Advanced Robotics

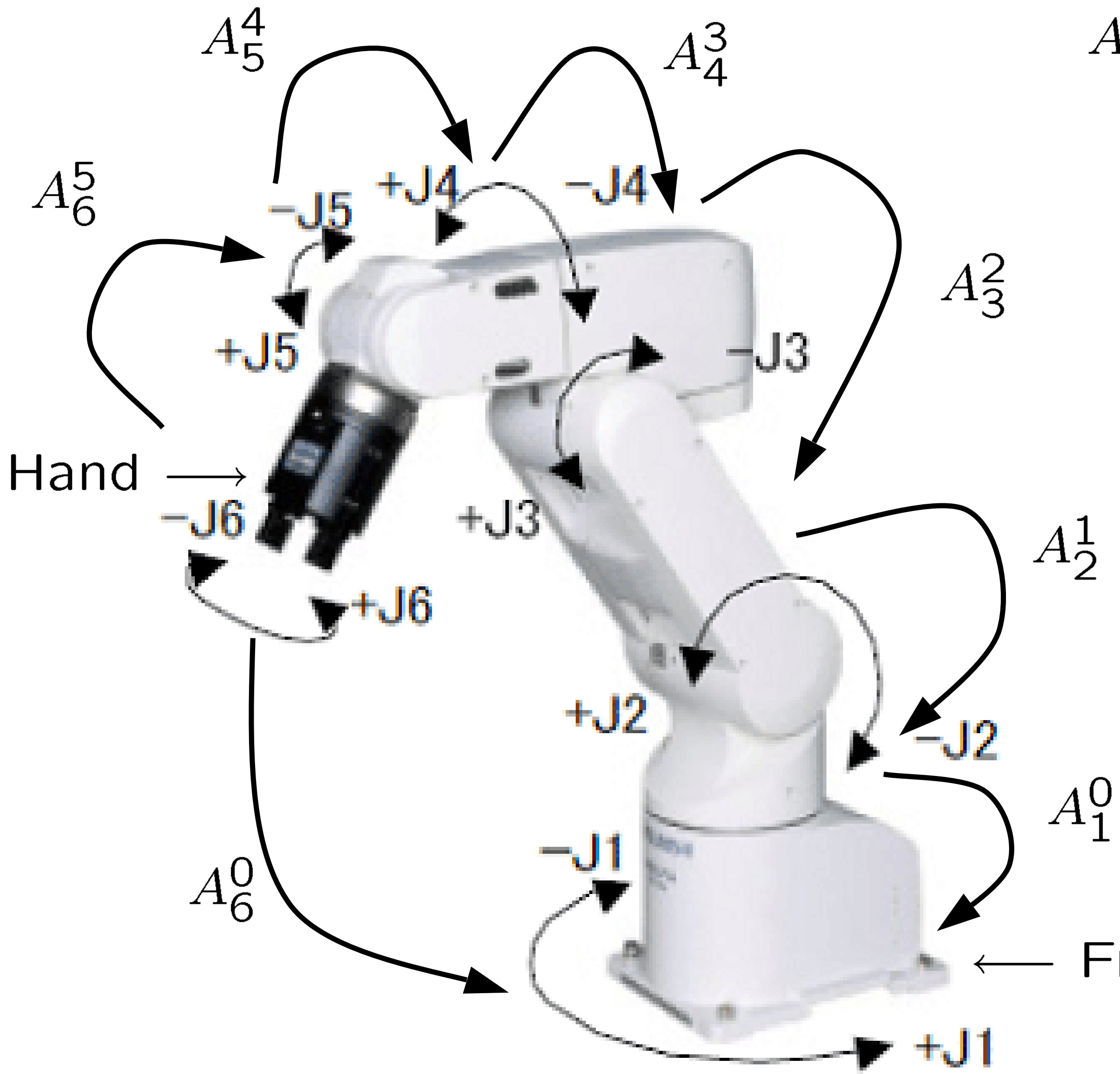
Lecture 3

pajdla@cmp.felk.cvut.cz

Kinematics of serial manipulators

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics



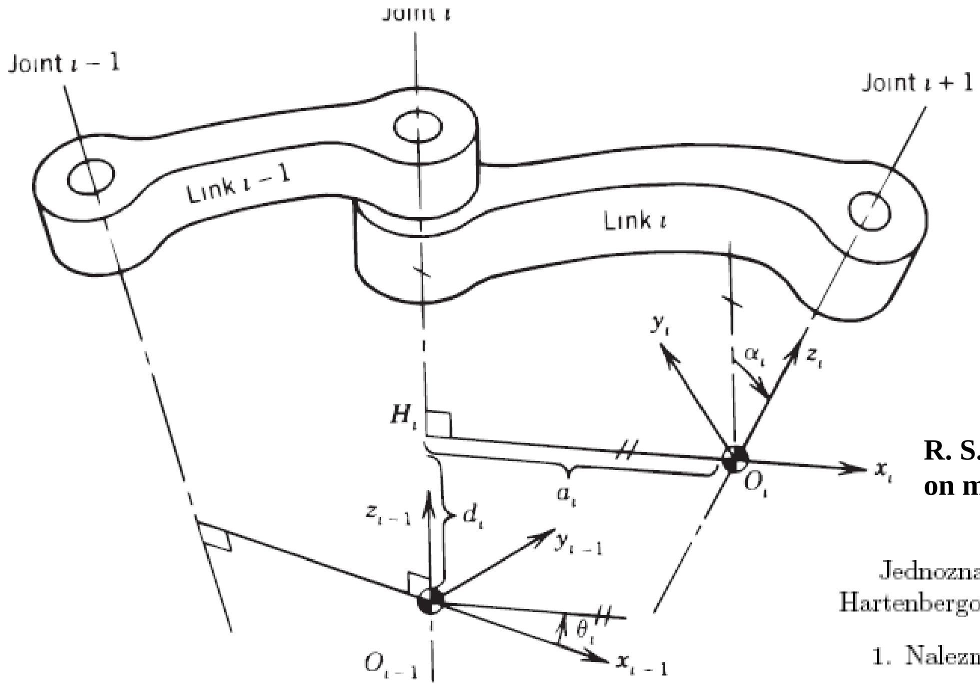
$$A_6^0 = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

6 transformations

7 coordinate systems
indices 0 . . . 6

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, "A kinematic notation for lower pair mechanisms based on matrices," *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

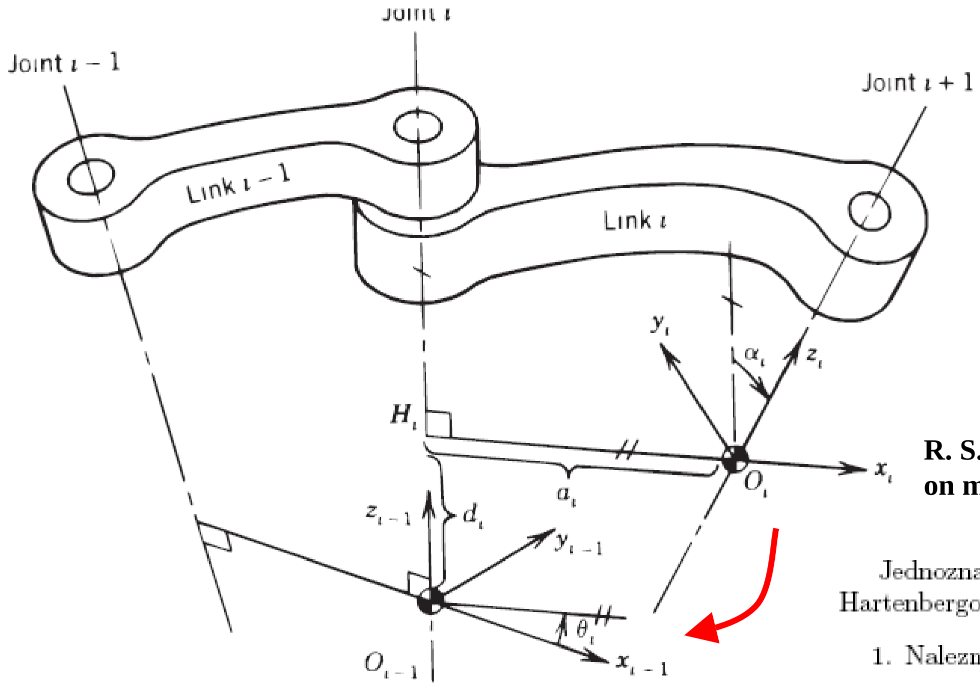
$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou–Hartenbergovou (Denavitova–Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i.

1. Nalezneme osy otáčení kloubů $i-1$, i, $i+1$.
2. Nalezneme příčku (společnou normálu) os kloubů $i-1$ a i a os kloubů i a $i+1$.
3. Nalezneme body O_{i-1}, H_i, O_i.
4. Osu z_i položme do osy kloubu $i+1$.
5. Osu x_i položme do prodloužení příčky H_iO_i.
6. Osa y_i tvoří s ostatními pravotočivou soustavu.
7. Označme vzdálenost bodů $|O_{i-1}, H_i| = d_i$.
8. Označme vzdálenost bodů $|H_i, O_i| = a_i$.
9. Označme úhel mezi příčkami θ_i.
10. Označme úhel mezi osami kloubů i, $i+1$ α_i.
11. Pro rám je možné zvolit polohu bodu O_o kdekoliv na ose kloubu a osu x_0 orientovat libovolně. Například tak, aby $d_1 = 0$.
12. Pro chapadlo je možné opět zvolit bod O_n a orientaci osy z_n při dodržení ostatních pravidel.
13. Jsou-li osy dvou po sobě jdoucích kloubů rovnoběžné, je možné polohu příčky zvolit, například tak, že $d_i = 0$.
14. Pro posuvné klouby lze polohu osy kloubu zvolit.

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, "A kinematic notation for lower pair mechanisms based on matrices," *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\mathbf{A}_i^{int},$$

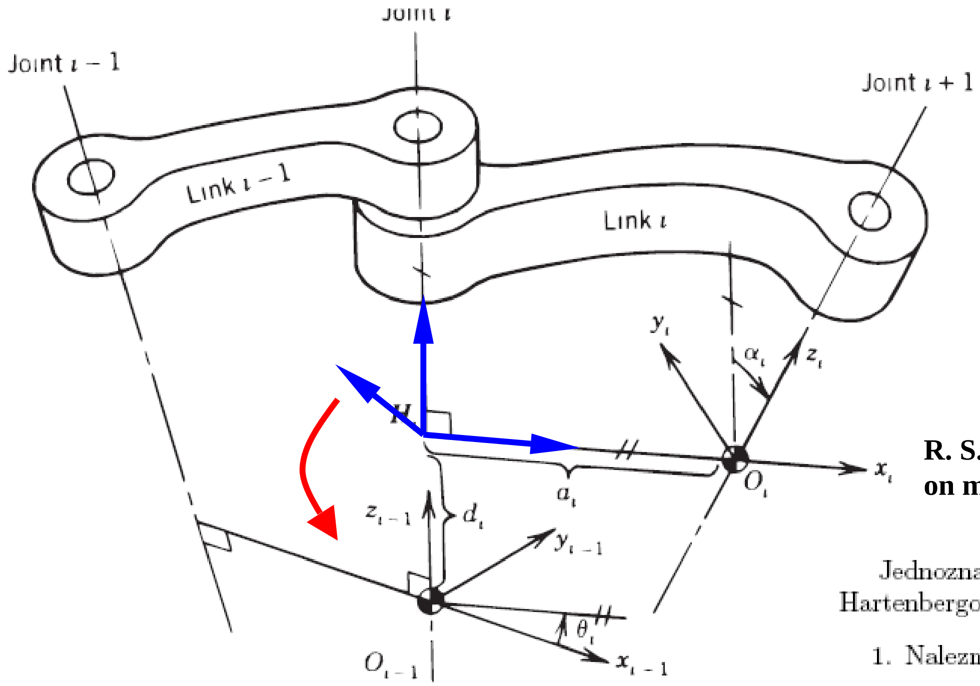
$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou–Hartenbergovou (Denavitova–Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i.

1. Nalezneme osy otáčení kloubů $i-1$, i, $i+1$.
2. Nalezneme příčku (společnou normálu) os kloubů $i-1$ a i a os kloubů i a $i+1$.
3. Nalezneme body O_{i-1}, H_i, O_i.
4. Osu z_i položme do osy kloubu $i+1$.
5. Osu x_i položme do prodloužení příčky H_iO_i.
6. Osa y_i tvoří s ostatními pravotočivou soustavu.
7. Označme vzdálenost bodů $|O_{i-1}, H_i| = d_i$.
8. Označme vzdálenost bodů $|H_i, O_i| = a_i$.
9. Označme úhel mezi příčkami θ_i.
10. Označme úhel mezi osami kloubů i, $i+1$ α_i.
11. Pro rám je možné zvolit polohu bodu O_o kdekoliv na ose kloubu a osu x_0 orientovat libovolně. Například tak, aby $d_1 = 0$.
12. Pro chapadlo je možné opět zvolit bod O_n a orientaci osy z_n při dodržení ostatních pravidel.
13. Jsou-li osy dvou po sobě jdoucích kloubů rovnoběžné, je možné polohu příčky zvolit, například tak, že $d_i = 0$.
14. Pro posuvné klouby lze polohu osy kloubu zvolit.

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, “A kinematic notation for lower pair mechanisms based on matrices,” *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

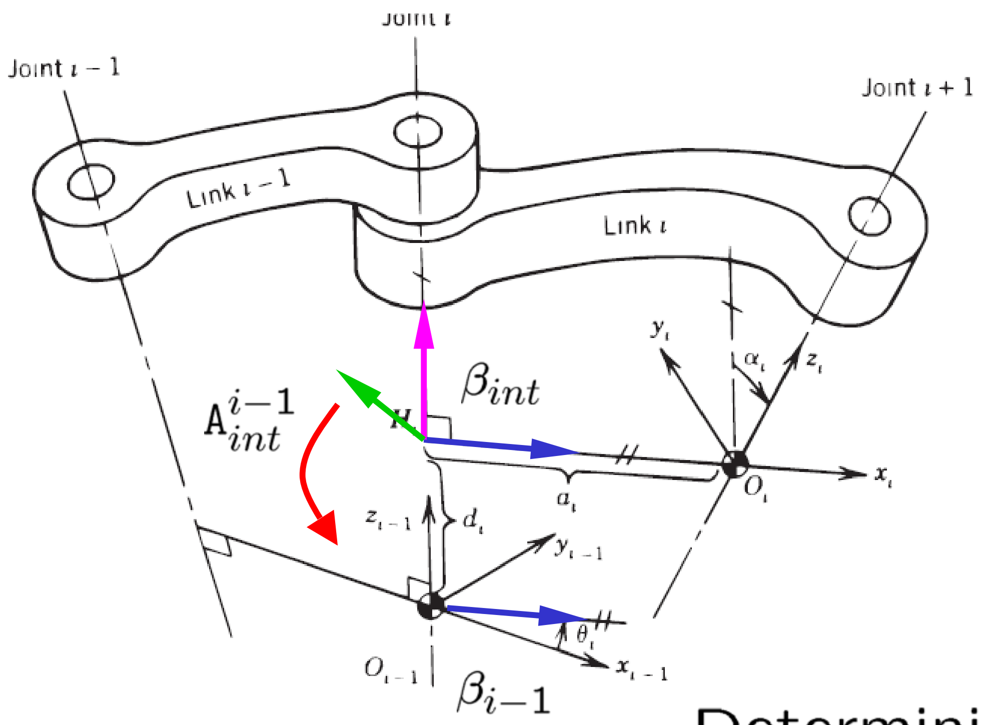
$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou–Hartenbergovou (Denavitova–Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i.

1. Nalezneme osy otáčení kloubů $i-1$, i, $i+1$.
2. Nalezneme příčku (společnou normálu) os kloubů $i-1$ a i a os kloubů i a $i+1$.
3. Nalezneme body O_{i-1}, H_i, O_i.
4. Osu z_i položme do osy kloubu $i+1$.
5. Osu x_i položme do prodloužení příčky H_iO_i.
6. Osa y_i tvoří s ostatními pravotočivou soustavu.
7. Označme vzdálenost bodů $|O_{i-1}, H_i| = d_i$.
8. Označme vzdálenost bodů $|H_i, O_i| = a_i$.
9. Označme úhel mezi příčkami θ_i.
10. Označme úhel mezi osami kloubů i, $i+1$ α_i.
11. Pro rám je možné zvolit polohu bodu O_o kdekoliv na ose kloubu a osu x_0 orientovat libovolně. Například tak, aby $d_1 = 0$.
12. Pro chapadlo je možné opět zvolit bod O_n a orientaci osy z_n při dodržení ostatních pravidel.
13. Jsou-li osy dvou po sobě jdoucích kloubů rovnoběžné, je možné polohu příčky zvolit, například tak, že $d_i = 0$.
14. Pro posuvné klouby lze polohu osy kloubu zvolit.

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention

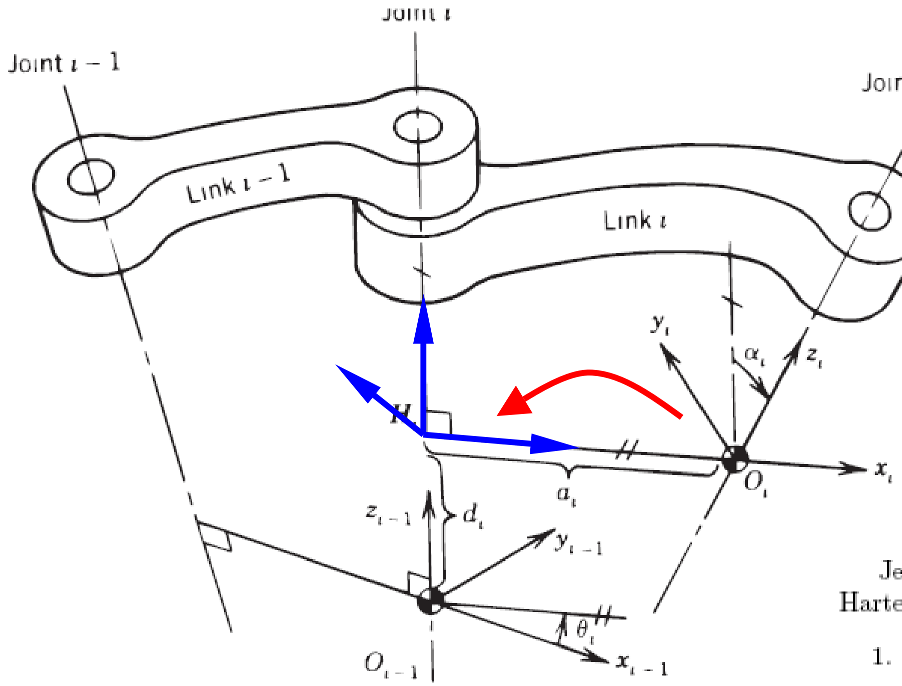


Determining θ_i

$$
\begin{aligned}
\mathbf{X}^{i-1} &= \mathtt{A}_{int}^{i-1}\,\mathbf{X}_{int} \\
&= \begin{bmatrix} \vec{e}_{int1_{\beta_{i-1}}} & \vec{e}_{int2_{\beta_{i-1}}} & \vec{e}_{int3_{\beta_{i-1}}} & \vec{d}_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{X}_{int} \\
&= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{X}_{int}
\end{aligned}
$$

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, "A kinematic notation for lower pair mechanisms based on matrices," *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

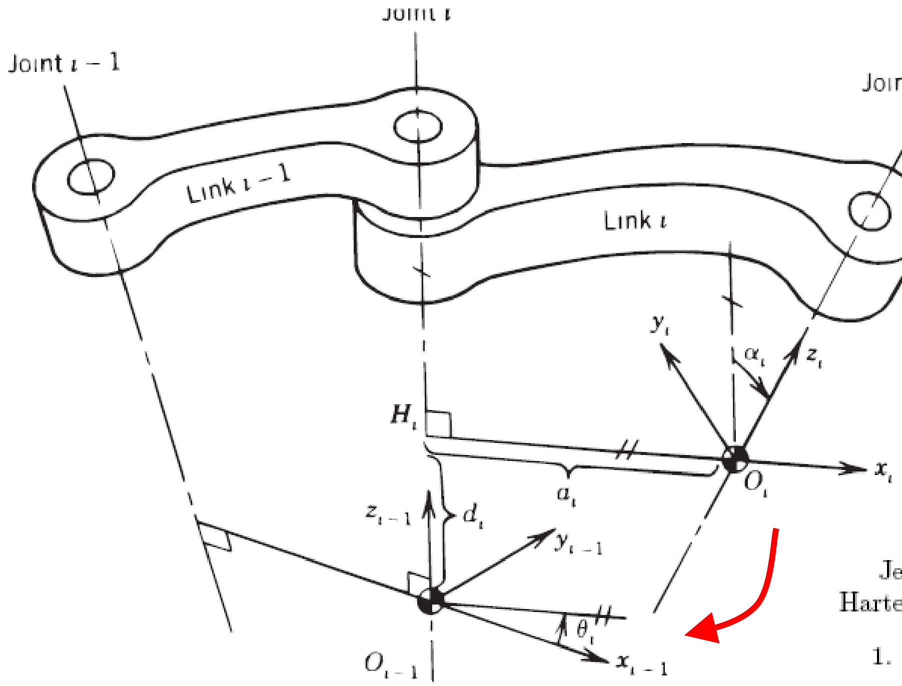
$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou–Hartenbergovou (Denavitova–Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i.

1. Nalezneme osy otáčení kloubů $i-1$, i, $i+1$.
2. Nalezneme příčku (společnou normálu) os kloubů $i-1$ a i a os kloubů i a $i+1$.
3. Nalezneme body O_{i-1}, H_i, O_i.
4. Osu z_i položme do osy kloubu $i+1$.
5. Osu x_i položme do prodloužení příčky H_iO_i.
6. Osa y_i tvoří s ostatními pravotočivou soustavu.
7. Označme vzdálenost bodů $|O_{i-1}, H_i| = d_i$.
8. Označme vzdálenost bodů $|H_i, O_i| = a_i$.
9. Označme úhel mezi příčkami θ_i.
10. Označme úhel mezi osami kloubů i, $i+1$ α_i.
11. Pro rám je možné zvolit polohu bodu O_o kdekoliv na ose kloubu a osu x_0 orientovat libovolně. Například tak, aby $d_1 = 0$.
12. Pro chapadlo je možné opět zvolit bod O_n a orientaci osy z_n při dodržení ostatních pravidel.
13. Jsou-li osy dvou po sobě jdoucích kloubů rovnoběžné, je možné polohu příčky zvolit, například tak, že $d_i = 0$.
14. Pro posuvné klouby lze polohu osy kloubu zvolit.

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R. S. Hartenberg and J. Denavit, "A kinematic notation for lower pair mechanisms based on matrices," *Journal of Applied Mechanics*, vol. 77, pp. 215–221, June 1955.

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Jednoznačný a efektivní popis jednotlivých transformací můžeme nalézt metodou Denavitovou–Hartenbergovou (Denavitova–Hartenbergova notace). Viz Obr. 9. Popisujeme kloub i.

1. Nalezneme osy otáčení kloubů $i-1$, i, $i+1$.
2. Nalezneme příčku (společnou normálu) os kloubů $i-1$ a i a os kloubů i a $i+1$.
3. Nalezneme body O_{i-1}, H_i, O_i.
4. Osu z_i položme do osy kloubu $i+1$.
5. Osu x_i položme do prodloužení příčky H_iO_i.
6. Osa y_i tvoří s ostatními pravotočivou soustavu.
7. Označme vzdálenost bodů $|O_{i-1}, H_i| = d_i$.
8. Označme vzdálenost bodů $|H_i, O_i| = a_i$.
9. Označme úhel mezi příčkami θ_i.
10. Označme úhel mezi osami kloubů i, $i+1$ α_i.
11. Pro rám je možné zvolit polohu bodu O_o kdekoliv na ose kloubu a osu x_0 orientovat libovolně. Například tak, aby $d_1 = 0$.
12. Pro chapadlo je možné opět zvolit bod O_n a orientaci osy z_n při dodržení ostatních pravidel.
13. Jsou-li osy dvou po sobě jdoucích kloubů rovnoběžné, je možné polohu příčky zvolit, například tak, že $d_i = 0$.
14. Pro posuvné klouby lze polohu osy kloubu zvolit.

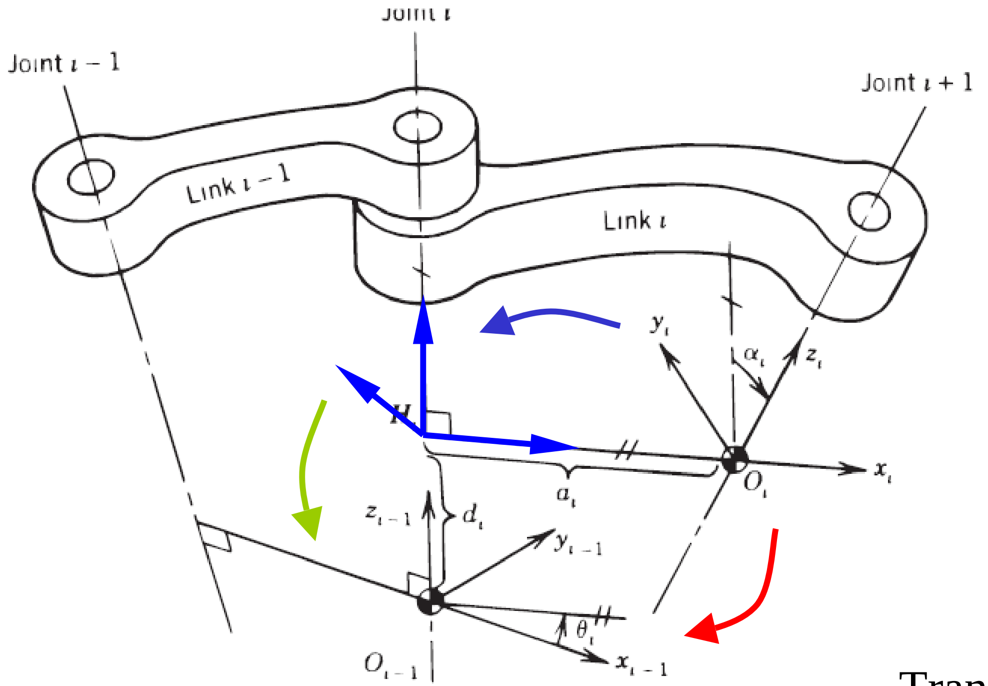
pajdla@cmp.felk.cvut.cz

Denavit-Hartenberg convention - step by step

R. S. Hartenberg and J. Denavit, A kinematic notation for lower pair mechanisms based on matrices, Journal of Applied Mechanics, vol. 77, pp. 215221, June 1955.

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Denavit-Hartenberg convention (for the joint i)

1. Find all motion axes $m_1, \ldots, m_{i-1}, m_i, m_{i+1}, \ldots$

2.1 Find the shortest transversals t_i between m_i and m_{i+1}.

2.2 If m_i is parallel to m_{i+1}, then t_i can be chosen arbitrarily, but the simplest is to make t_i intersect t_{i-1}.

2.3 If m_i intersects m_{i+1}, the t_i becomes the intersection point and the direction perpendicular to m_i, m_{i+1}.

3.2 O_0 can, in principle, be placed anywhere on m_1, but the simplest choice is $O_0 = H_1$.

3.3 Find origins $O_i = t_i \wedge m_{i+1}$ and points $H_i = m_i \wedge t_i$.

3.4 If m_i intersects m_{i+1}, then $O_i = H_i$

3.5 O_N can, in principle, be placed anywhere, but the simplest choice is $O_N = H_N = O_{N-1}$.

4.1 Choose axis $\vec{z}_0$ along the motion axis m_1. There are two choices of the orientation, which are equivalent and can be chosen at will.

4.2 Choose axis $\vec{x}_0$ along t_1 in the direction from O_0 to O_1. at will.

4.3 Choose axis $\vec{y}_0$ to form a right-handed coordinate system.

4.4 Place $\vec{z}_i$ axis alongh the m_{i+1} axis, preferably to contain a sharp angle with the $\vec{z}_{i-1}$.

4.5 Place $\vec{x}_i$ axis along t_i in the direction from H_i to O_i.

4.6 If m_i intersects m_{i+1}, then place $\vec{x}_i$ in the direction perpendicular to m_i, m_{i+1}, preferably to contain a sharp angle with $\vec{x}_{i-1}$.

4.7 Choose axis $\vec{y}_i$ to form a right-handed coordinate system.

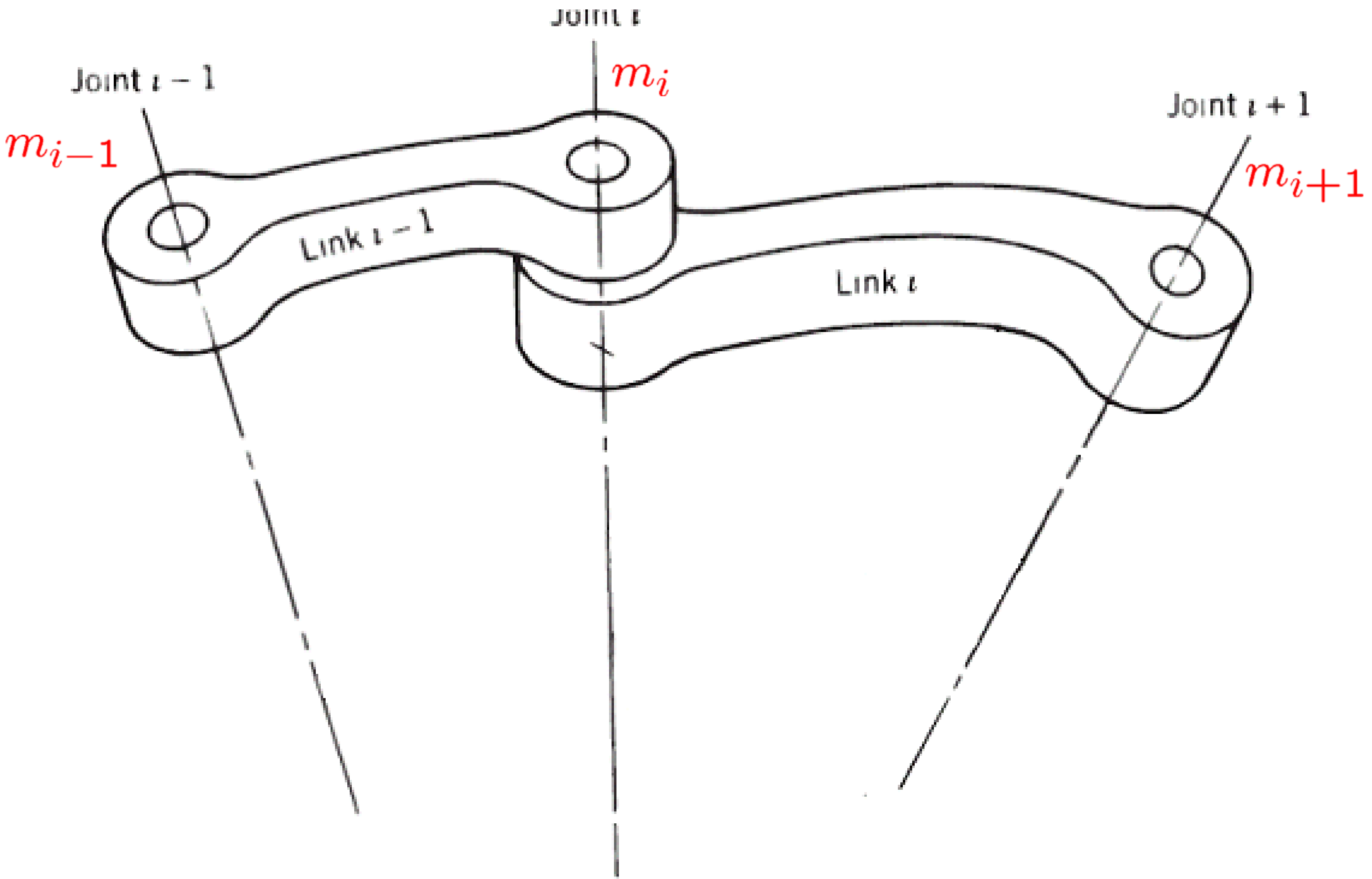
4.8 Construct the *int*ermediate coordinate system $(H_i, \vec{x}_{int} = \vec{x}_i, \vec{y}_{int} = \vec{z}_{i-1} \times \vec{x}_i, \vec{z}_{int} = \vec{z}_{i-1})$ and define α_i such that $\vec{y}_i = \cos(\alpha_i)\,\vec{y}_{int} + \sin(\alpha_i)\,\vec{z}_{int}$.

4.9 Define θ_i such that $\vec{x}_{int} = \cos(\theta_i)\,\vec{x}_{i-1} + \sin(\theta_i)\,\vec{y}_{i-1}$

5.1 Define a_i such that $O_i = H_i + a_i\,\vec{x}_{int}$

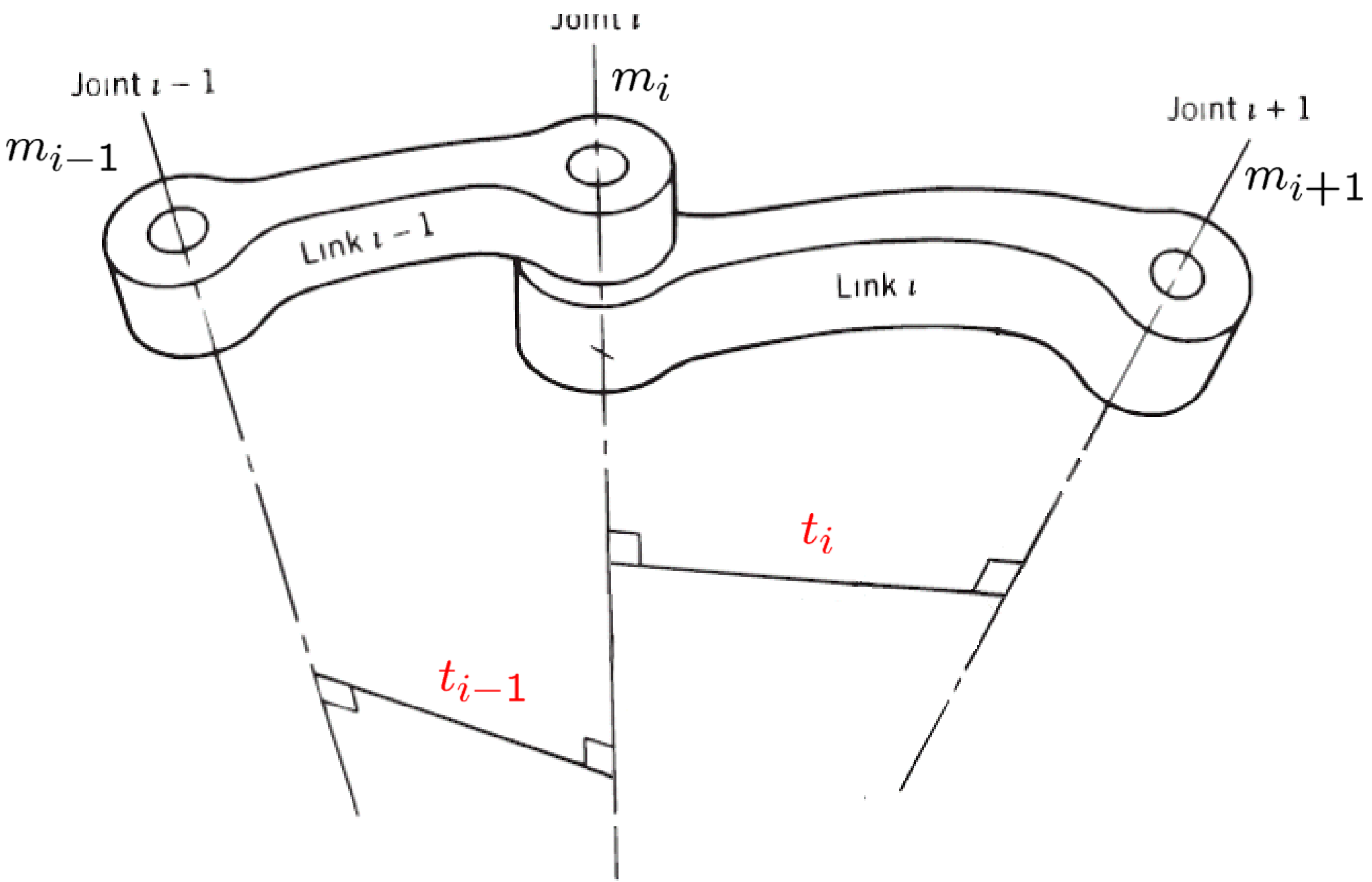
5.2 Define d_i such that $H_i = O_{i-1} + d_i\,\vec{z}_{i-1}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



1. Find all motion axes $m_1, \ldots, m_{i-1}, m_i, m_{i+1}, \ldots$

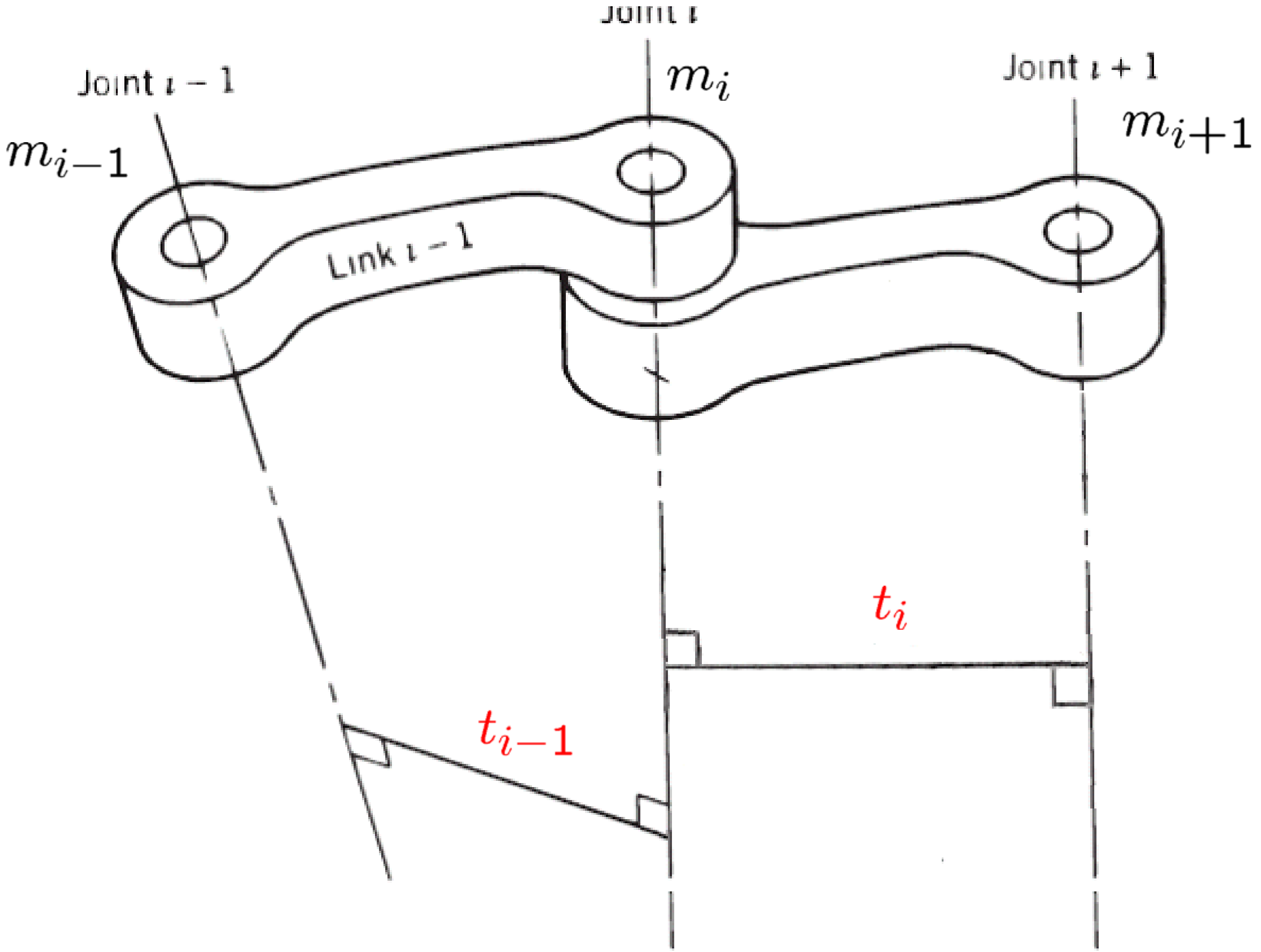
Serial manipulator kinematics in the Denavit-Hartenberg convention



2.1 Find the shortest transversals t_i between m_i and m_{i+1}.

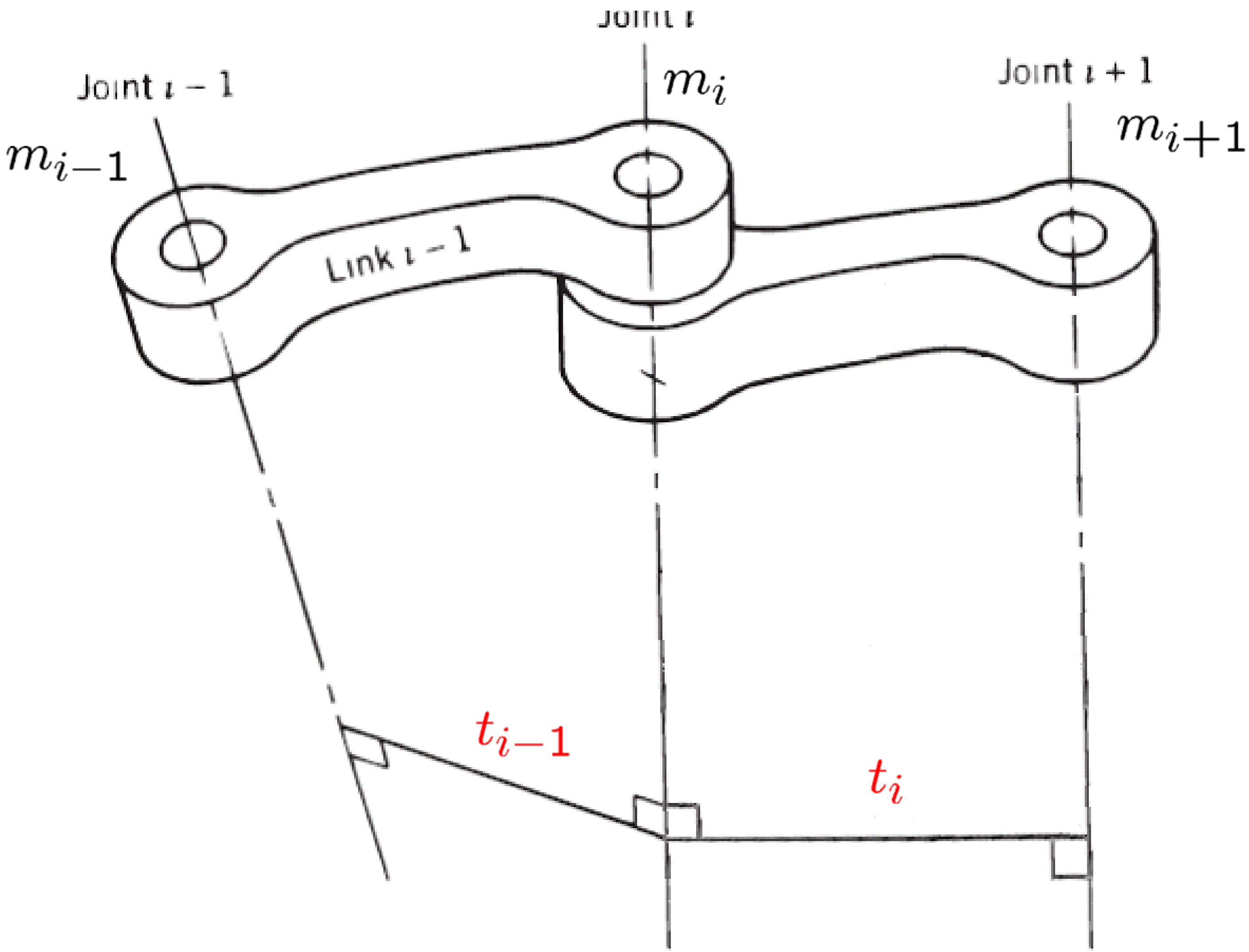
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



2.2 If m_i is parallel to m_{i+1}, then t_i can be chosen arbitrarily, but . . .

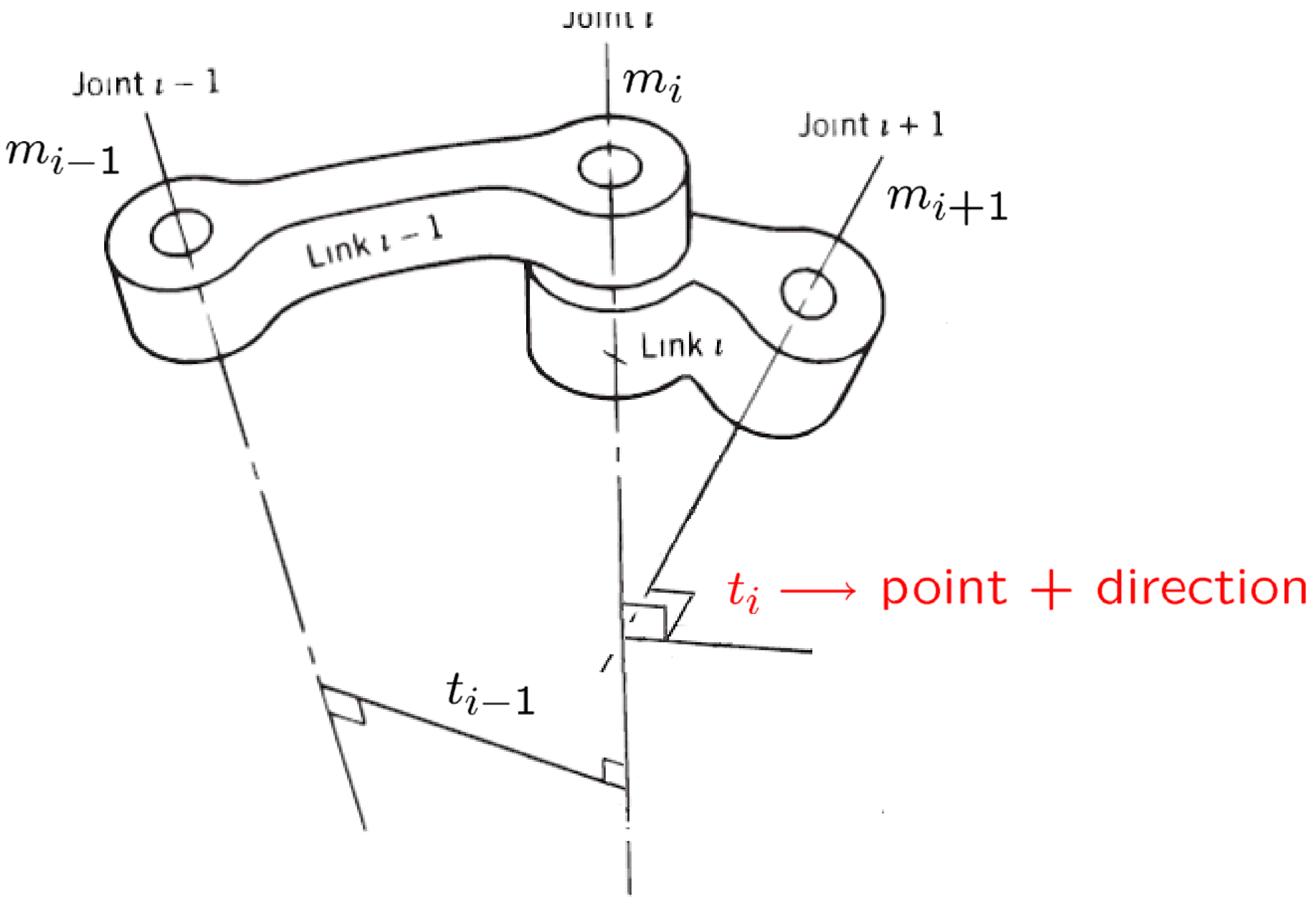
Serial manipulator kinematics in the Denavit-Hartenberg convention



...the simplest is to make t_i intersect t_{i-1}.

pajdla@cmp.felk.cvut.cz

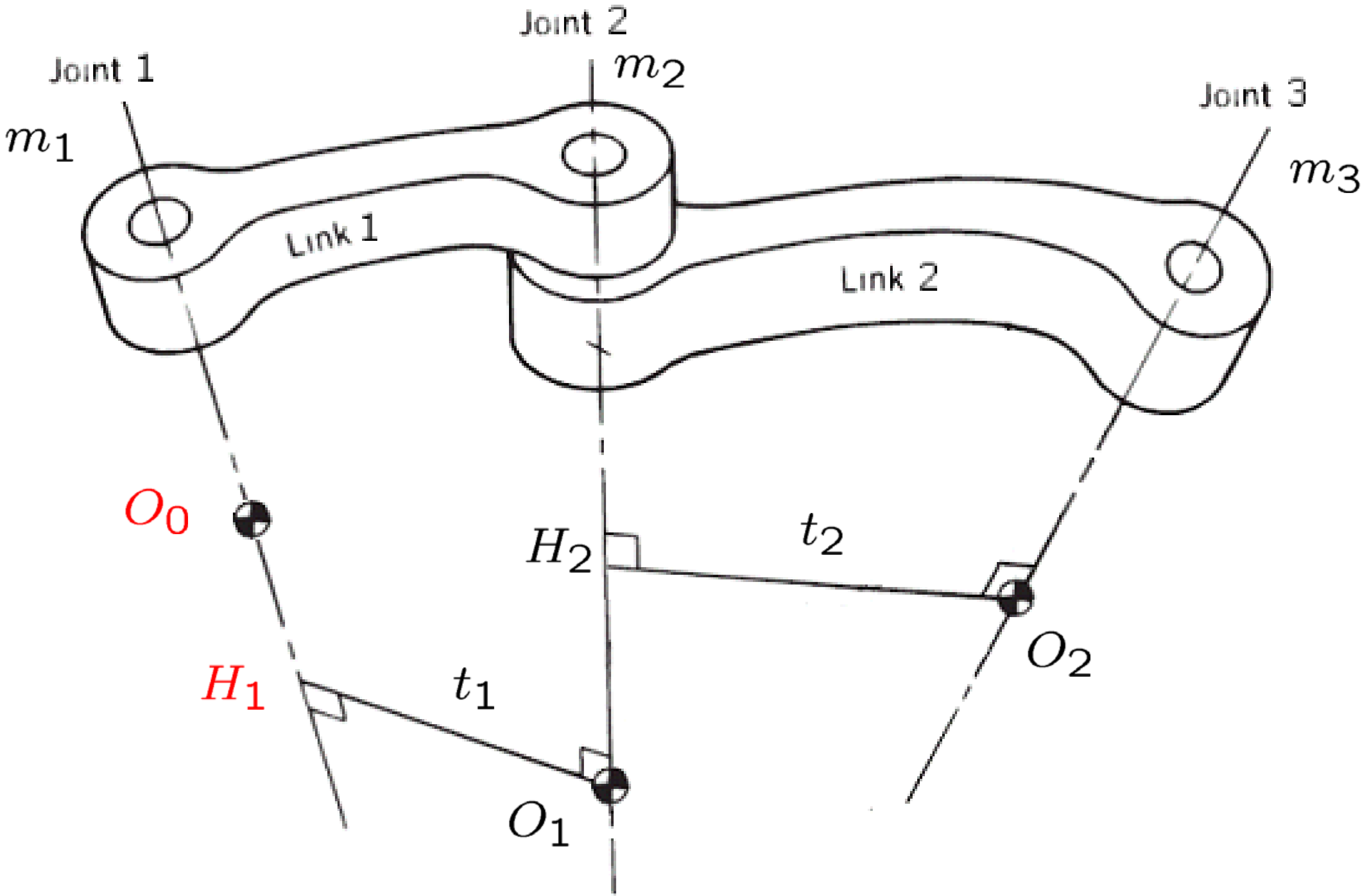
Serial manipulator kinematics in the Denavit-Hartenberg convention



2.3 If m_i intersects m_{i+1}, the t_i becomes the intersection point and the direction perpendicular to m_i, m_{i+1}.

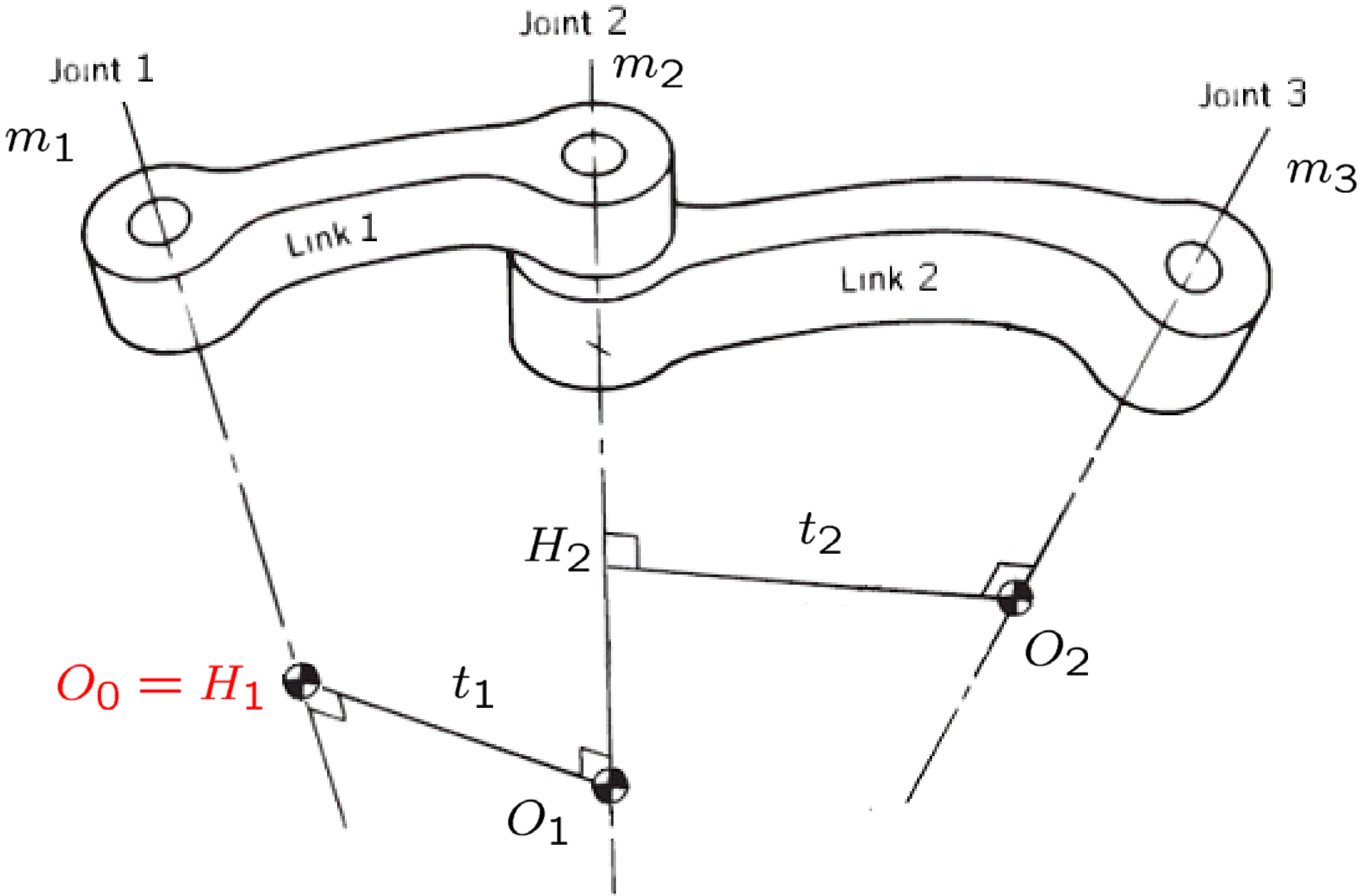
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



3.2 O_0 can, in principle, be placed anywhere on m_1, but . . .

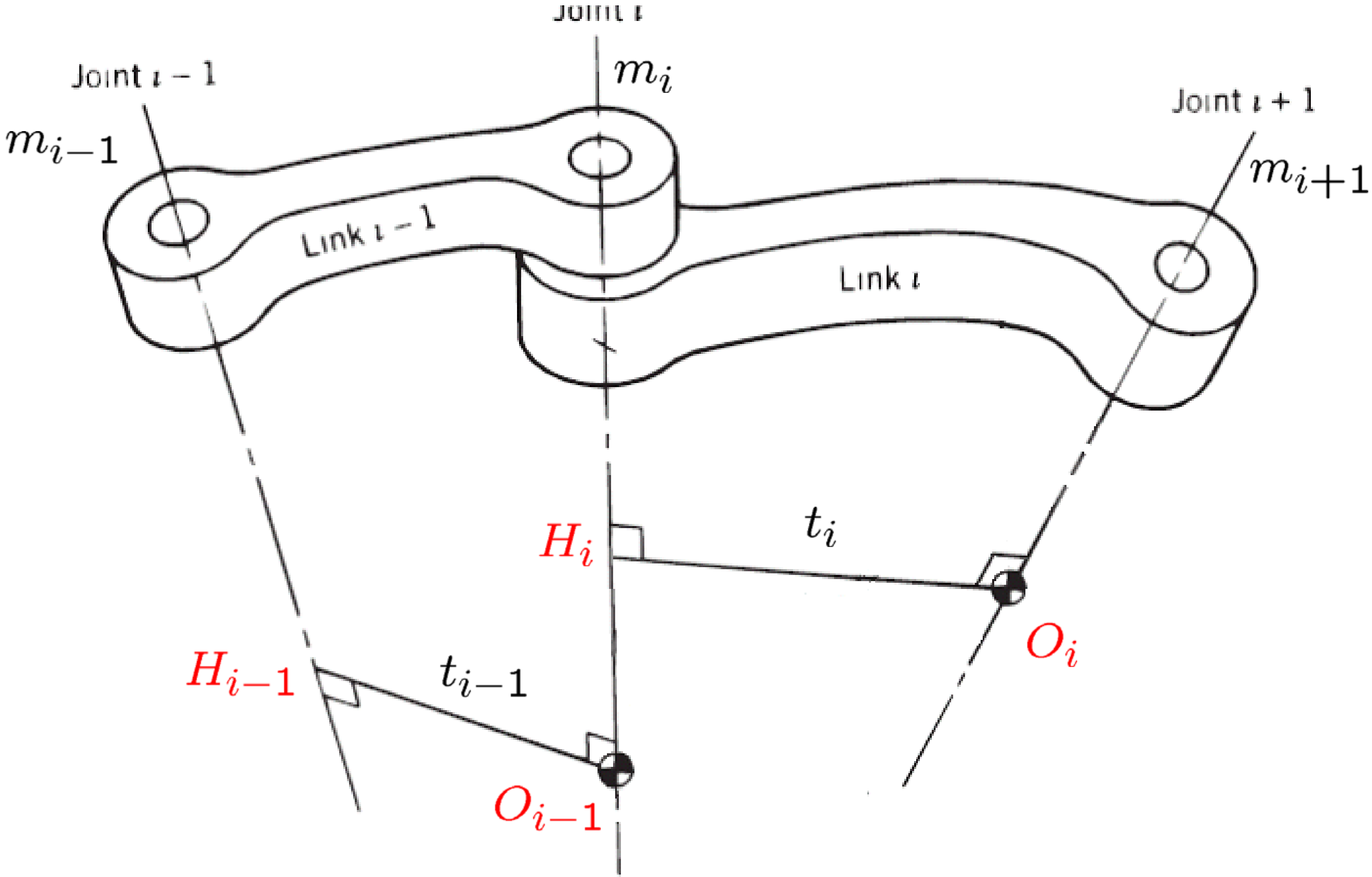
Serial manipulator kinematics in the Denavit-Hartenberg convention



. . . the simplest choice is $O_0 = H_1$.

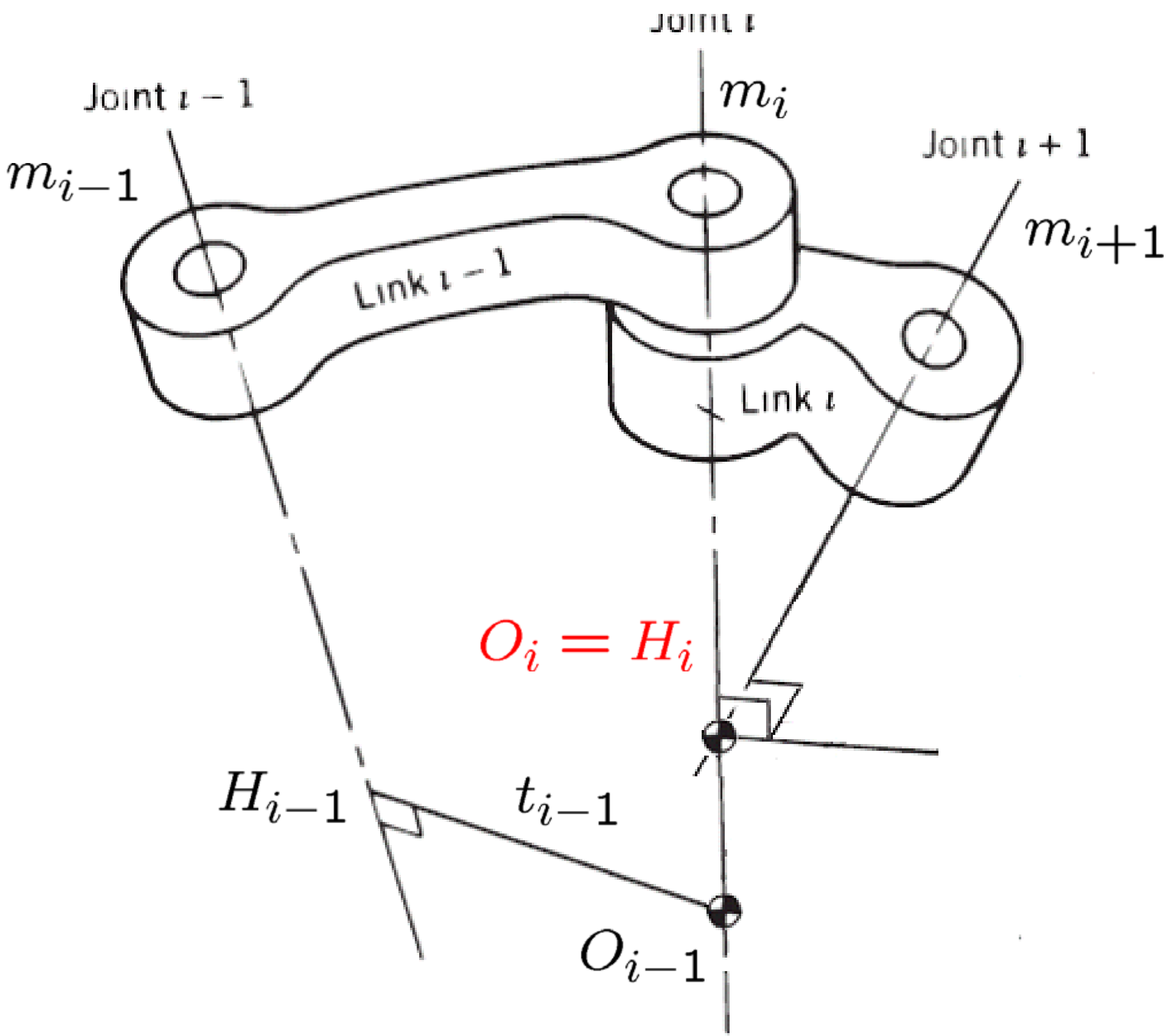
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



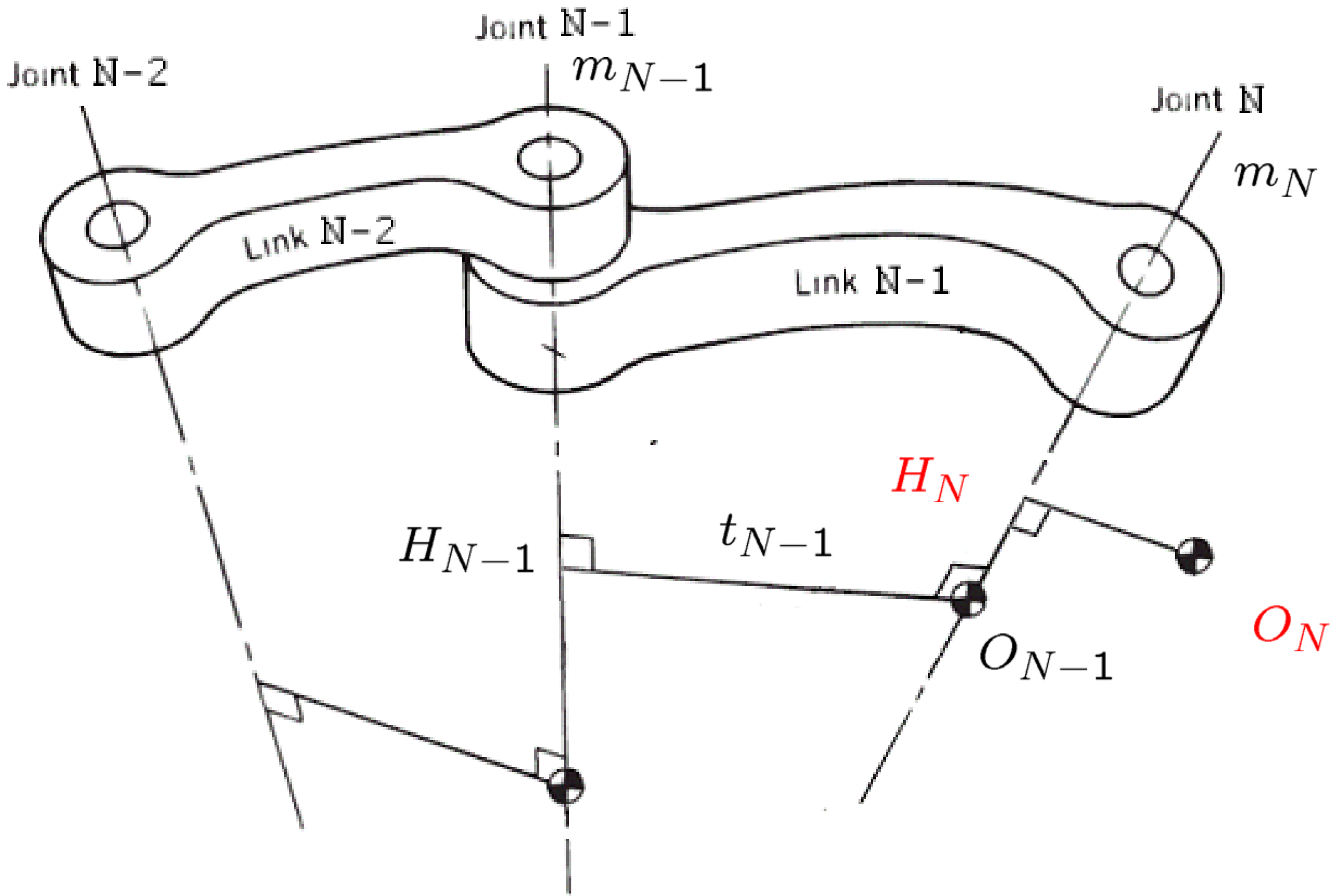
3.3 Find origins $O_i = t_i \wedge m_{i+1}$ and points $H_i = m_i \wedge t_i$.

pajdla@cmp.felk.cvut.cz



3.4 If m_i intersects m_{i+1}, then $O_i = H_i$

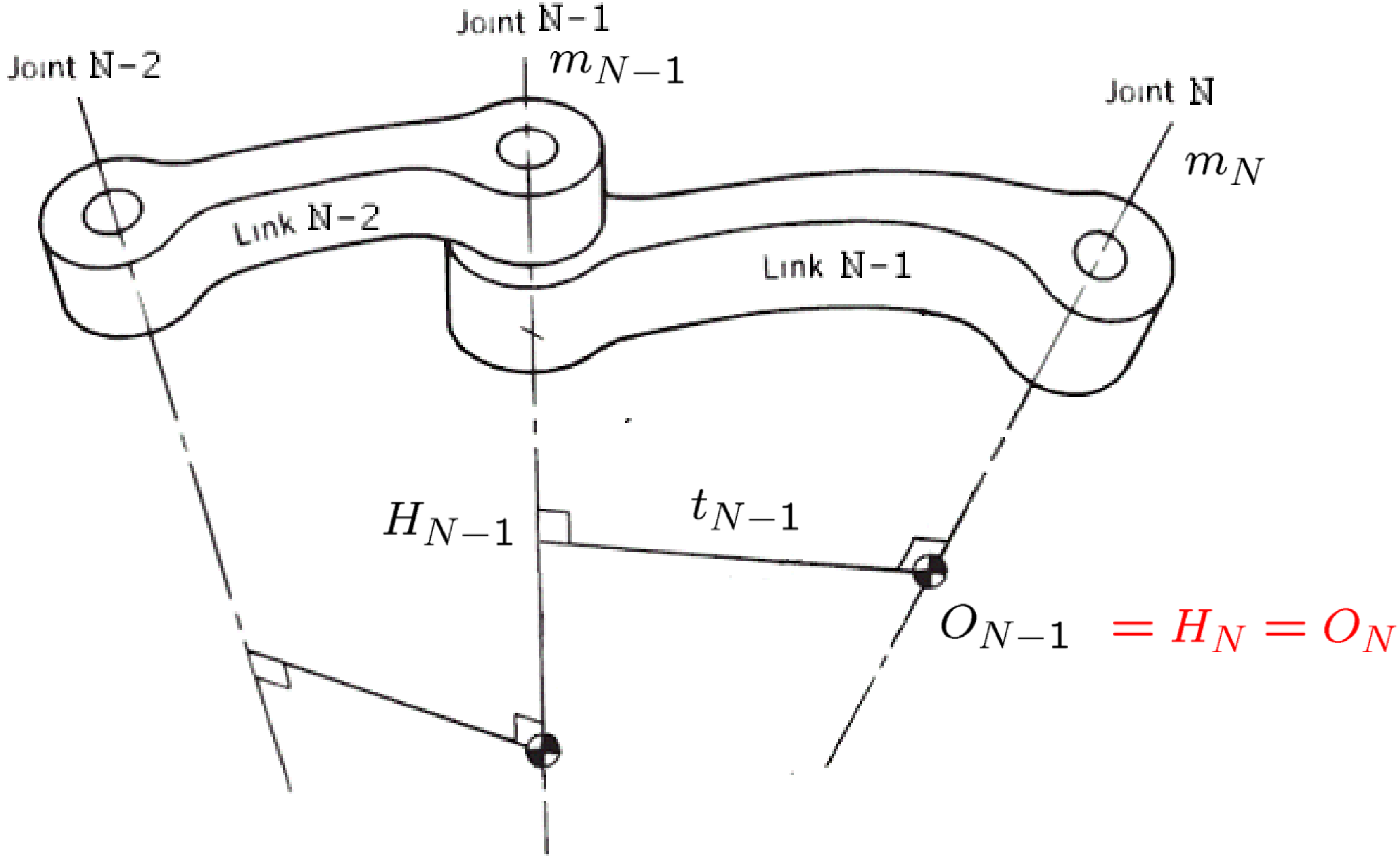
Serial manipulator kinematics in the Denavit-Hartenberg convention



3.5 O_N can, in principle, be placed anywhere, but . . .

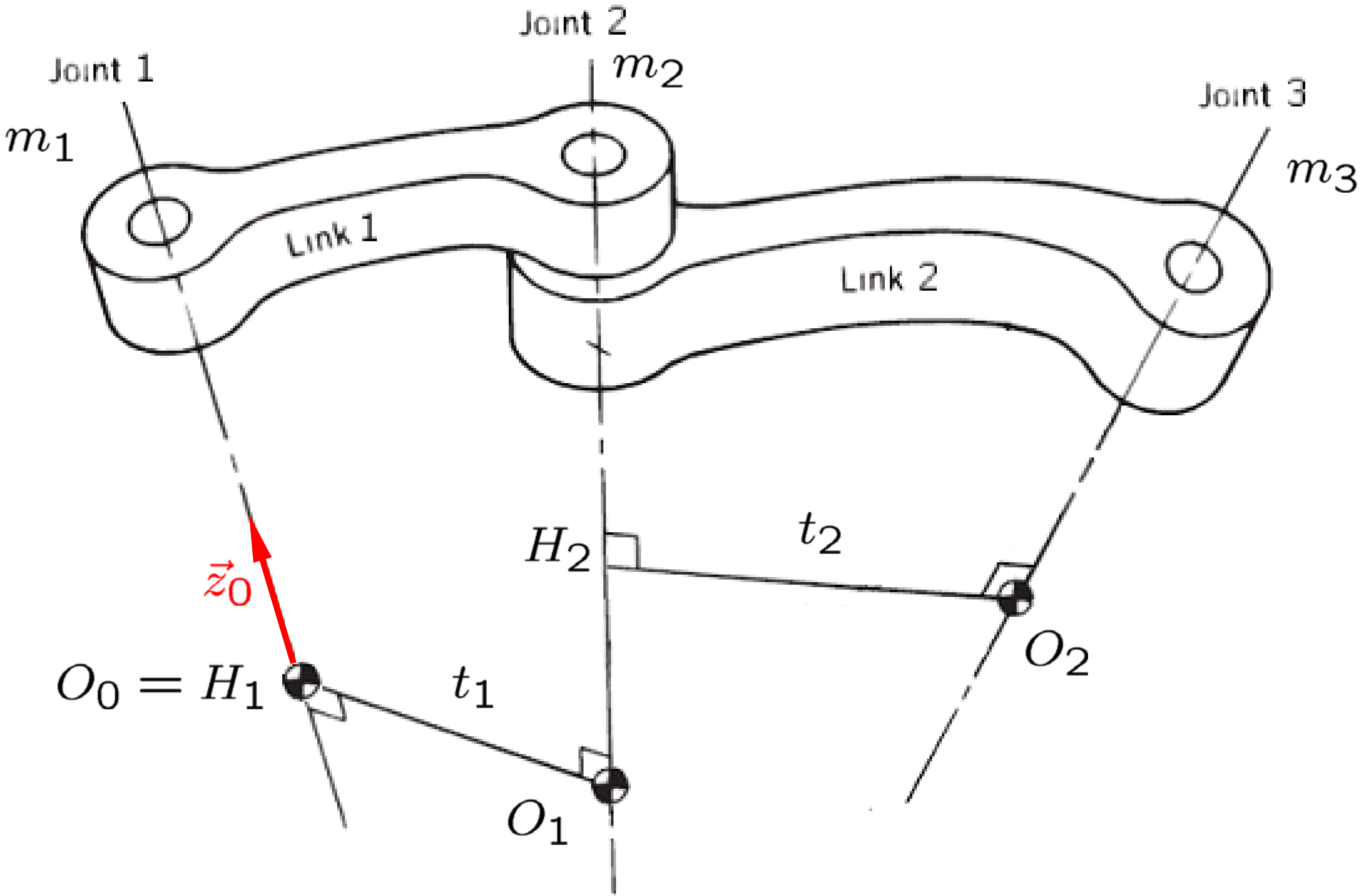
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



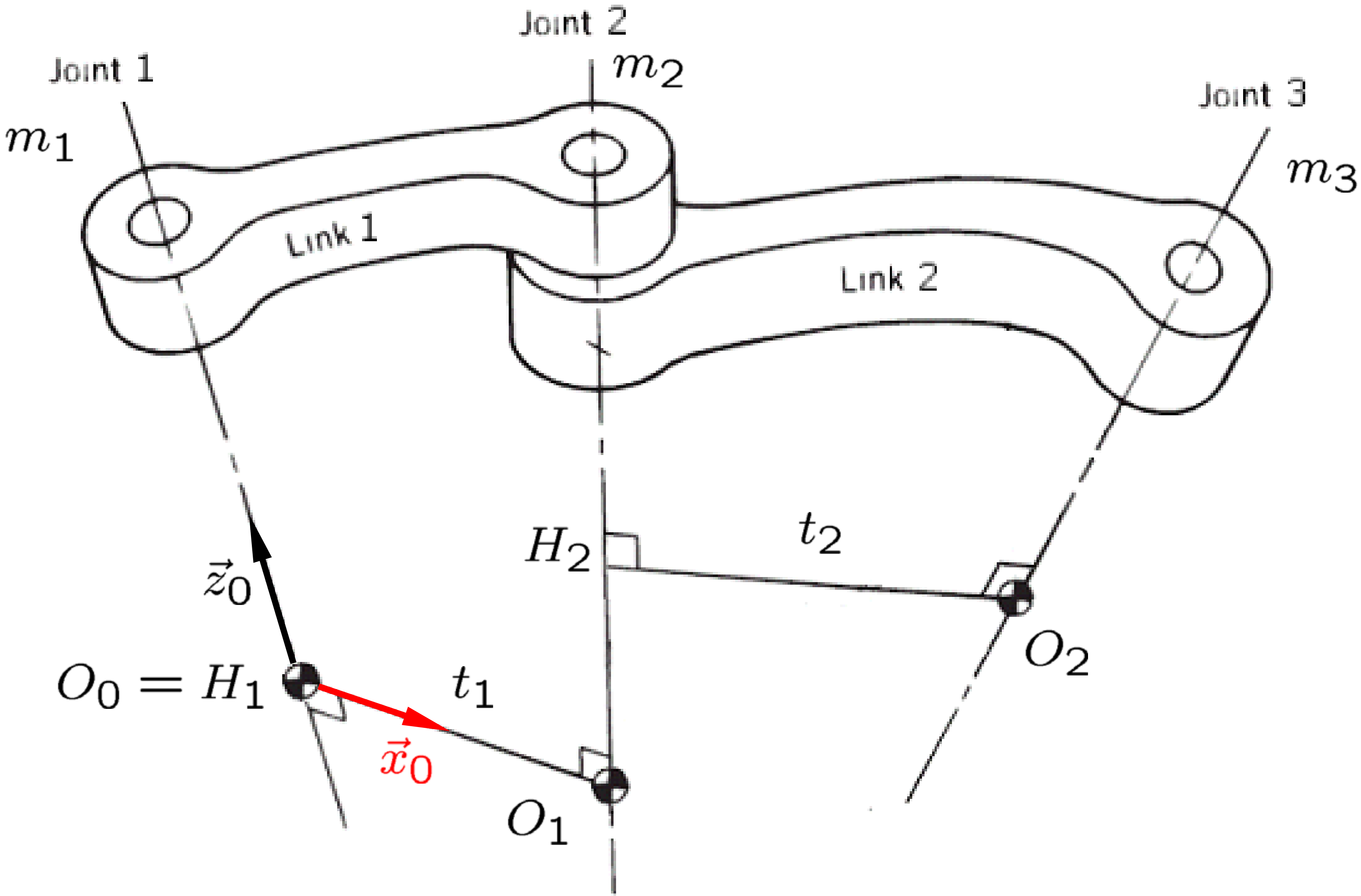
... the simplest choice is $O_N = H_N = O_{N-1}$.

Serial manipulator kinematics in the Denavit-Hartenberg convention



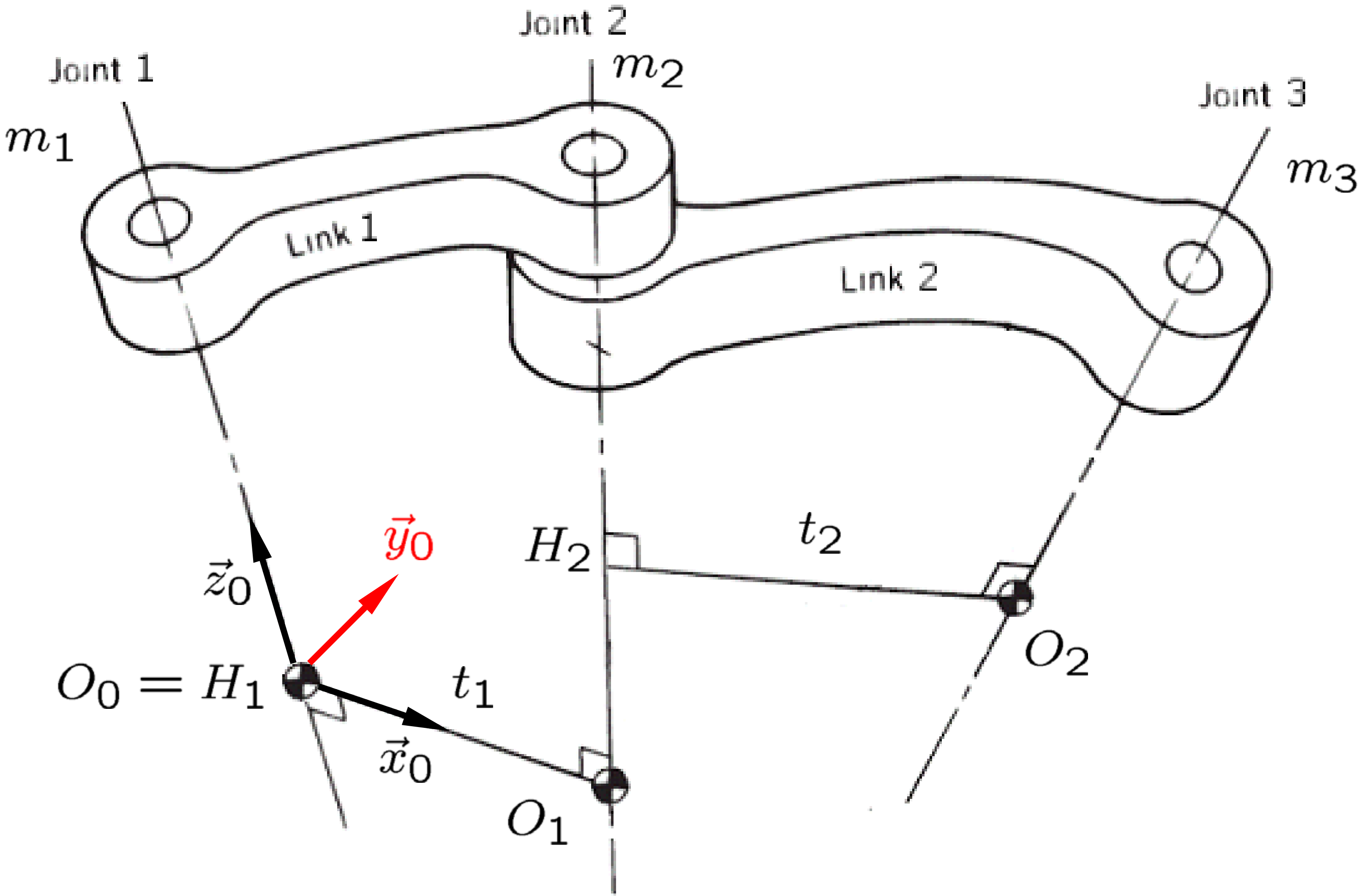
4.1 Choose axis $\vec{z}_0$ along the motion axis m_1. There are two choices of the orientation, which are equivalent and can be chosen at will.

pajdla@cmp.felk.cvut.cz



4.2 Choose axis $\vec{x}_0$ along t_1 in the direction from O_0 to O_1. at will.

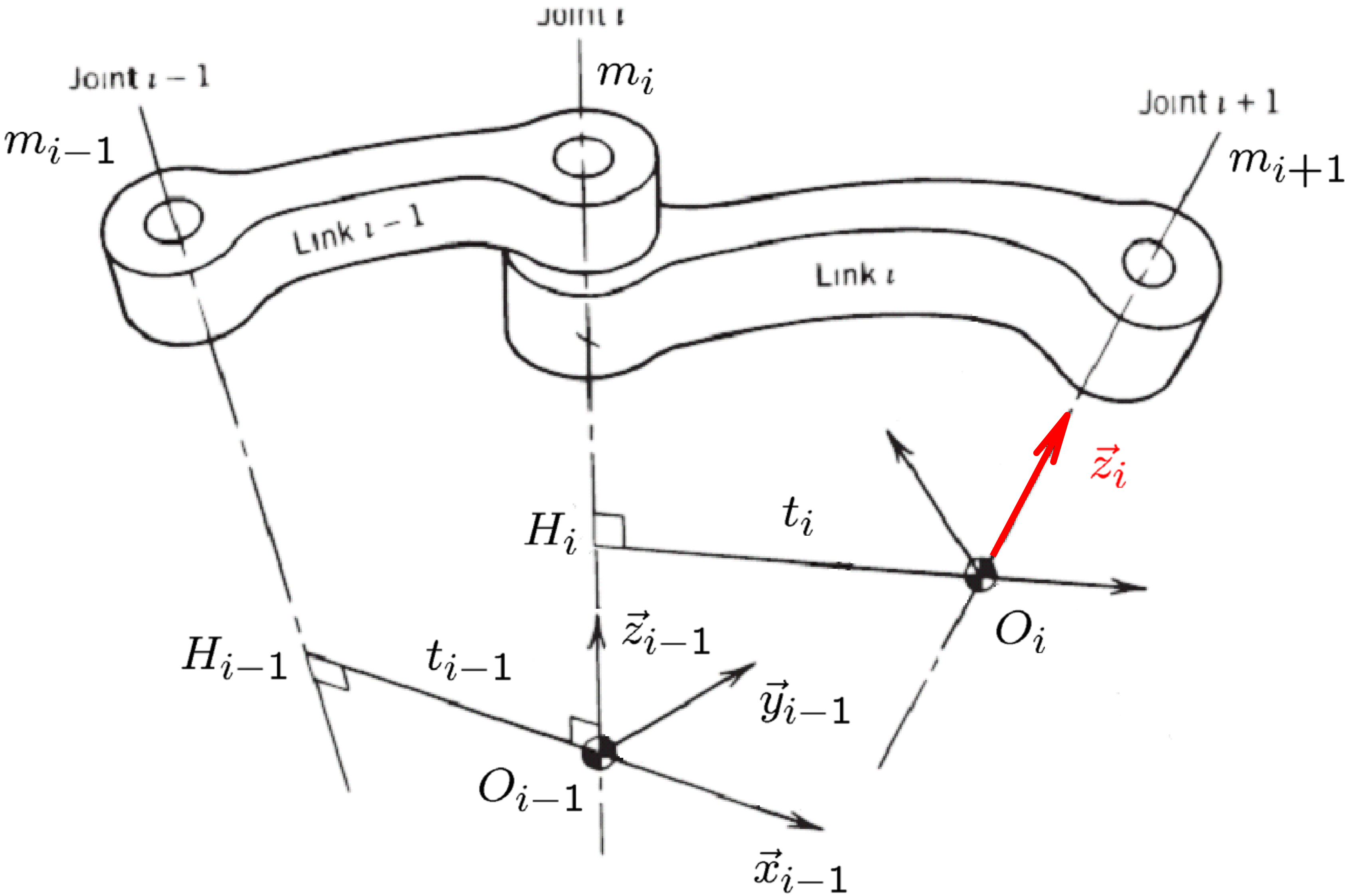
Serial manipulator kinematics in the Denavit-Hartenberg convention



4.3 Choose axis $\vec{y}_0$ to form a right-handed coordinate system.

pajdla@cmp.felk.cvut.cz

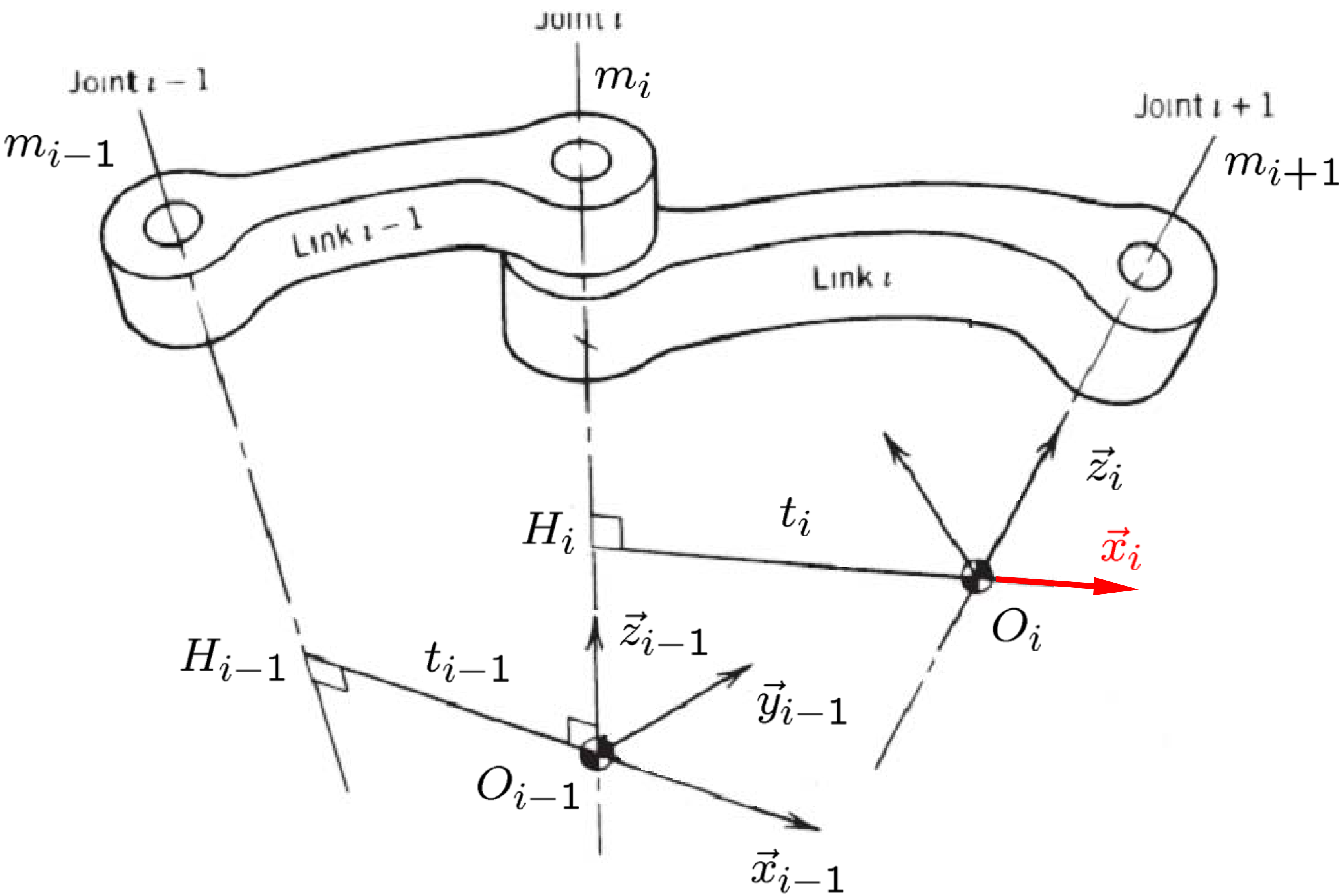
Serial manipulator kinematics in the Denavit-Hartenberg convention



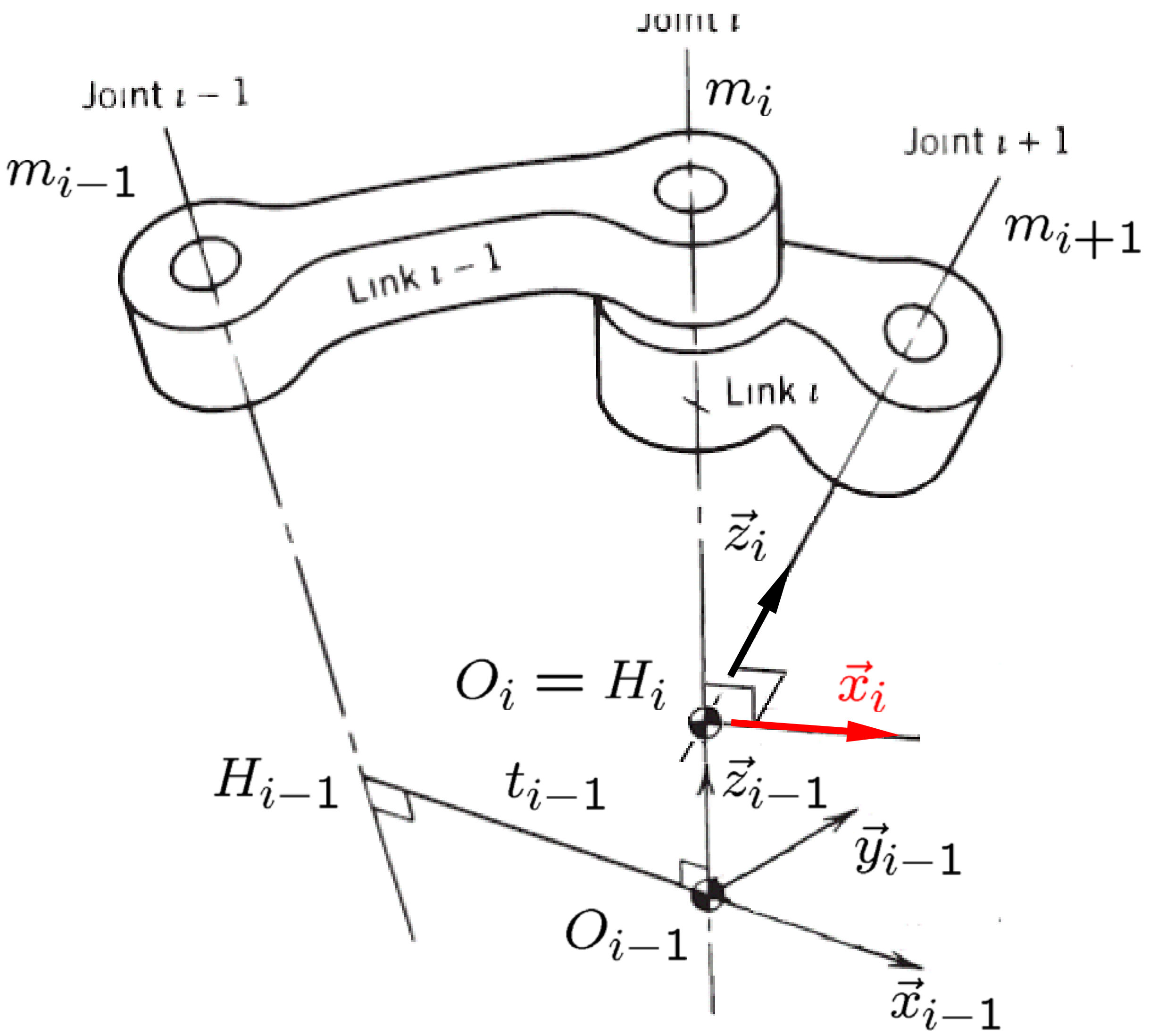
4.4 Place $\vec{z}_i$ axis alongh the m_{i+1} axis, preferably to contain a sharp angle with the $\vec{z}_{i-1}$.

pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



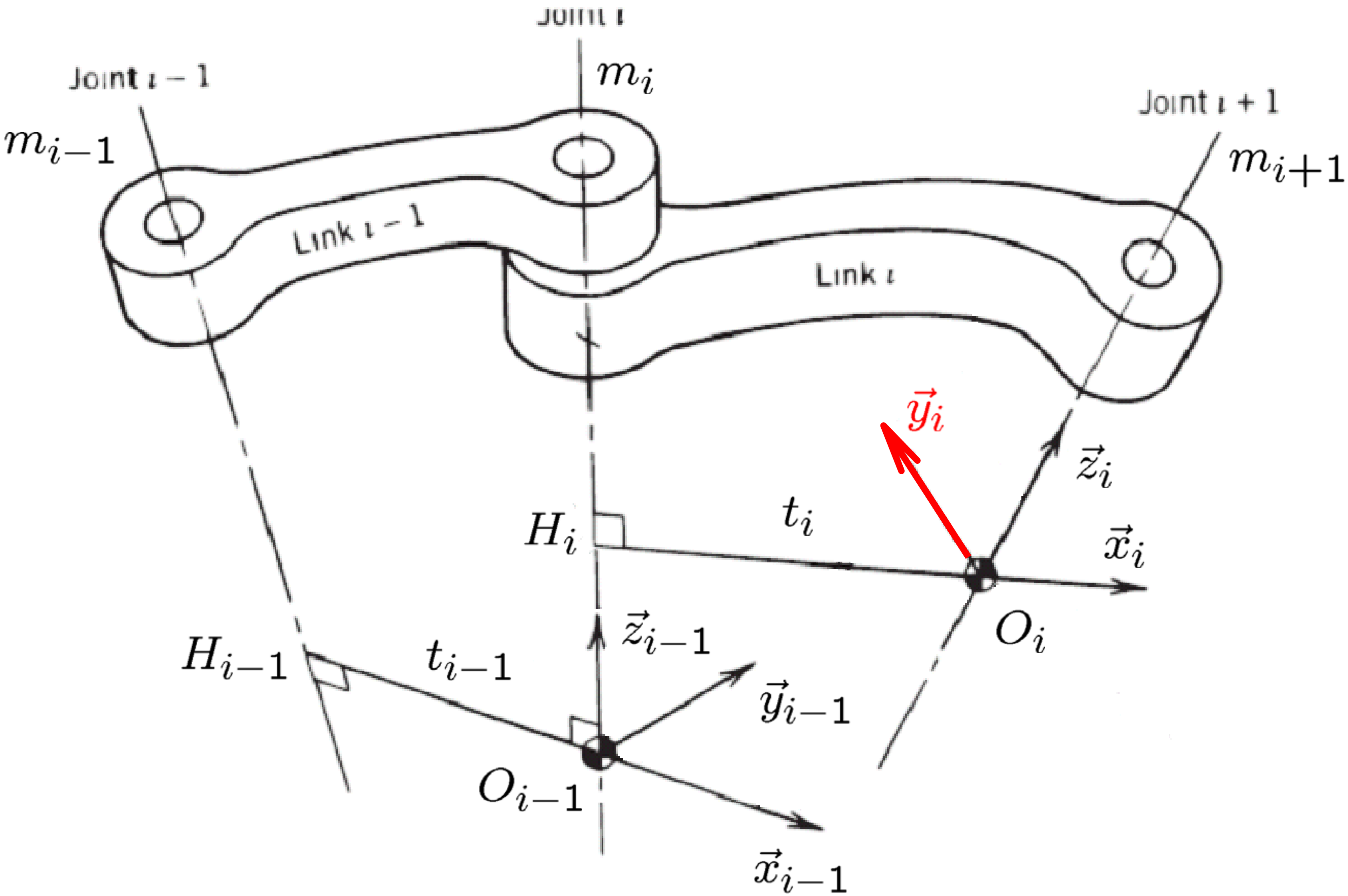
4.5 Place $\vec{x}_i$ axis along t_i in the direction from H_i to O_i.



4.6 If m_i intersects m_{i+1}, then place $\vec{x}_i$ in the direction perpendicular to m_i, m_{i+1}, preferably to contain a sharp angle with $\vec{x}_{i-1}$.

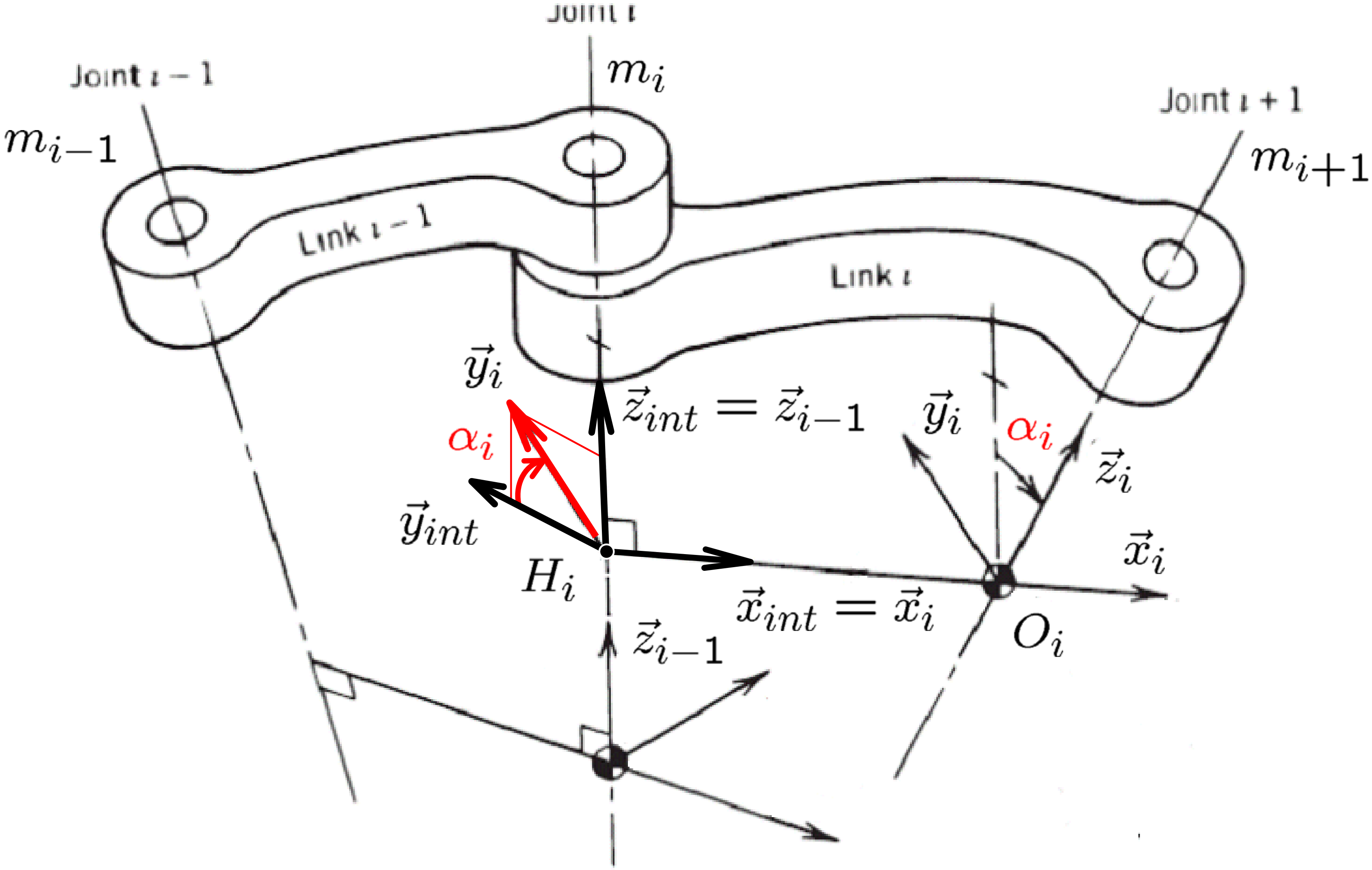
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



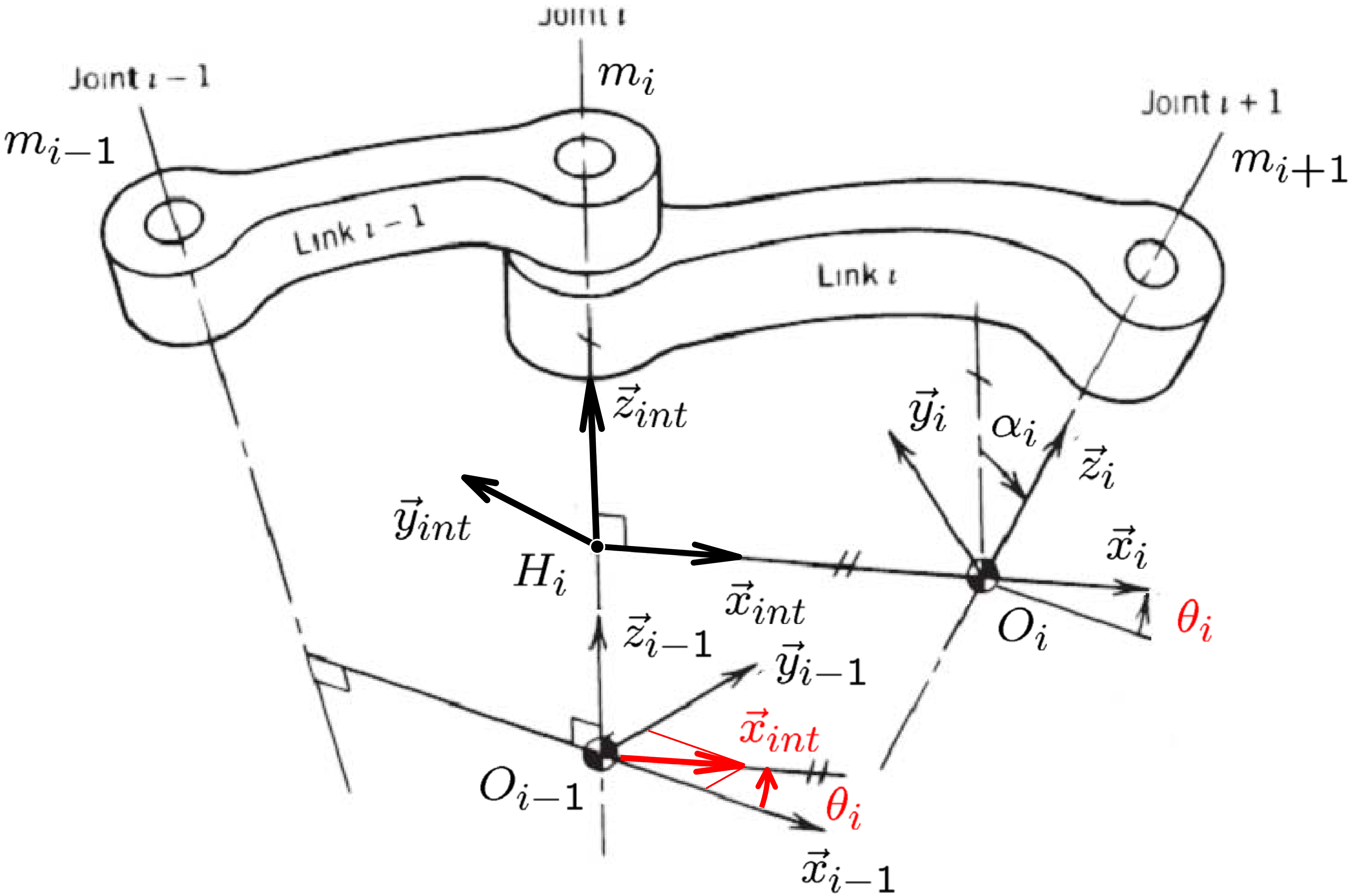
4.7 Choose axis $\vec{y}_i$ to form a right-handed coordinate system.

pajdla@cmp.felk.cvut.cz



4.8 Construct the *int*ermediate coordinate system $(H_i, \vec{x}_{int} = \vec{x}_i, \vec{y}_{int} = \vec{z}_{i-1} \times \vec{x}_i, \vec{z}_{int} = \vec{z}_{i-1})$ and define α_i such that $\vec{y}_i = \cos(\alpha_i)\, \vec{y}_{int} + \sin(\alpha_i)\, \vec{z}_{int}$

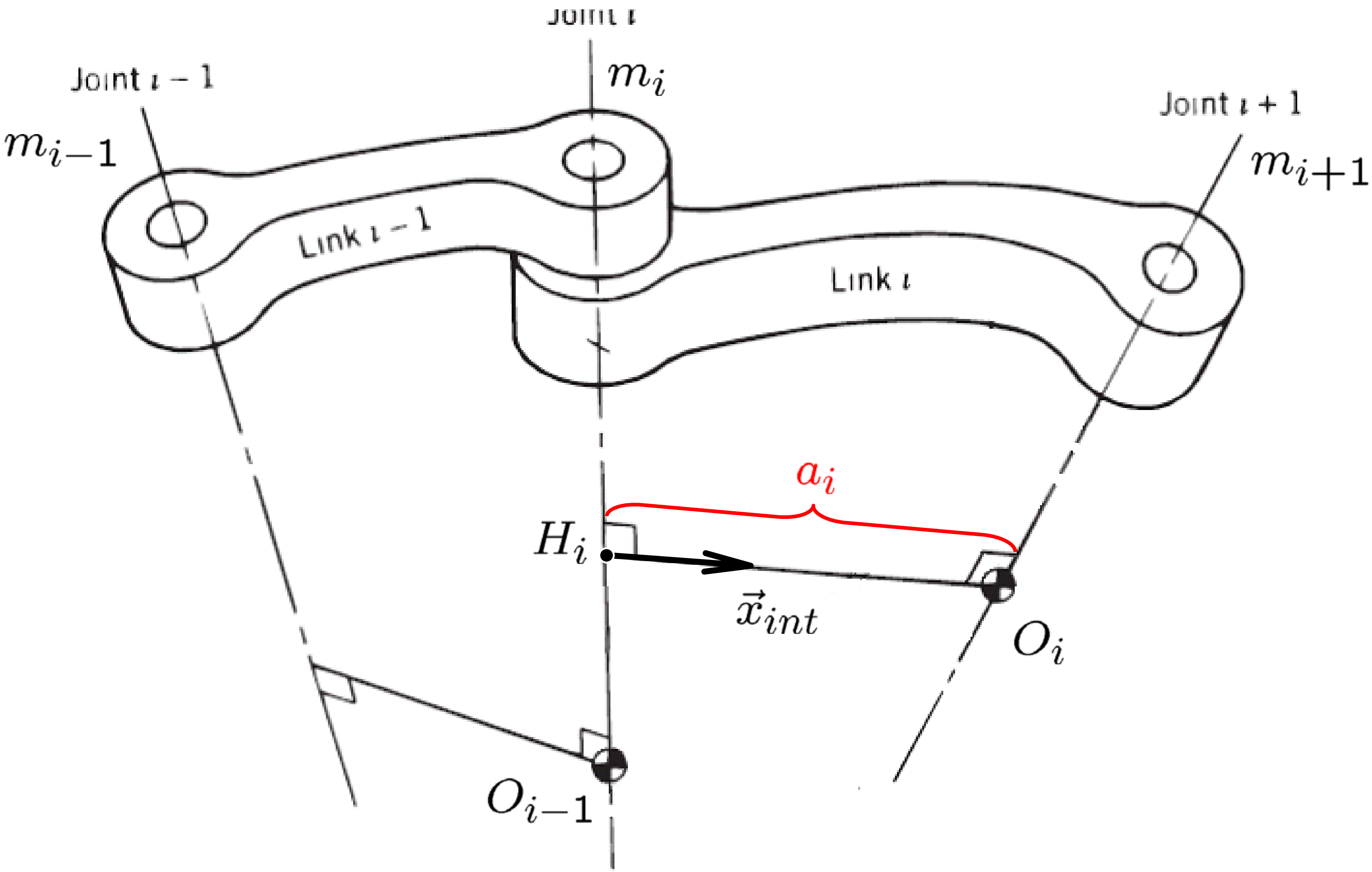
Serial manipulator kinematics in the Denavit-Hartenberg convention



4.9 Define θ_i such that $\vec{x}_{int} = \cos(\theta_i)\,\vec{x}_{i-1} + \sin(\theta_i)\,\vec{y}_{i-1}$

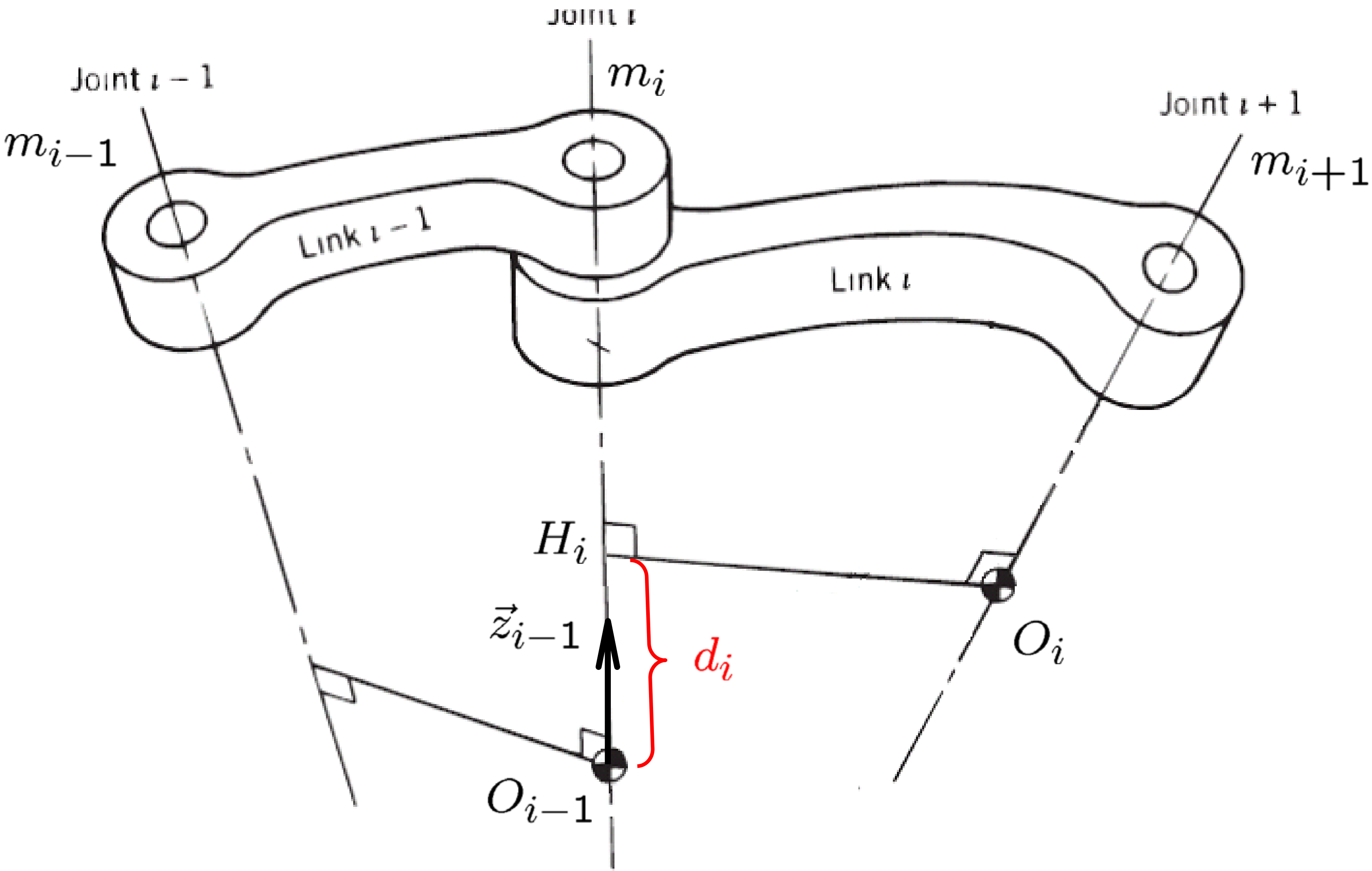
pajdla@cmp.felk.cvut.cz

Serial manipulator kinematics in the Denavit-Hartenberg convention



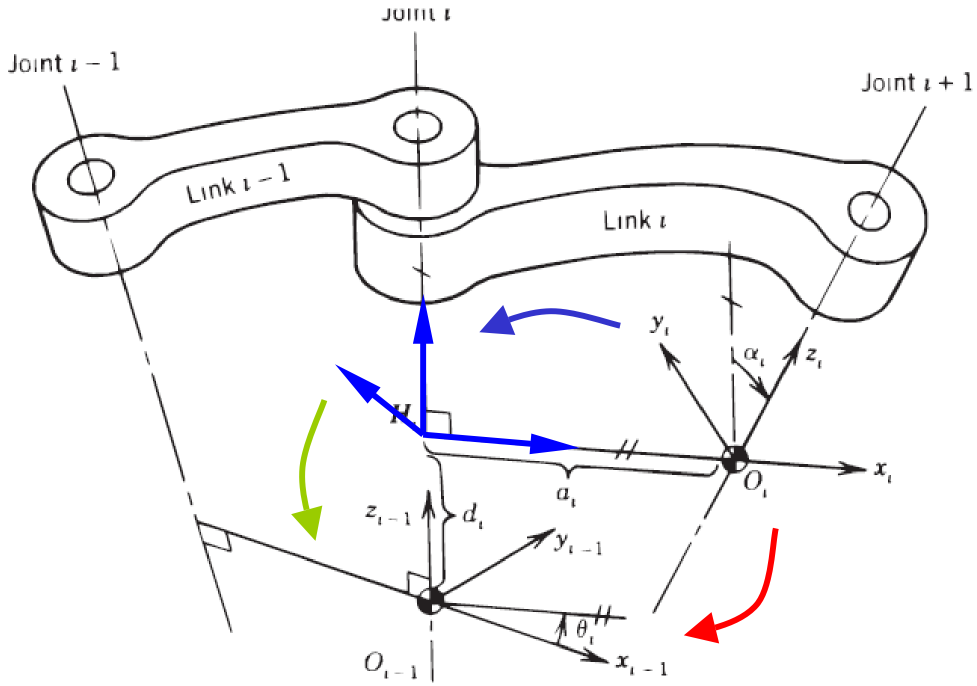
5.1 Define a_i such that $O_i = H_i + a_i\,\vec{x}_{int}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



5.2 Define d_i such that $H_i = O_{i-1} + d_i\,\vec{z}_{i-1}$

Serial manipulator kinematics in the Denavit-Hartenberg convention



Transformation in a joint is described by 4 parameters

$$\mathbf{A}_i^{i-1} = \mathbf{A}_{int}^{i-1}\,\mathbf{A}_i^{int},$$

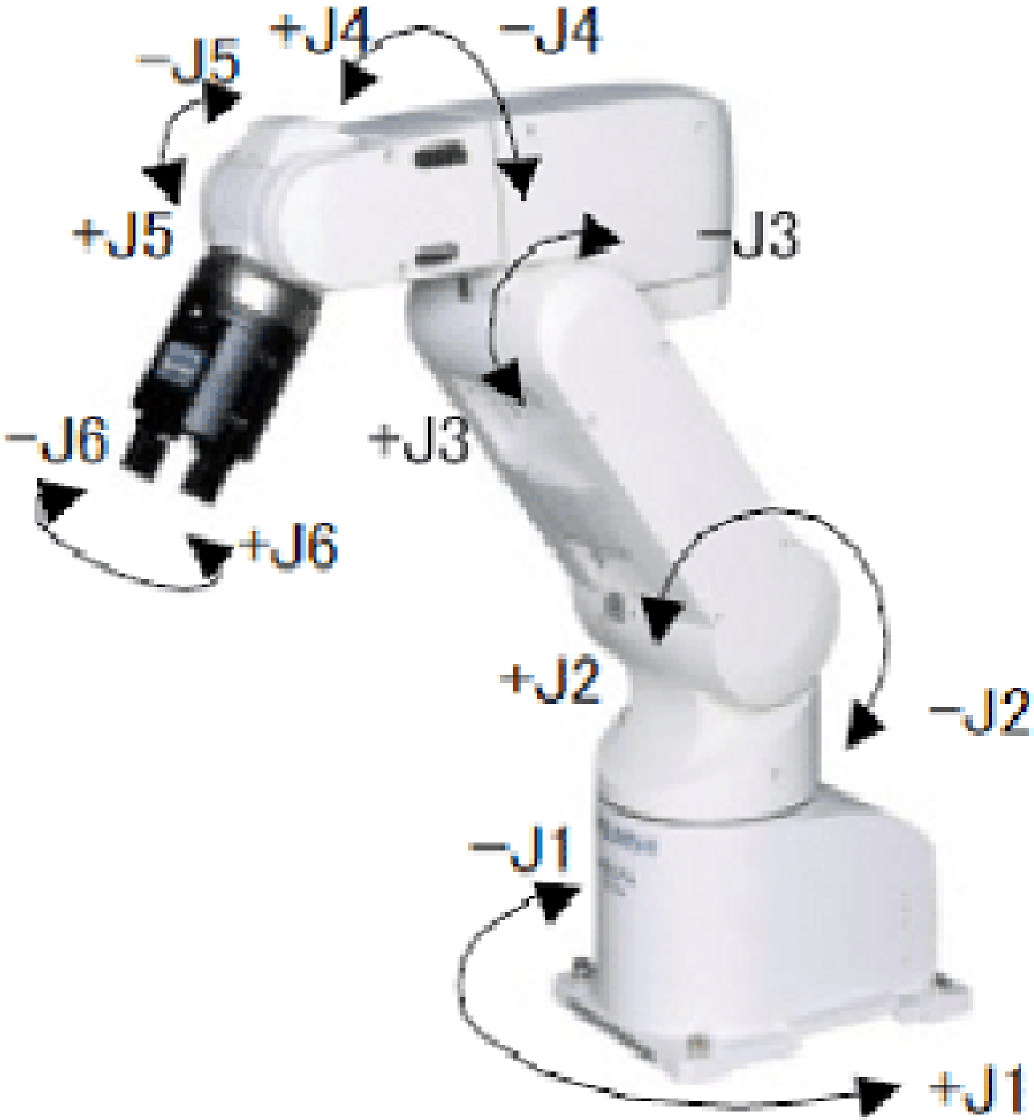
$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$\mathbf{A}_{int}^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_i^{int} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

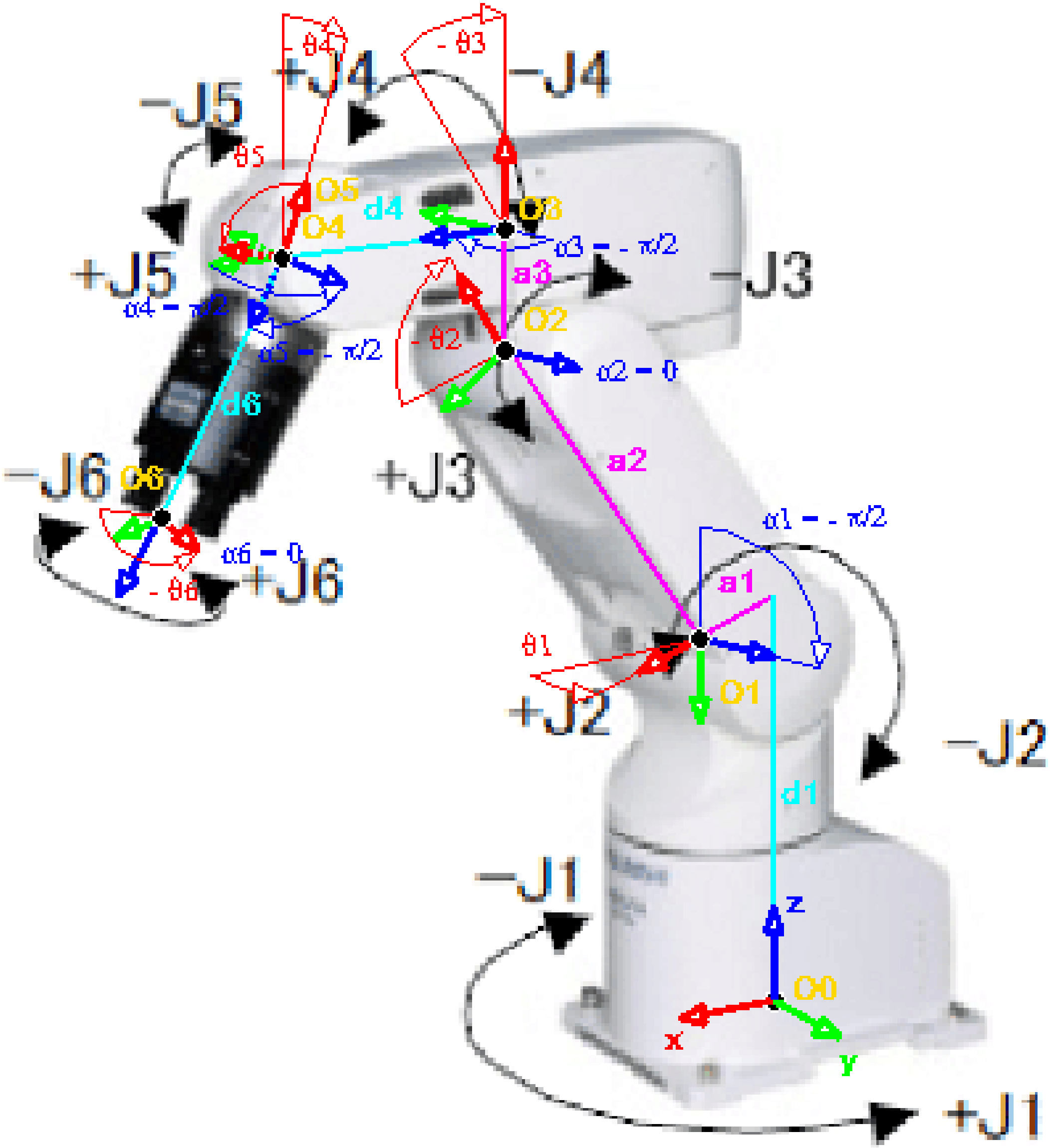
Kinematics of Mitsubishi RV-6S/6SC Robot



RV-6S/6SC

$$\alpha_i \mid a_i \mid \theta_i \mid d_i$$

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

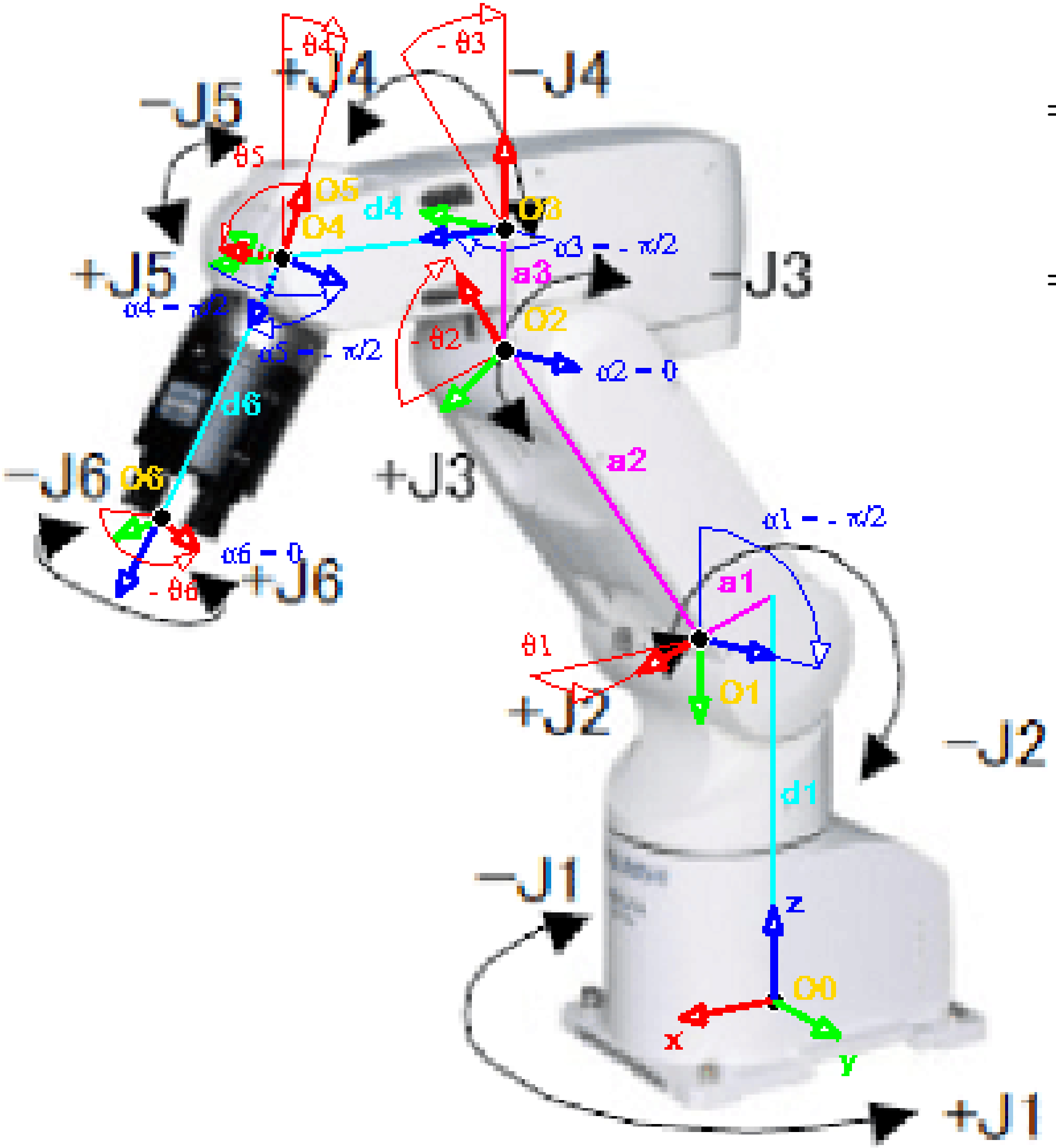


$$X_{i-1} = A_i^{i-1} X_i$$

pajdla@cmp.felk.cvut.cz

A_1^0

α_1	a_1	θ_1	d_1
$-\frac{\pi}{2}$	a_1	θ_1	d_1



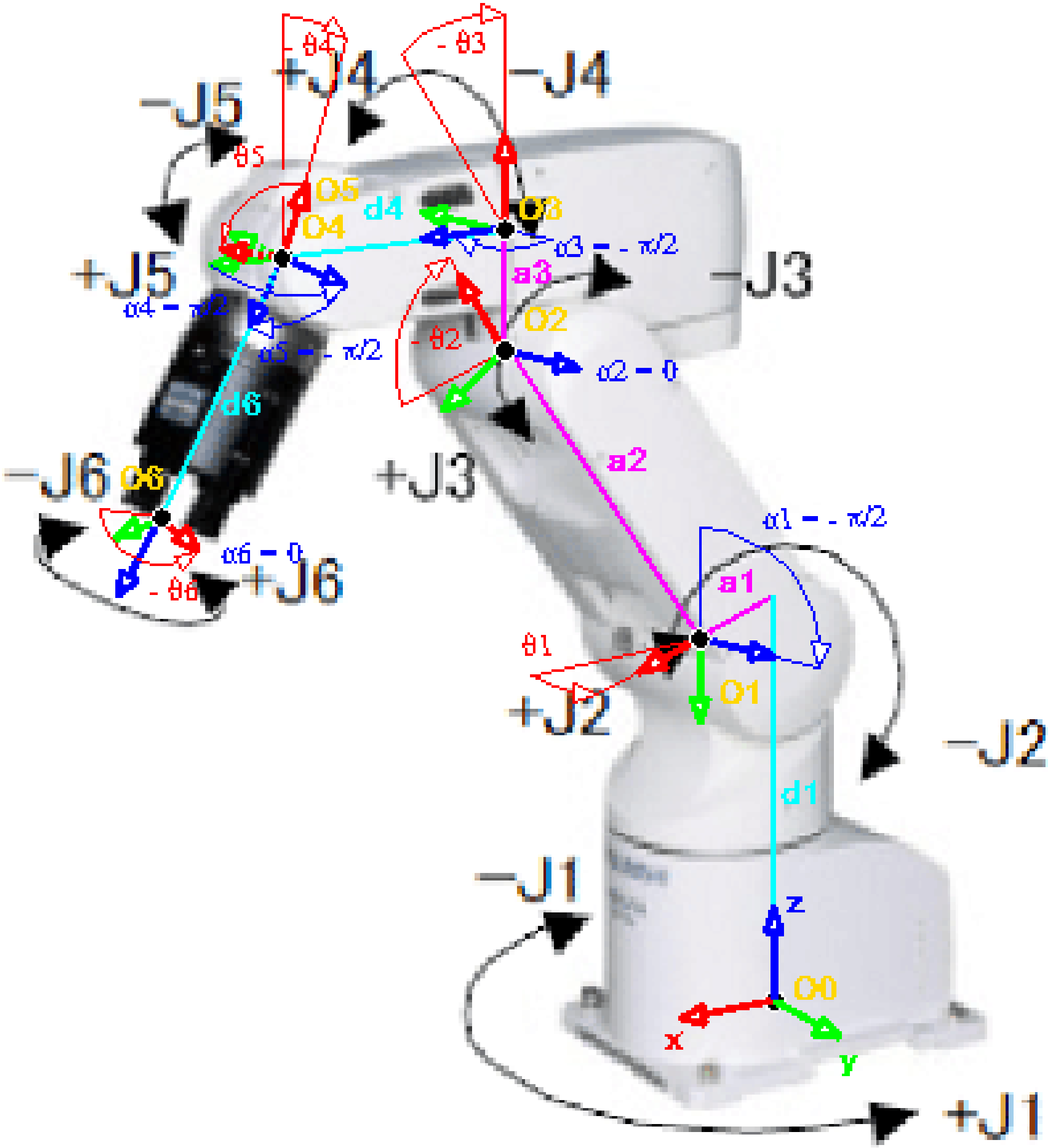
$$A_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos\alpha_1 & \sin\theta_1\sin\alpha_1 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos\alpha_1 & -\cos\theta_1\sin\alpha_1 & a_1\sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos-\pi/2 & \sin\theta_1\sin-\pi/2 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos-\pi/2 & -\cos\theta_1\sin-\pi/2 & a_1\sin\theta_1 \\ 0 & \sin-\pi/2 & \cos-\pi/2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pajdla@cmp.felk.cvut.cz

A_2^1



α_2	a_2	θ_2	d_2
0	a_2	θ_2	0

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\alpha_2 & \sin\theta_2\sin\alpha_2 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos\alpha_2 & -\cos\theta_2\sin\alpha_2 & a_2\sin\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

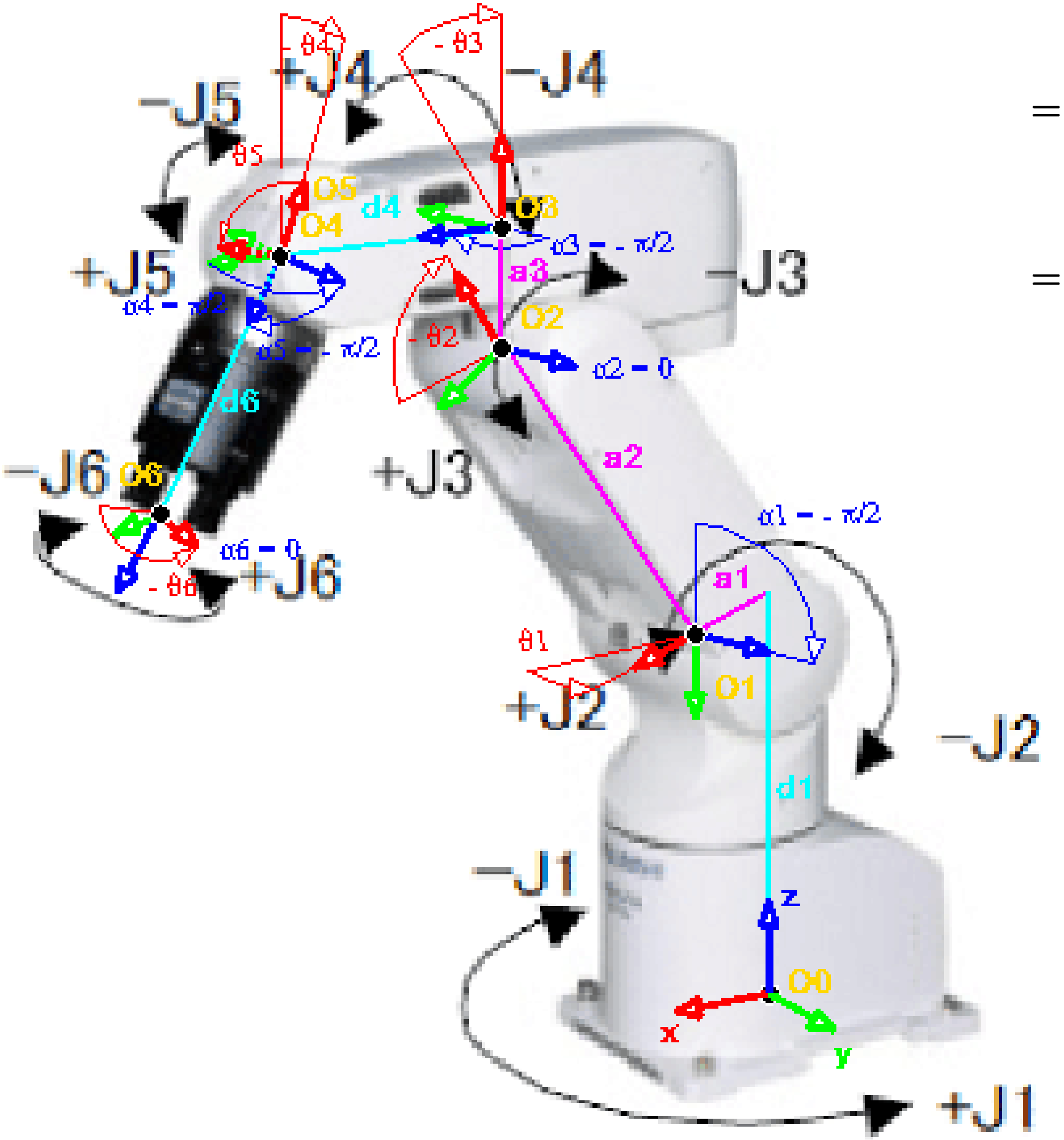
$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos 0 & \sin\theta_2\sin 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos 0 & -\cos\theta_2\sin 0 & a_2\sin\theta_2 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pajdla@cmp.felk.cvut.cz

A_3^2

α_3	a_3	θ_3	d_3
$-\frac{\pi}{2}$	a_3	θ_3	0



$$A_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos\alpha_3 & \sin\theta_3\sin\alpha_3 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos\alpha_3 & -\cos\theta_3\sin\alpha_3 & a_3\sin\theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos-\pi/2 & \sin\theta_3\sin-\pi/2 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos-\pi/2 & -\cos\theta_3\sin-\pi/2 & a_3\sin\theta_3 \\ 0 & \sin-\pi/2 & \cos-\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pajdla@cmp.felk.cvut.cz

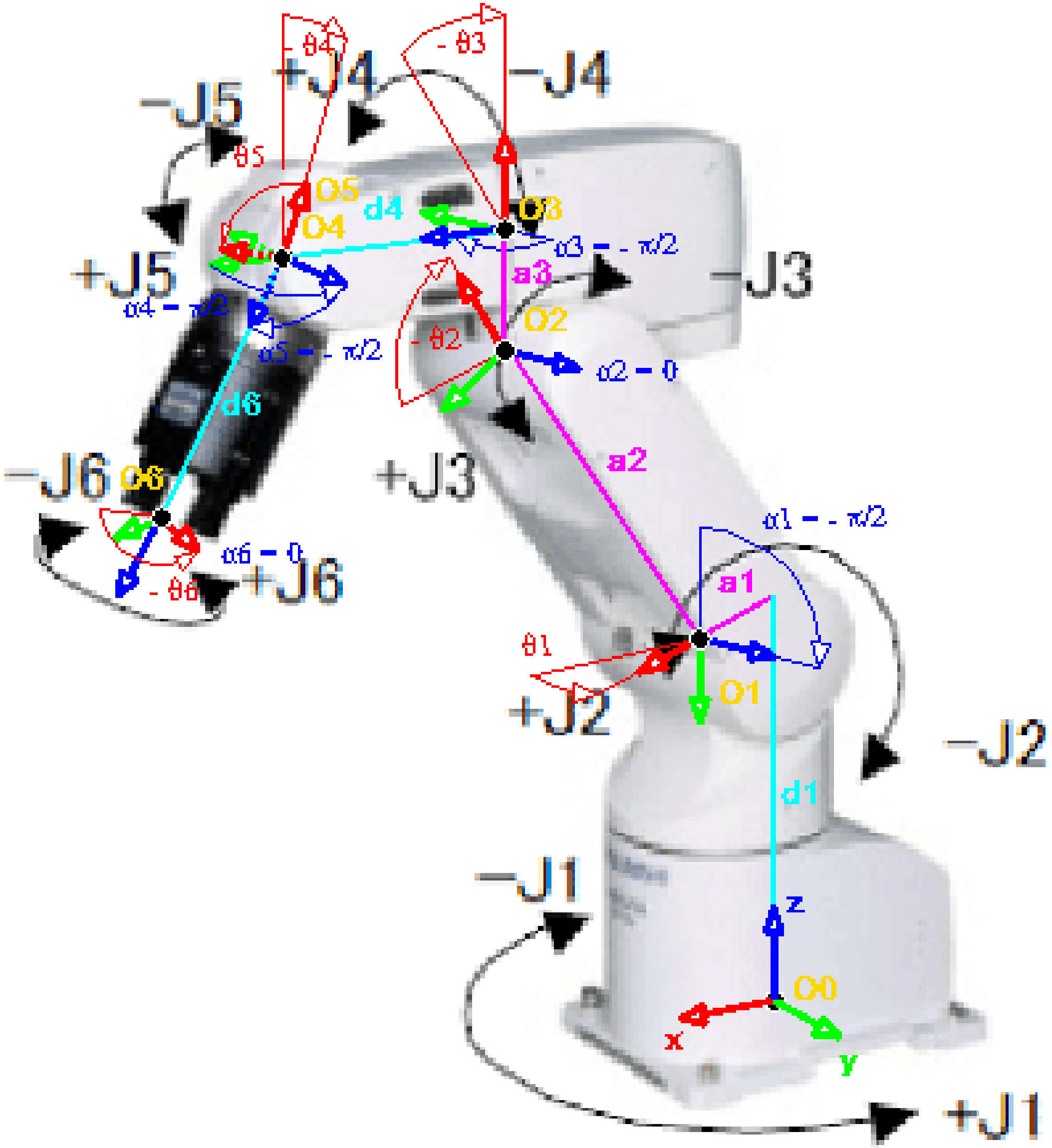
A_4^3

α_4	a_4	θ_4	d_4
$\frac{\pi}{2}$	0	θ_4	d_4

$$A_4^3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4\cos\alpha_4 & \sin\theta_4\sin\alpha_4 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4\cos\alpha_4 & -\cos\theta_4\sin\alpha_4 & a_4\sin\theta_4 \\ 0 & \sin\alpha_4 & \cos\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_4 & -\sin\theta_4\cos\pi/2 & \sin\theta_4\sin\pi/2 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4\cos\pi/2 & -\cos\theta_4\sin\pi/2 & a_4\sin\theta_4 \\ 0 & \sin\pi/2 & \cos\pi/2 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

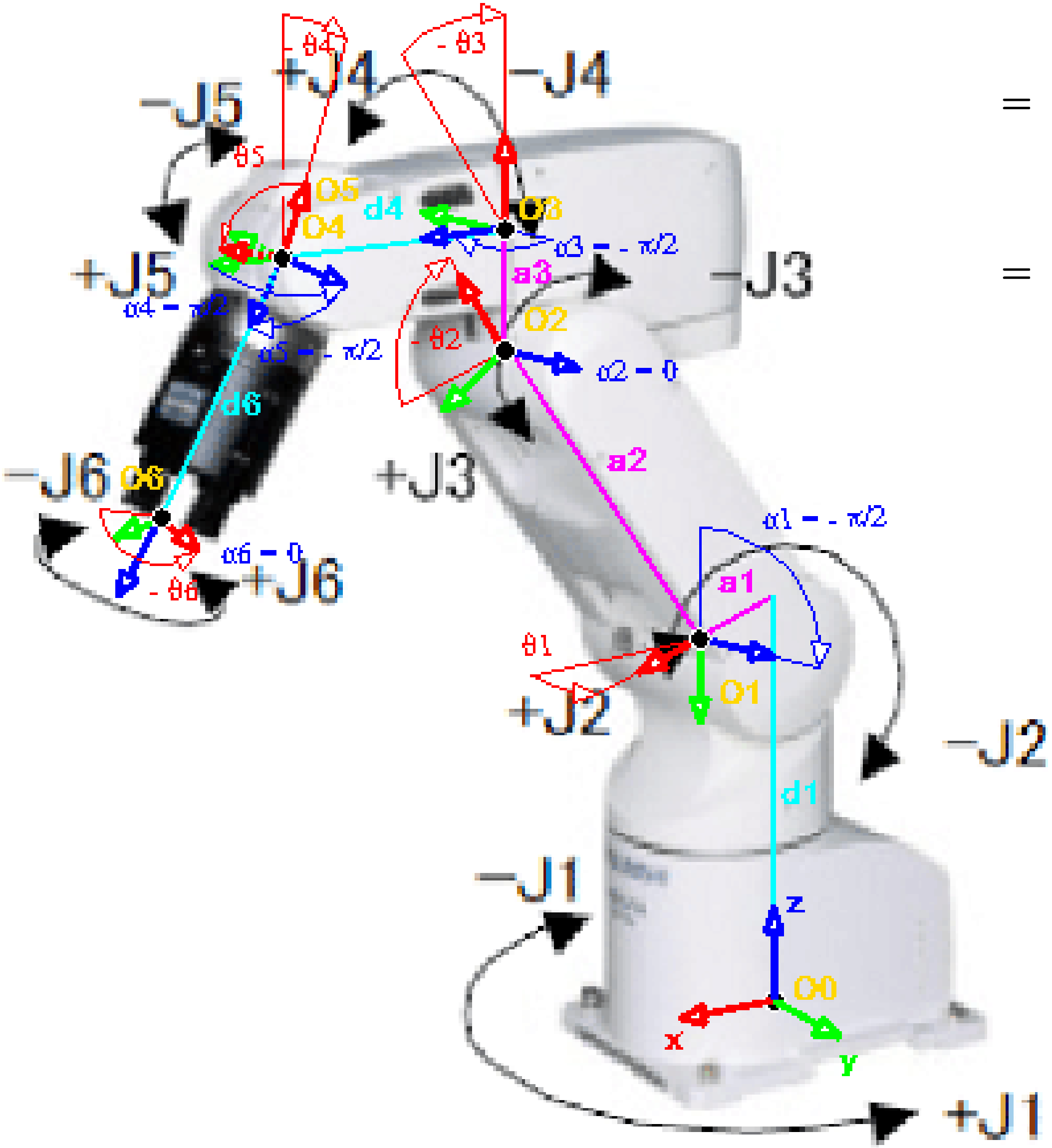
$$= \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



pajdla@cmp.felk.cvut.cz

A_5^4

α_5	a_5	θ_5	d_5
$\frac{-\pi}{2}$	0	θ_5	0



$$A_5^4 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5\cos\alpha_5 & \sin\theta_5\sin\alpha_5 & a_5\cos\alpha_5 \\ \sin\theta_5 & \cos\theta_5\cos\alpha_5 & -\cos\theta_5\sin\alpha_5 & a_5\sin\alpha_5 \\ 0 & \sin\alpha_5 & \cos\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_5 & -\sin\theta_5\cos-\pi/2 & \sin\theta_5\sin-\pi/2 & 0\cos-\pi/2 \\ \sin\theta_5 & \cos\theta_5\cos-\pi/2 & -\cos\theta_5\sin-\pi/2 & 0\sin-\pi/2 \\ 0 & \sin-\pi/2 & \cos-\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pajdla@cmp.felk.cvut.cz

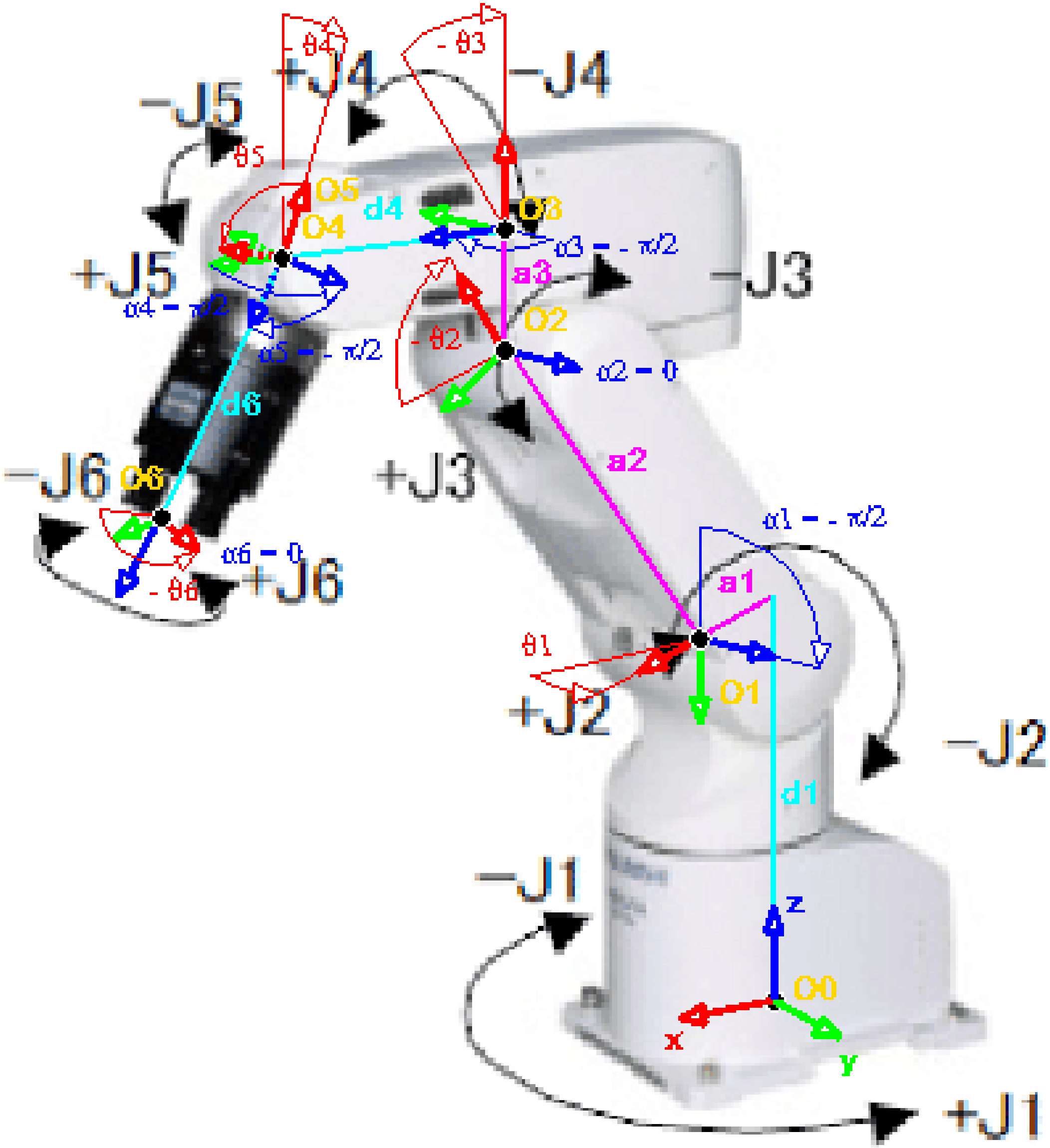
A_6^5

α_6	a_6	θ_6	d_6
0	0	θ_6	d_6

$$A_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6\cos\alpha_6 & \sin\theta_6\sin\alpha_6 & a_6\cos\alpha_6 \\ \sin\theta_6 & \cos\theta_6\cos\alpha_6 & -\cos\theta_6\sin\alpha_6 & a_6\sin\alpha_6 \\ 0 & \sin\alpha_6 & \cos\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

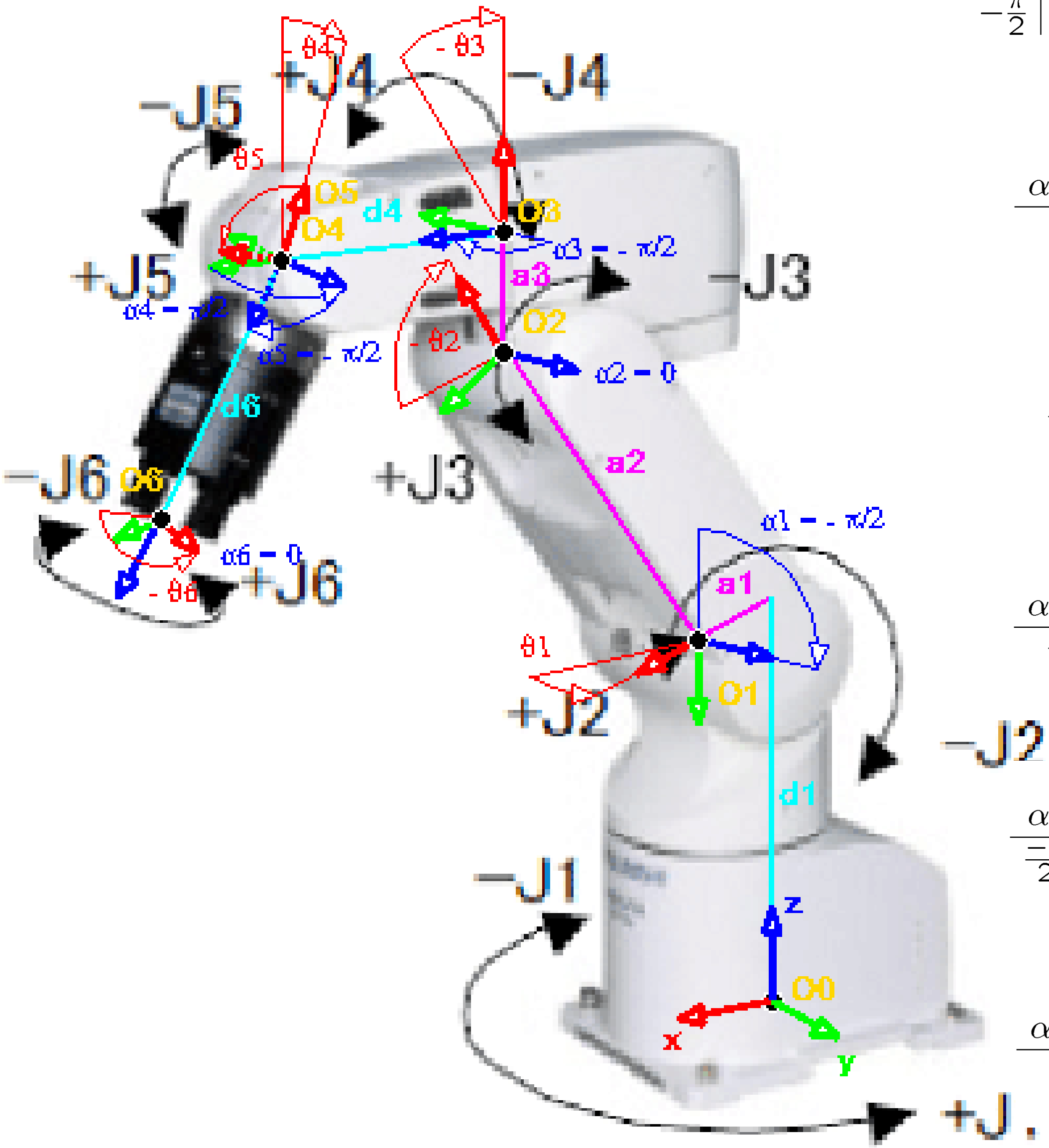
$$= \begin{bmatrix} \cos\theta_6 & -\sin\theta_6\cos 0 & \sin\theta_6\sin 0 & 0\cos 0 \\ \sin\theta_6 & \cos\theta_6\cos 0 & -\cos\theta_6\sin 0 & 0\sin 0 \\ 0 & \sin 0 & \cos 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



pajdla@cmp.felk.cvut.cz

$$G = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$



α_1	a_1	θ_1	d_1
$-\frac{\pi}{2}$	a_1	θ_1	d_1

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1 cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1 sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_2	a_2	θ_2	d_2
0	a_2	θ_2	0

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_3	a_3	θ_3	d_3
$-\frac{\pi}{2}$	a_3	θ_3	0

$$A_3^2 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_4	a_4	θ_4	d_4
$\frac{\pi}{2}$	0	θ_4	d_4

$$A_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α_5	a_5	θ_5	d_5
$\frac{-\pi}{2}$	0	θ_5	0

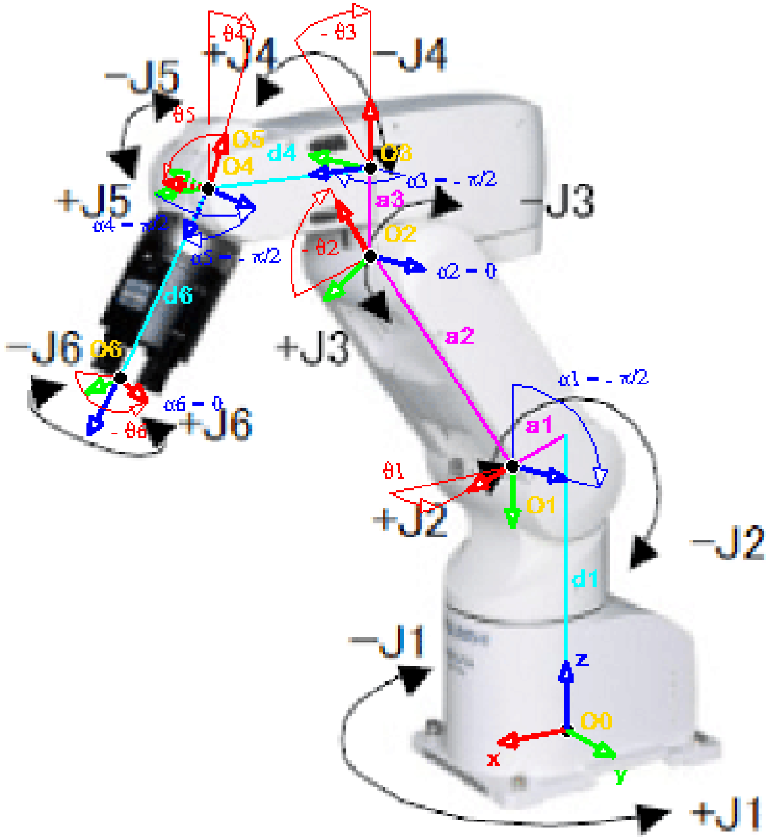
$$A_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

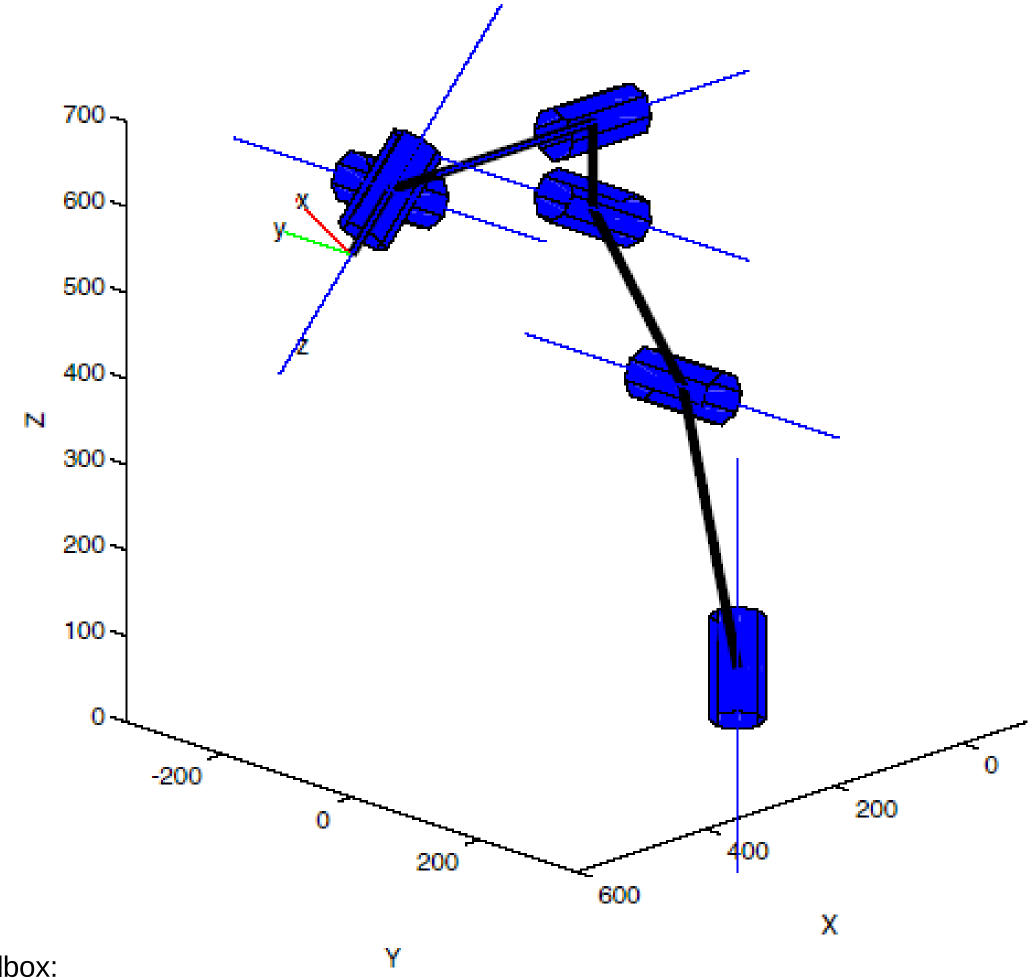
α_6	a_6	θ_6	d_6
0	0	θ_6	d_6

$$A_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pajdla@cmp.felk.cvut.cz

Robot Description — Matlab Robotic Toolbox



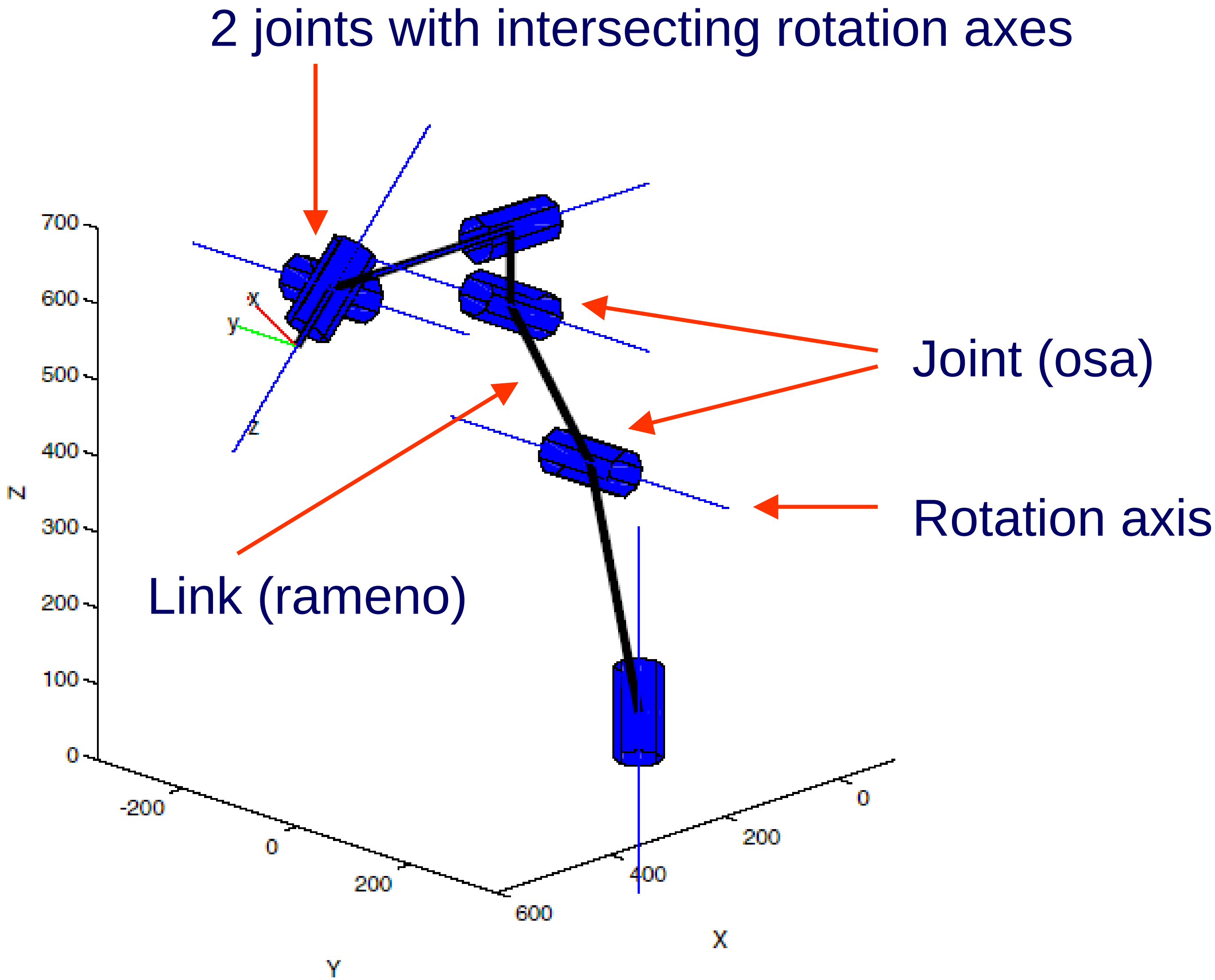


MATLAB simulation in ROBOT toolbox:

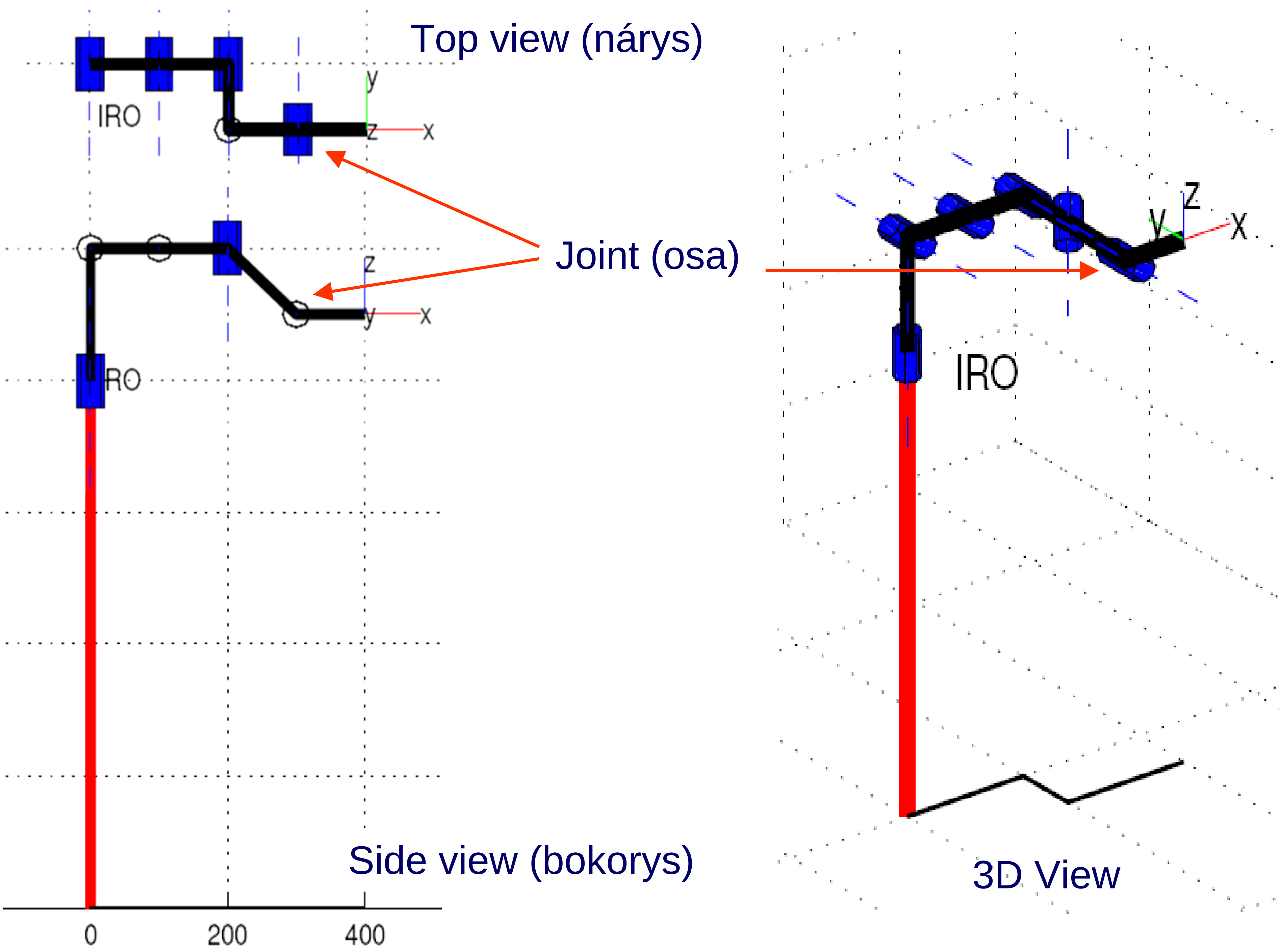
mRV6S =

RV-6S (6 axis, RRRRRR) [Mitsubishi] <home = [0.000000 -1.047198 -0.523599 0.000000 0.785398 0.000000]>
grav = [0.00 0.00 9.81]

alpha_i	a_i	theta_i	d_i	R/P	standard D&H parameters
-1.570796	85.000	parameter	350.000	R	(std)
0.000000	280.000	parameter	0.000	R	(std)
-1.570796	100.000	parameter	0.000	R	(std)
1.570796	0.000	parameter	315.000	R	(std)
-1.570796	0.000	parameter	0.000	R	(std)
0.000000	0.000	parameter	85.000	R	(std)


2 joints with intersecting rotation axes
Joint (osa)
Rotation axis
Link (rameno)
700
600
500
400
300
200
100
0
Z
x
y
z
-200
0
200
Y
0
200
400
600
X

Robot Description — Drawings



 pajdla@cmp.felk.cvut.cz