(s,a,s') + \gamma\, v^*(s')\right]$$

$$v^*(s) = \max_a q^*(s,a)$$

Notes

Deterministic robot/agent with four actions possible

Maze: v^* vs. q^*, $\gamma = 1$, $T = [0.8, 0.1, 0.1, 0]$

0.81	0.87	0.92	1.00
0.76		0.66	-1.00
0.71	0.66	0.61	0.39

0.78 / 0.77 / 0.81 / 0.74	0.83 / 0.78 / 0.87 / 0.83	0.88 / 0.81 / 0.92 / 0.68	0.00 / 0.00 / 0.00 / 0.00
0.76 / 0.72 / 0.72 / 0.68		0.66 / 0.64 / -0.69 / 0.42	0.00 / 0.00 / 0.00 / 0.00
0.71 / 0.67 / 0.63 / 0.66	0.62 / 0.66 / 0.58 / 0.62	0.59 / 0.61 / 0.40 / 0.55	-0.74 / 0.39 / 0.21 / 0.37

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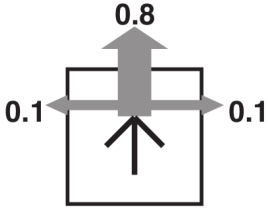
This is the $R = -0.04$ for nonterminal states maze (AIMA Fig. 17.3).



, $\gamma = 1$

Note that the Value of a state takes into account a number of things:

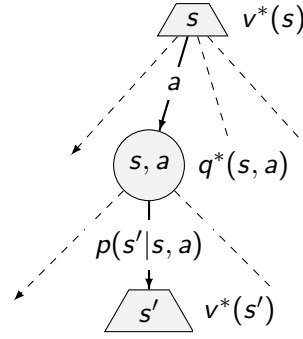
- the policy – which action will chosen in s
- the fact that the goal may be far away and
  - there will be a number of living penalties incured before reaching it
  - the final reward will be discounted
- the transition probabilities

Q-values - useful for choosing the best action – getting the policy.

Value iteration

- Bellman equations **characterize** the optimal values

$$v^*(s) = \max_{a \in A(s)} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma v^*(s') \right]$$



- Value iteration **computes** them:

$$V_{k+1}(s) \leftarrow \max_{a \in A(s)} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_k(s') \right]$$

Value iteration is a fixed point solution method.

Notes

Bellman equations:

1. Take correct first action (1 ply of Expectimax)
2. Keep being optimal (recursion $v^*(s')$)

Recall that we may simplify equations by marginalizing rewards if all $r(s,a,s')$ are the same.

$$r(s) = \sum_{s'} p(s'|a,s) r(s,a,s')$$

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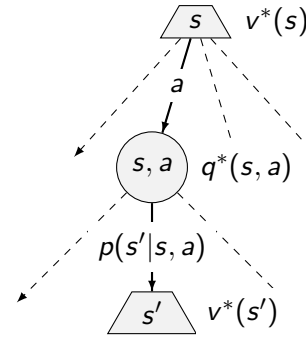

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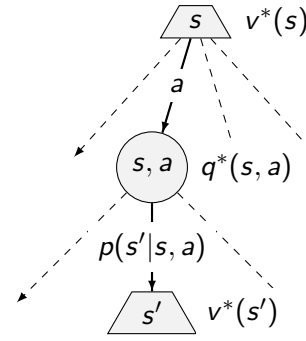

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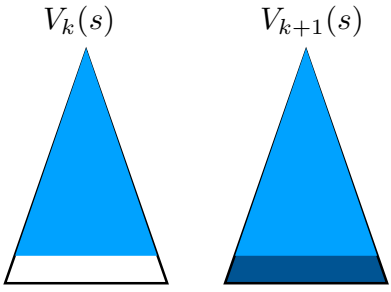
Convergence

$$V_{k+1}(s) \leftarrow \max_{a \in A(s)} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_k(s') \right]$$

- Thinking about special cases: deterministic world, $\gamma = 0$, $\gamma = 1$.
- For all s, $V_k(s)$ and $V_{k+1}(s)$ can be seen as expectimax search trees of depth k and $k+1$



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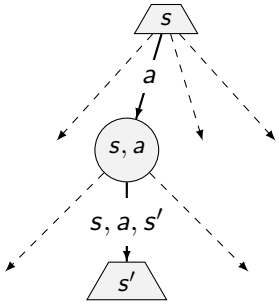
- Bottom (last) layer, zeros for $V_k(s)$, true rewards for $V_{k+1}(s)$
- Last layer $\langle R_{min}, R_{max} \rangle$
- But the last layer is γ^k discounted ...
- hence, V_k and V_{k+1} are no more than $\gamma^k \max |R|$ apart.
- The k increases, the values converge.

From Values to Policy

Notes

Policy extraction - computing actions from Values



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- What is the optimal action?
- We need a one-step expectimax:

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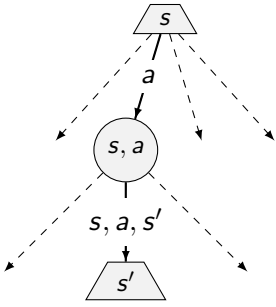

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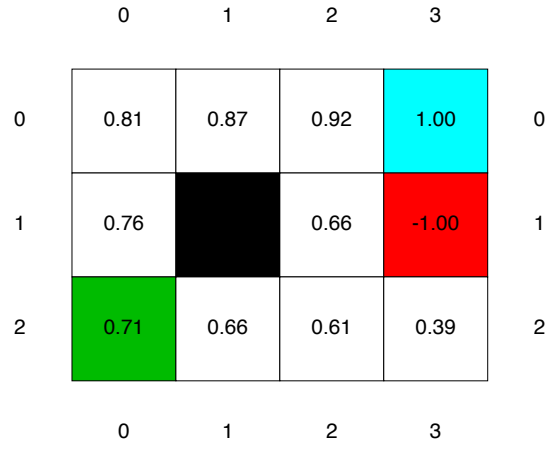

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Policy extraction - computing actions from q-Values

- Assume we have $q^*(s, a)$
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- Just take the (arg) max:

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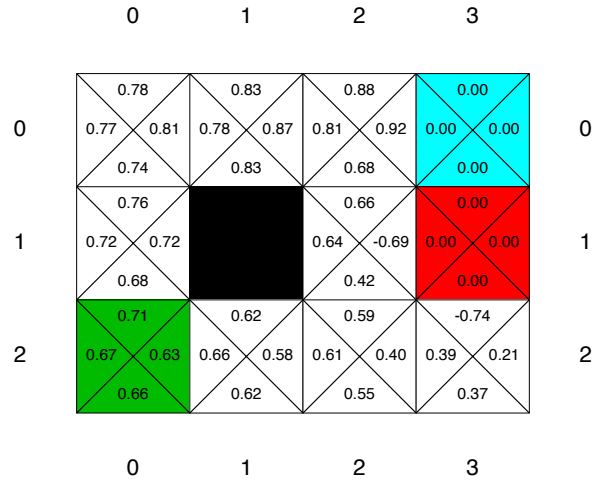
Actions are easier to extract from q-values.



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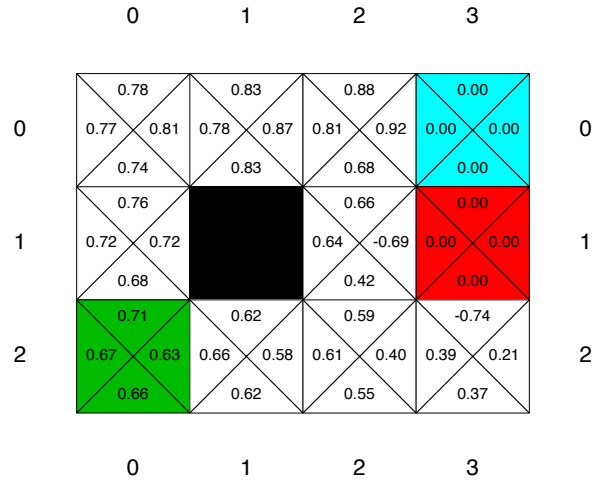

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What is wrong with the Value iteration?

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- What is complexity of one iteration - over all S states?
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- Complexity: $O(AS^2)$ per iteration For every state (LHS), there can be up to $\sharp S$ also on RHS – if every other state was reachable from the current state. In addition, all actions from every state need to be considered.
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Run "AIMA Fig. 17.2 / 17.3 demo" with $R = -0.04$
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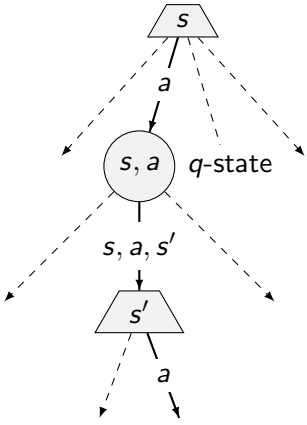
Policy evaluation

- Assume $\pi(s)$ given.
- How to evaluate (compare)?

Notes

Remember last week's quizz?

Fixed policy, do what π says



- Expectimax trees "max" over all actions ...
- Fixed π for each state $\rightarrow$ no "max" operator!

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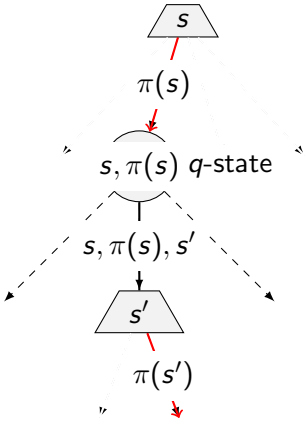

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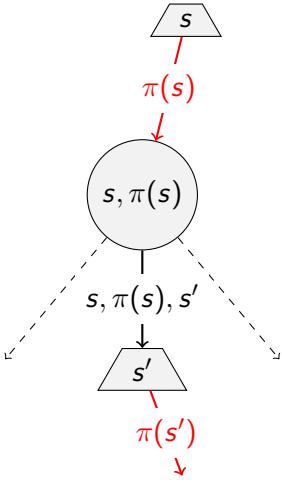
State values under a fixed policy



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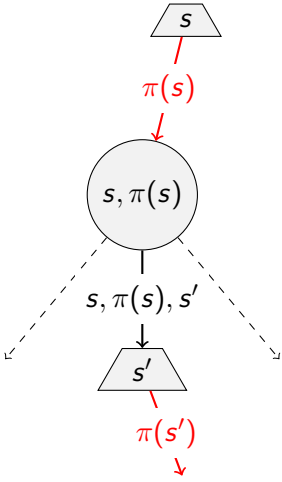
$v^{\pi}(s) = \sum_{s'} p(s' \mid s, \pi(s)) \left[r(s, \pi(s), s') + \gamma v^{\pi}(s') \right]$

Notes

Recall that $v^{\pi}(s)$ quantity contains all the future – expected discounted sum of rewards – executing policy from the state s onwards.

State values under a fixed policy



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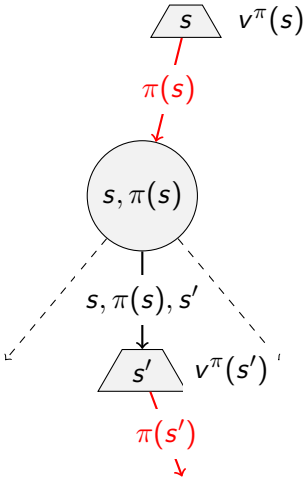

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How to compute $v^{\pi}(s)$?

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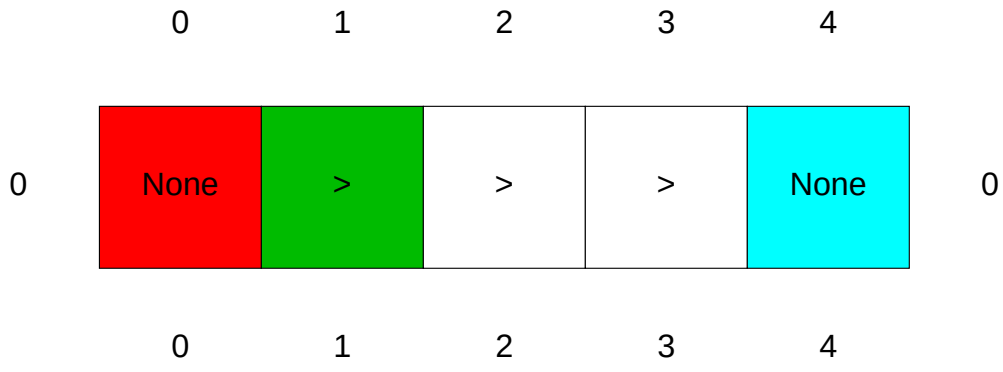

Notes

- by iteration
- solving set of equations

Policy iteration

- Start with a random policy.
- Step 1: Evaluate it.
- Step 2: Improve it.
- Repeat steps until policy converges.

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Policy iteration

- Policy π evaluation. Solve equations or iterate until convergence.

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} p(s' \mid s, \pi(s)) \left[r(s, \pi(s), s') + \gamma V_k^{\pi_i}(s') \right]$$

- Policy improvement. Look-ahead and keep optimality. Policy extraction from fixed values.

$$\pi_{i+1}(s) = \arg\max_{a \in \mathcal{A}(s)} \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V_k^{\pi_i}(s') \right]$$

Notes

A few demo runs of `mdp_agents.py`.

Note that the value is taken from "old policy" on RHS.

Policy iteration algorithm

function POLICY-ITERATION(env) **returns:** policy π
  input: env - MDP problem
  $\pi(s) \leftarrow$ random $a \in A(s)$ in all states
  $V(s) \leftarrow 0$ in all states
  repeat ▷ iterate values until no change in policy
    $V \leftarrow$ POLICY-EVALUATION(π, V, env)
    unchanged $\leftarrow$ True
    for each state s **in** S **do**
      if $\max_{a \in A(s)} \sum_{s'} P(s'|a,s)V(s') > \sum_{s'} P(s'|s,\pi(s))V(s')$ **then**
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Policy vs. Value iteration

- Value iteration.
  - Iteration updates values and policy. Although policy implicitly – extracted from values
  - No track of policy.
- Policy iteration.
  - Update utilities is fast – only one action per state.
  - New policy from values (slower)
  - New policy is better or done.
- Both methods belong to Dynamic programming realm.

Notes

Complexity (of one iteration step):
Value iteration: $O(S^2 * A)$
For every state (LHS), there can be up to $\sharp S$ also on RHS – if every other state was reachable from the current state.
In addition, all actions from every state need to be considered.
$Max(A)$ **does not** change often.
Policy often converges long before the values.
Policy evaluation: $O(S^3)$ (after AIMA, pg. 657)
The Bellman equations are *linear* because the max operator is gone.
For $\sharp S$ states, we have $\sharp S$ equations, which can be solved exactly in time $O(S^3)$ using standard linear algebra methods.
For small state spaces - ok.
For large state spaces - may be prohibitive $\rightarrow$ *modified policy iteration* with only a certain number of simplified Bellman update.

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References

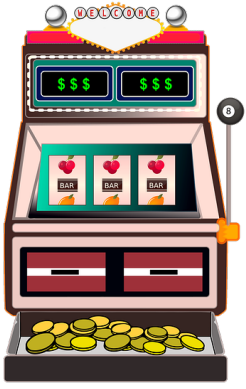
Further reading: Chapter 17 of [1] however, policy iteration is quite compact there. More detailed discussion can be found in chapter Dynamic programming in [2] with slightly different notation, though. This lecture has been also greatly inspired by the 9th lecture of CS 188 at http://ai.berkeley.edu as it convincingly motivates policy search and offers an alternative convergence proof of the value iteration method.

[1] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

[2] Richard S. Sutton and Andrew G. Barto.
Reinforcement Learning; an Introduction.
MIT Press, 2nd edition, 2018.
http://www.incompleteideas.net/book/the-book-2nd.html.

Notes

Bandits



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