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k-th order Voronoi diagram

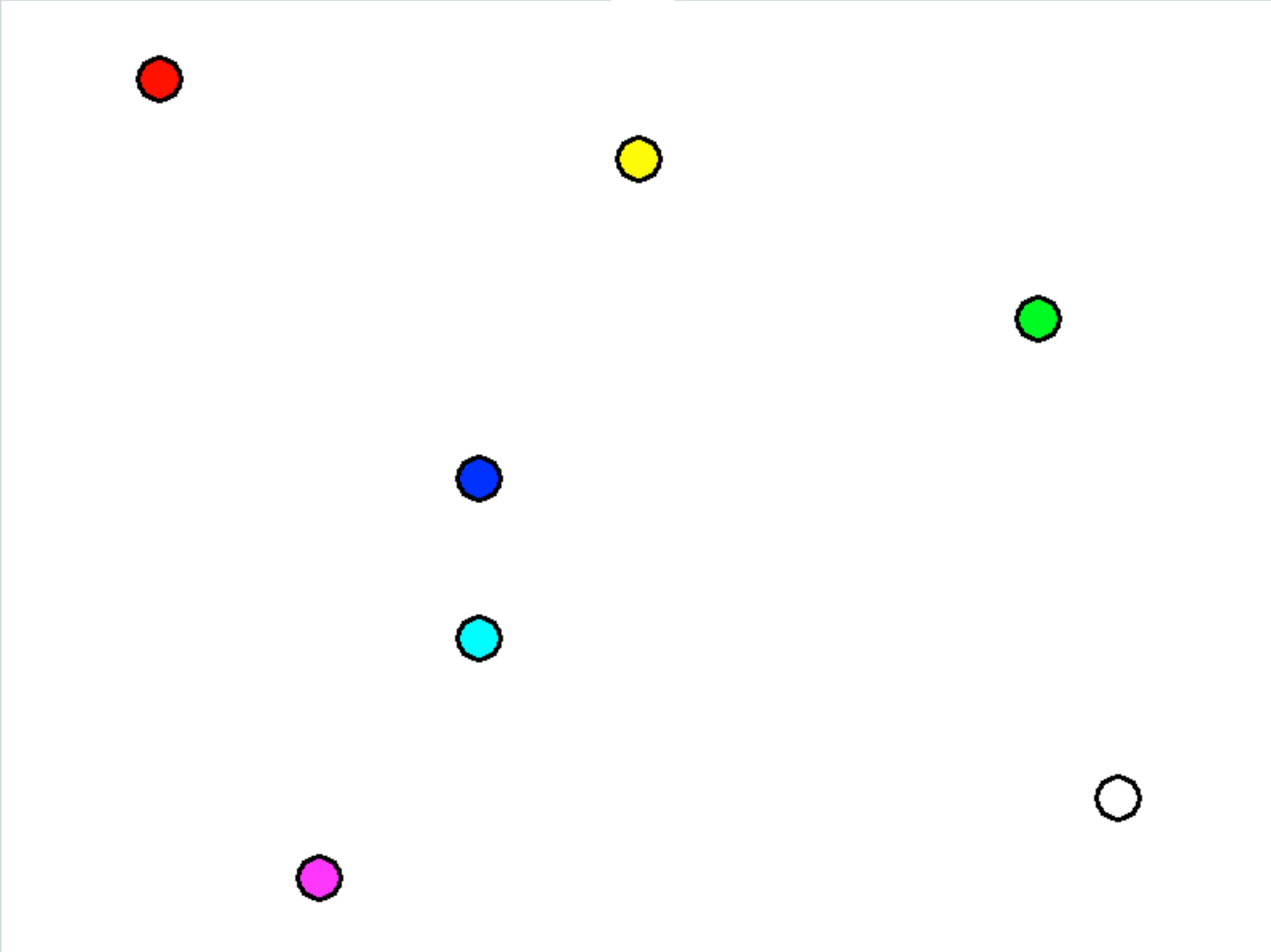
ADAM POSPÍŠIL
CTU FEE

Quick recapitulation

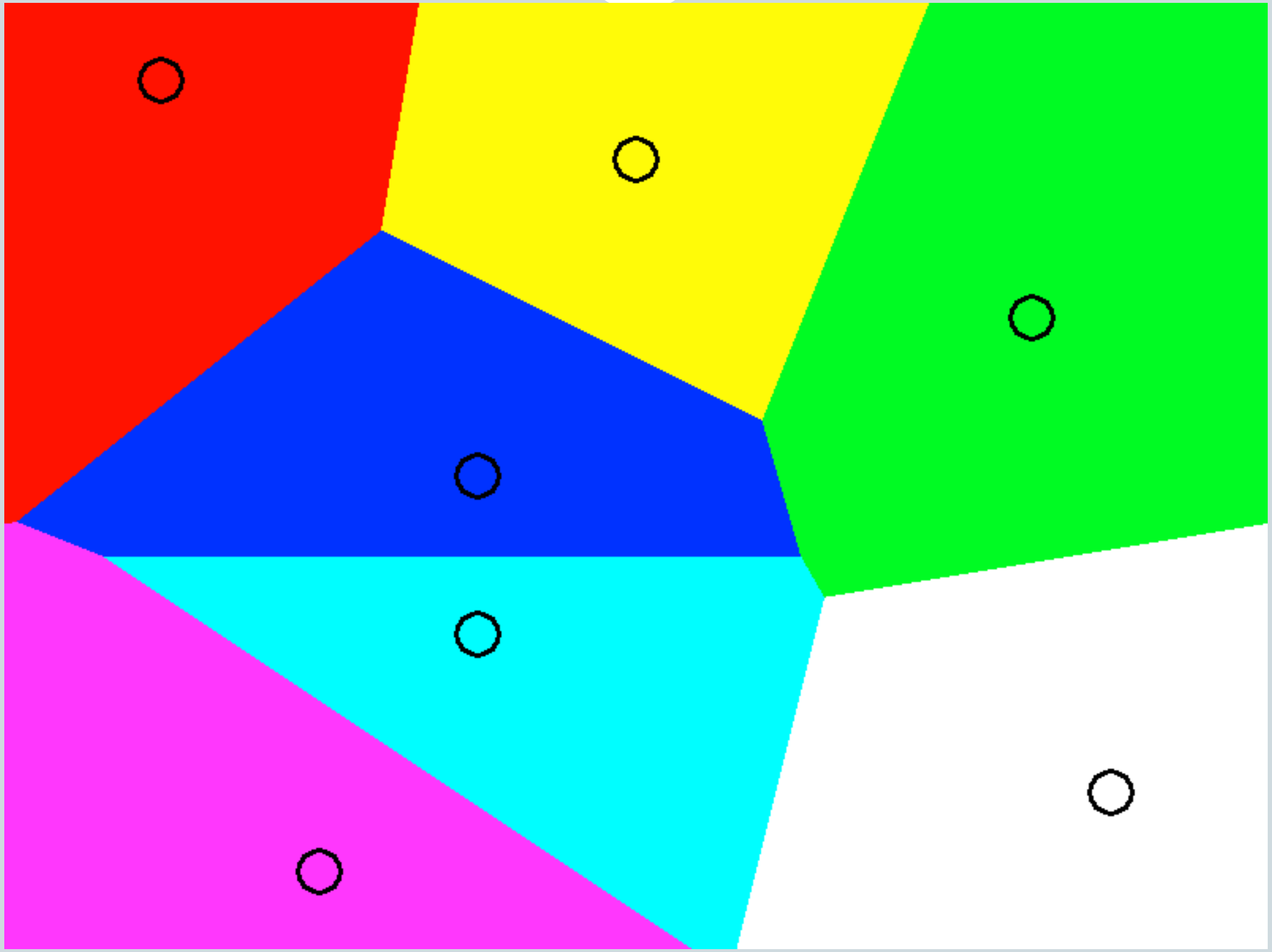
- sites
- Euclidean metrics
- circles
- convex hull

- Disclaimer: „Colours are for your convenience only!“

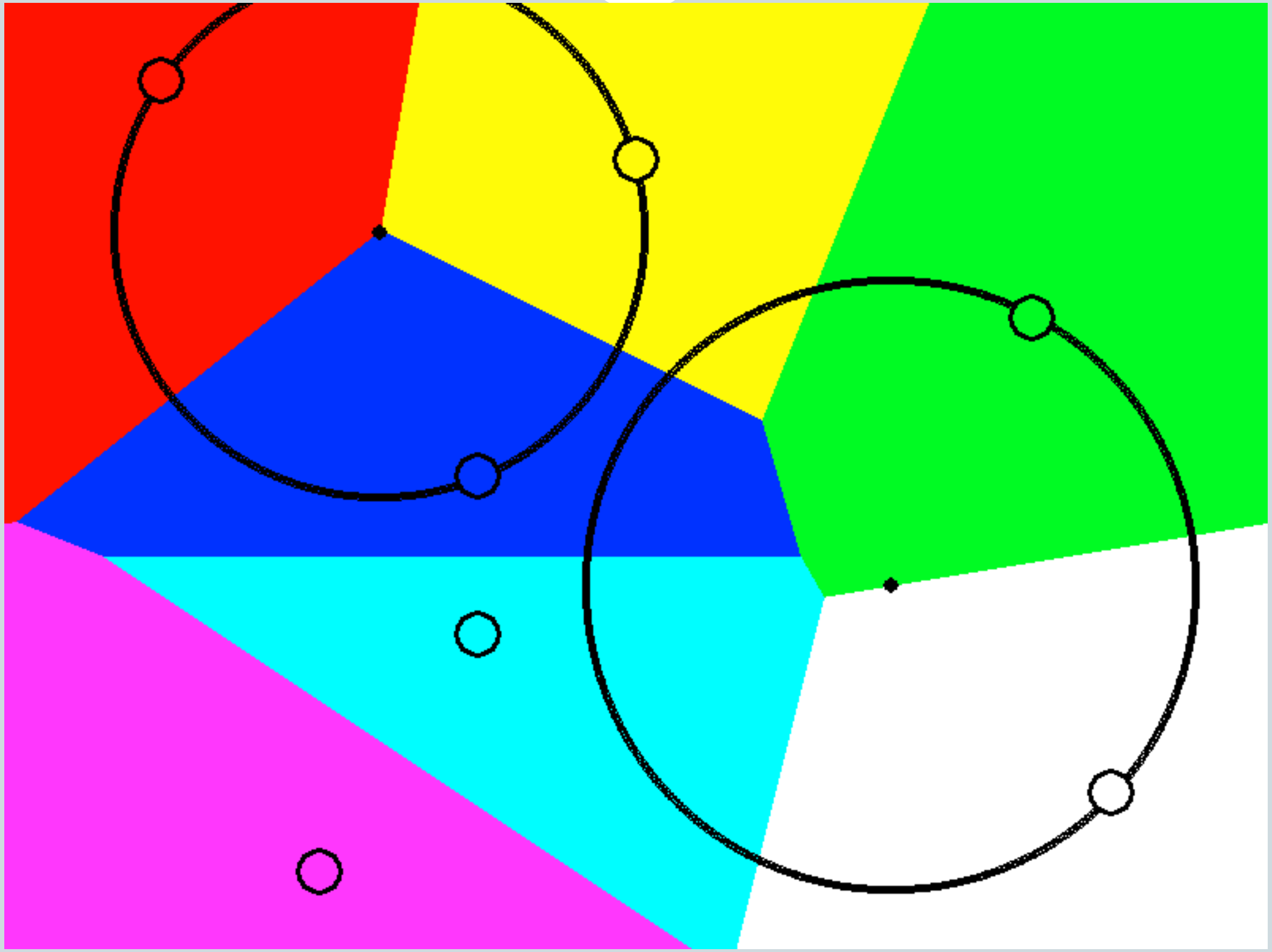
Sites



Voronoi diagram



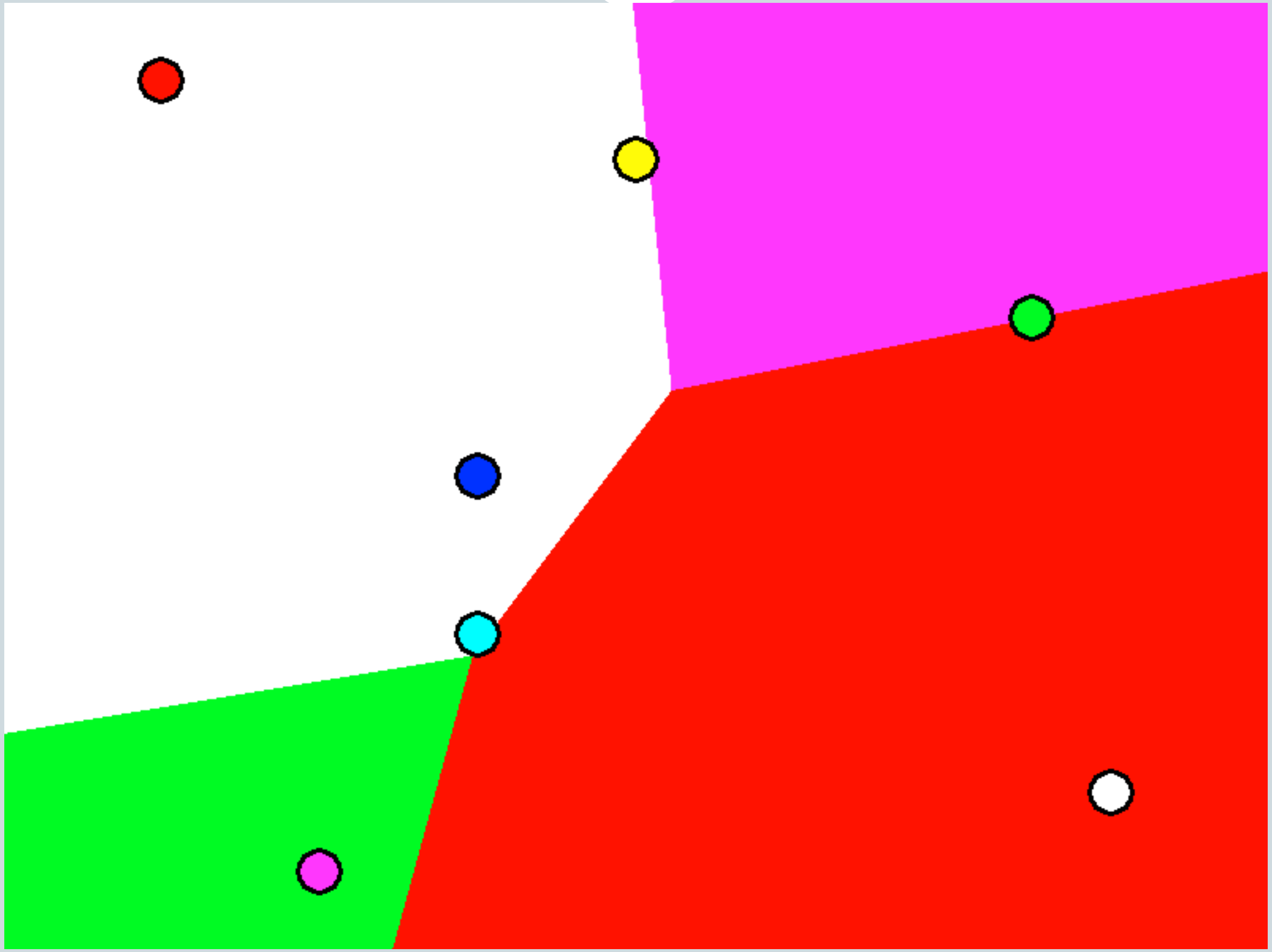
(Empty) Circles



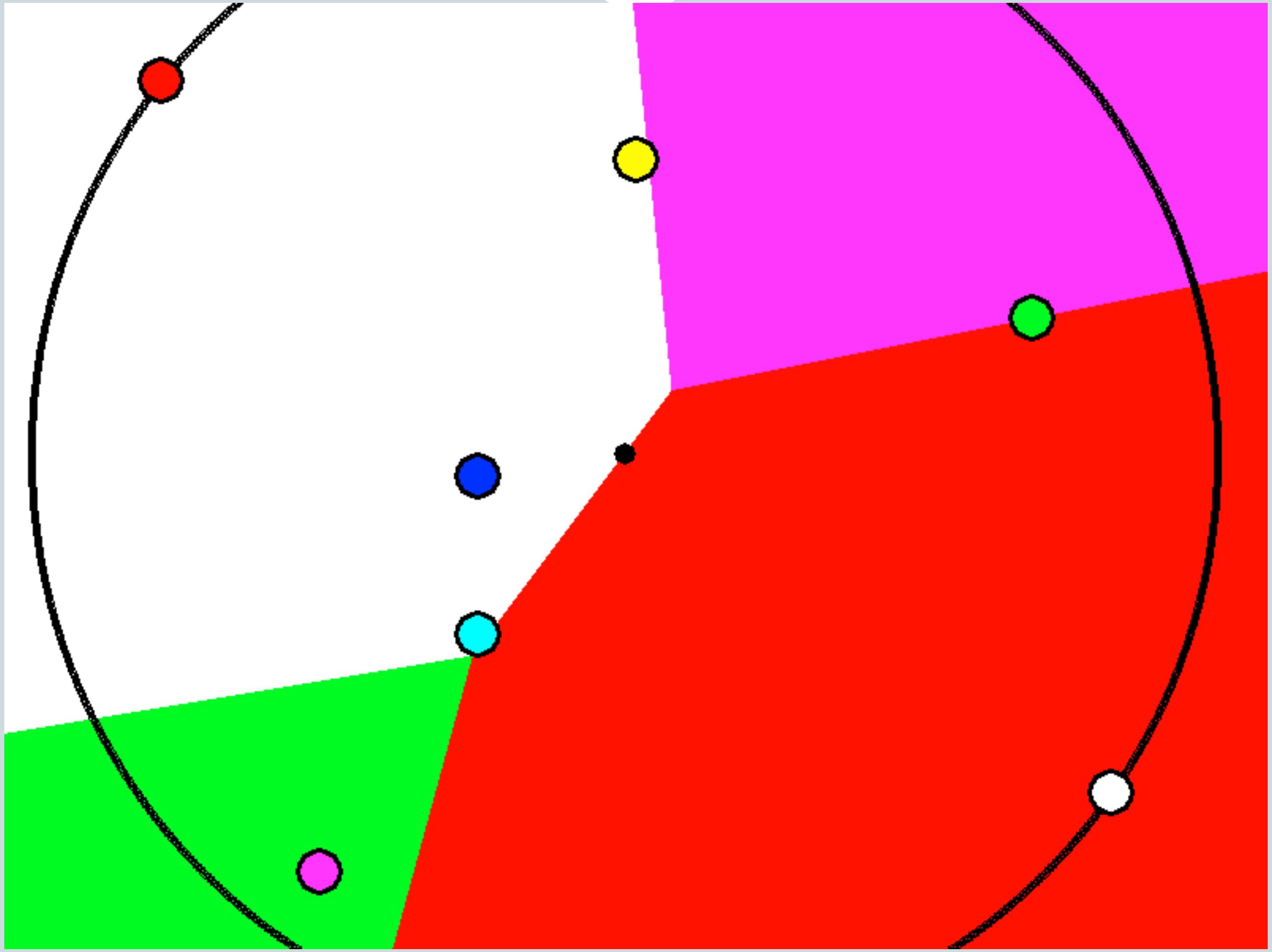
What is k-th order VD?

- k-th order = k-th nearest site
- (nearest point) VD = first order VD

Farthest point = N-th order VD



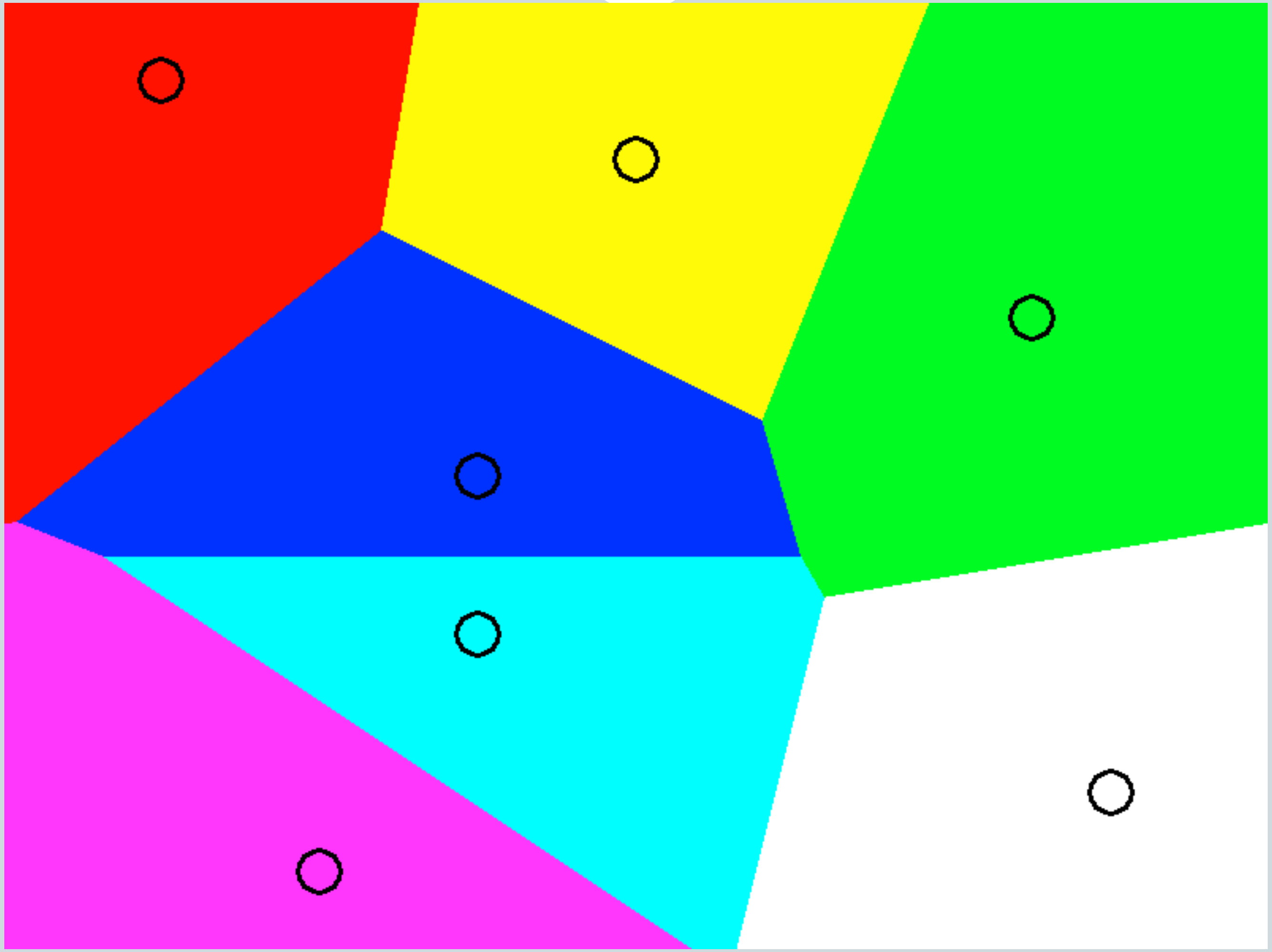
Bounding circle



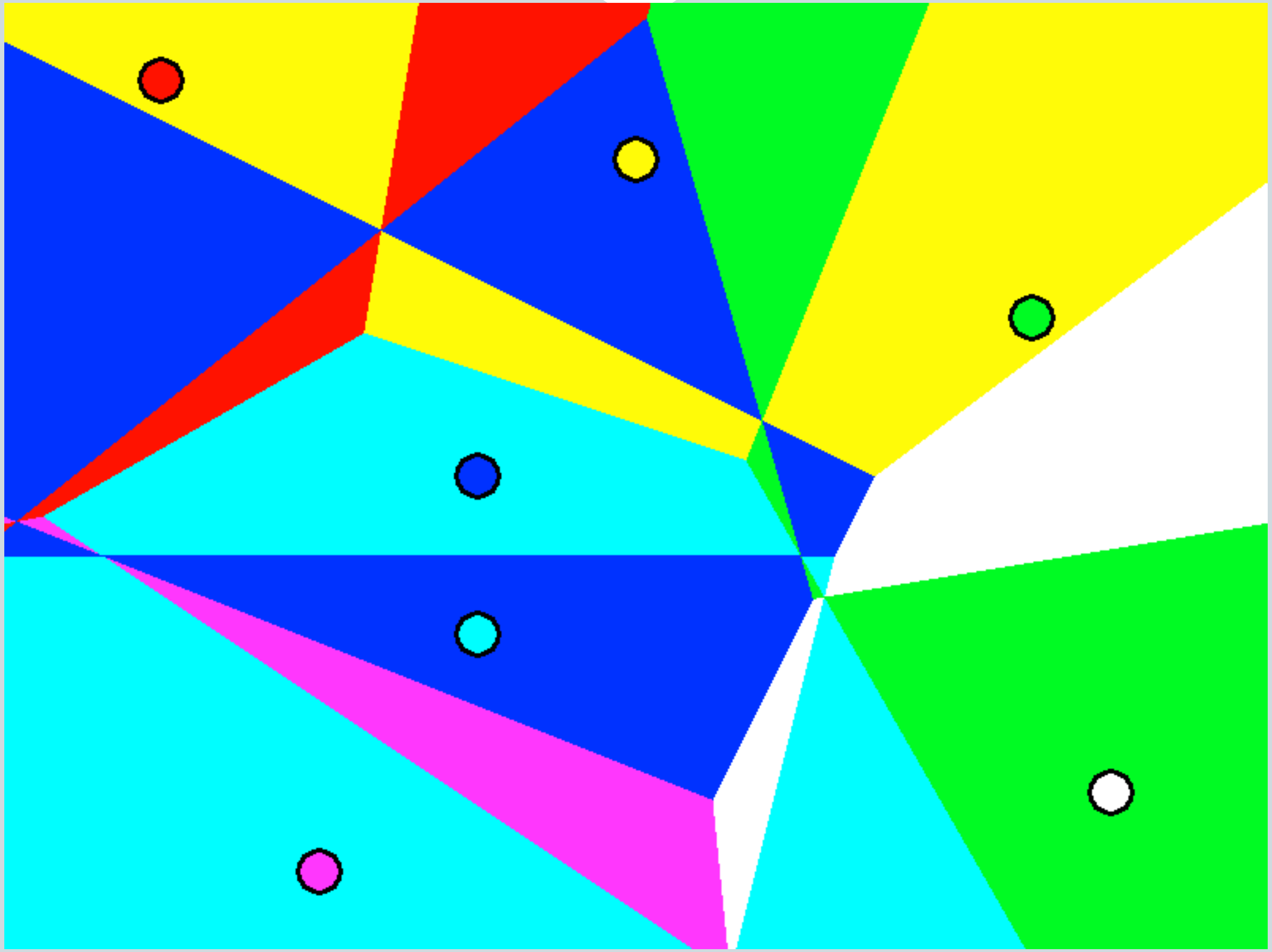
What about the others

- Example
- k-th order
- Generalized Voronoi polygon
  - V(T) T is a subset of S and has a size k
- lets have a look for a while first…

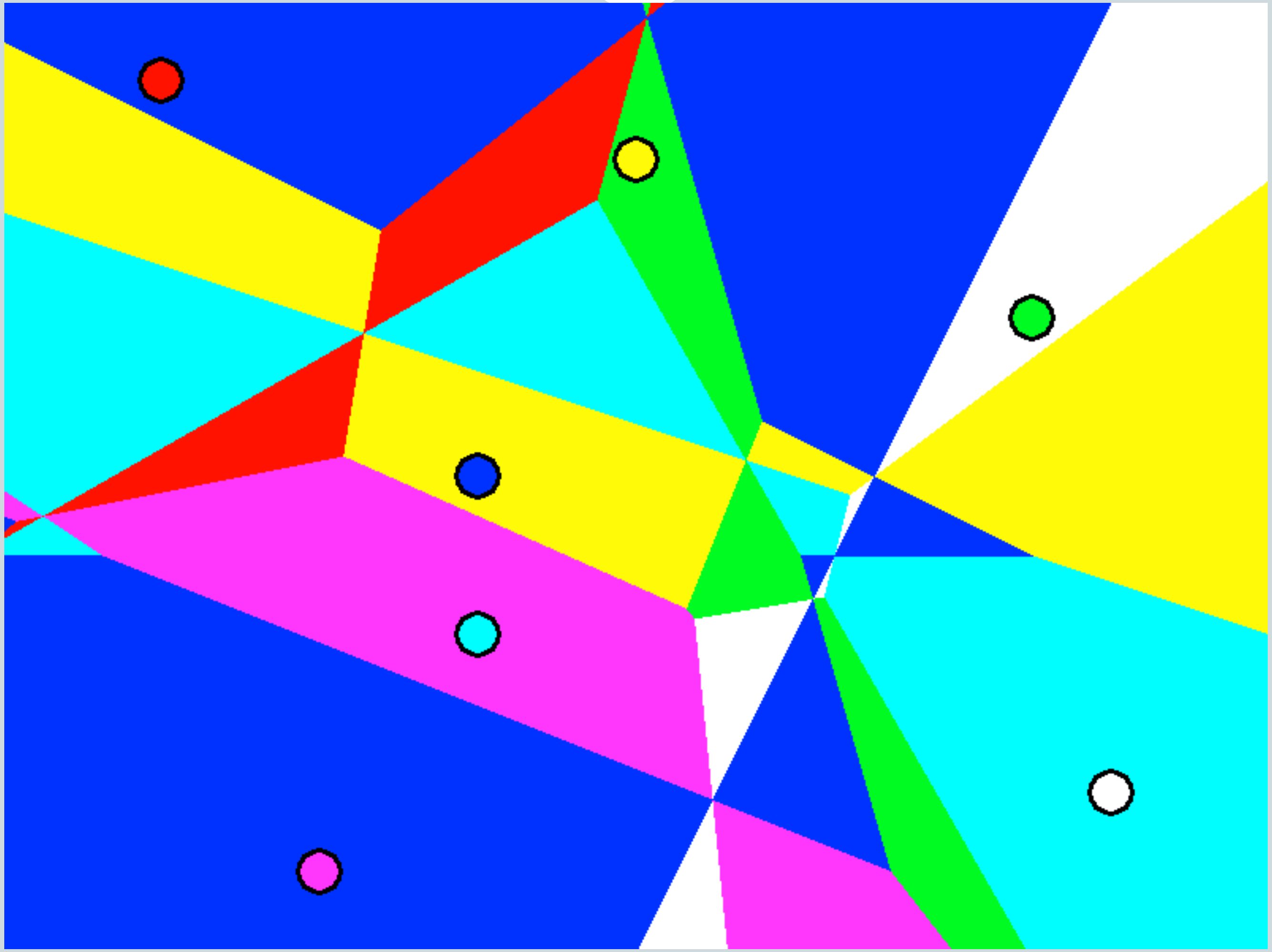
1st order VD



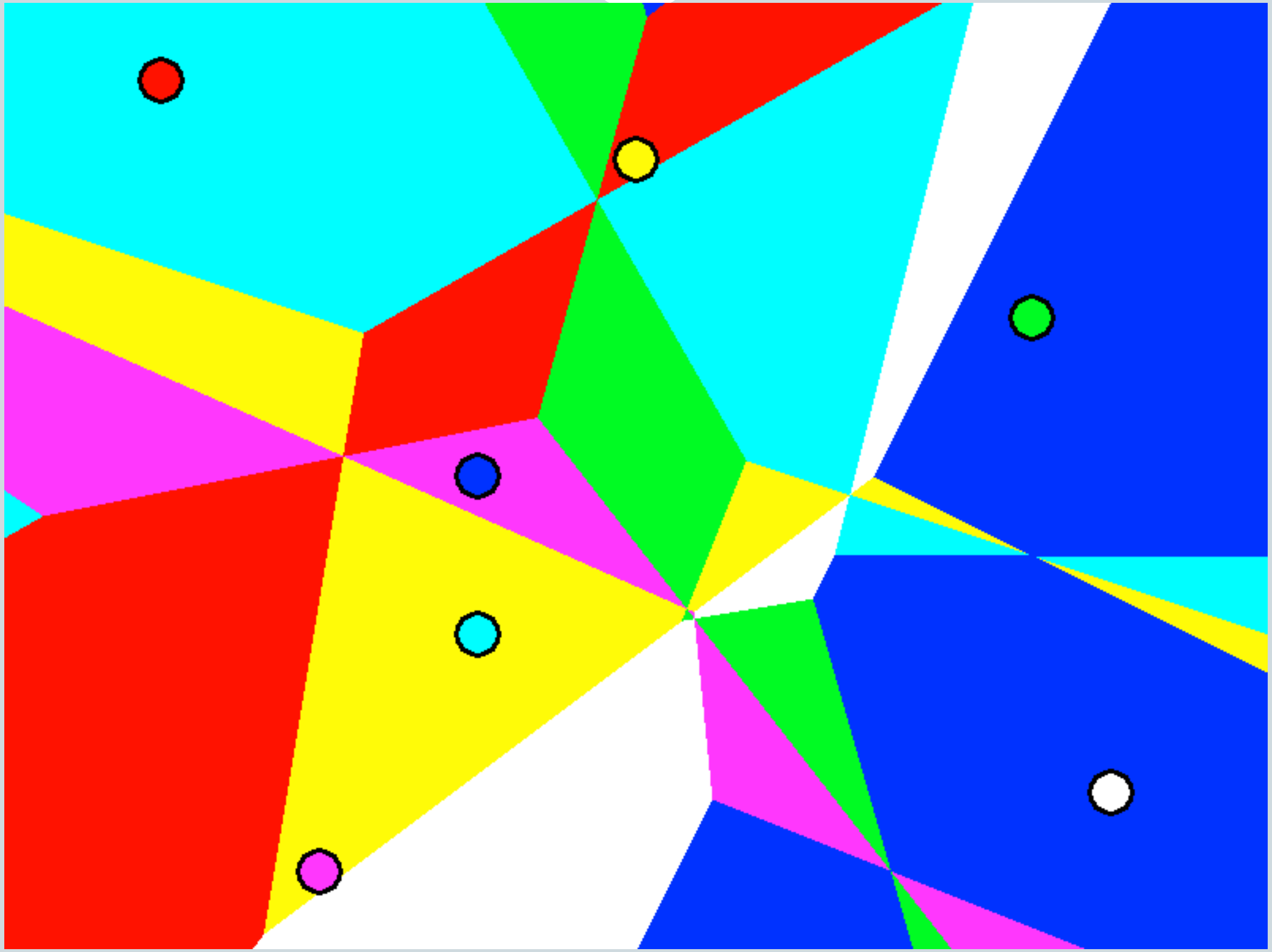
2nd order VD



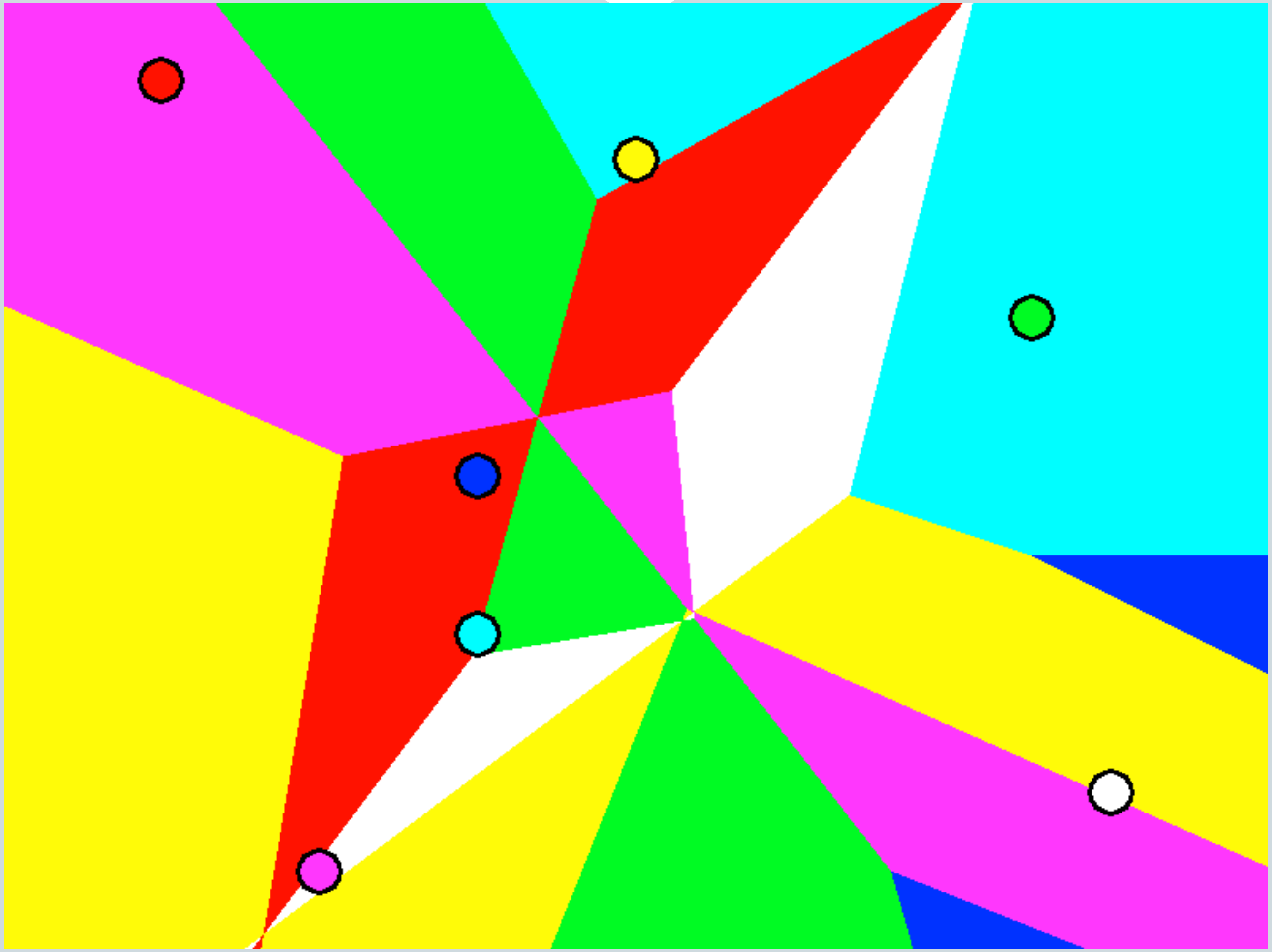
3rd order VD



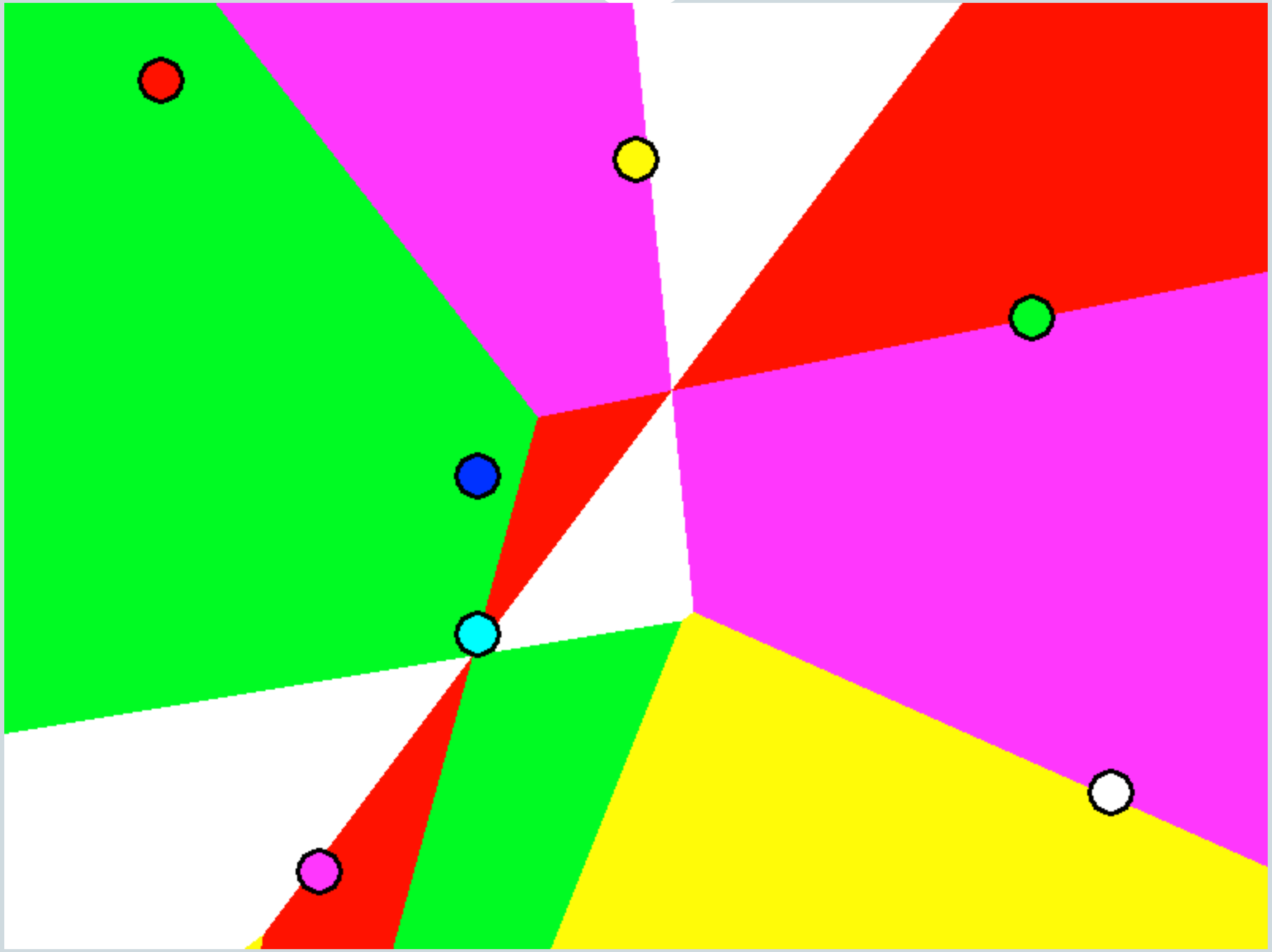
4th order VD



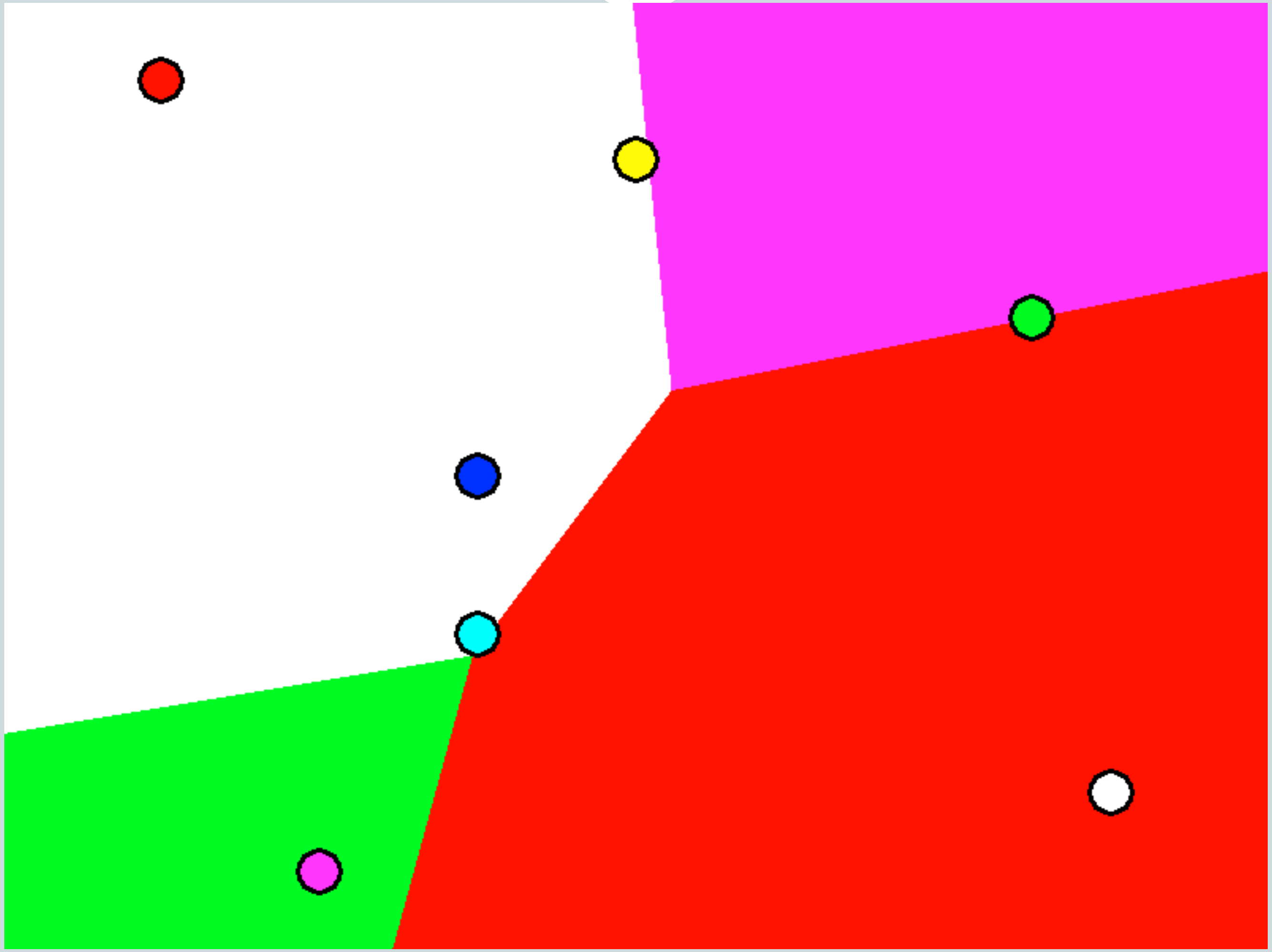
5th order VD



6th order VD



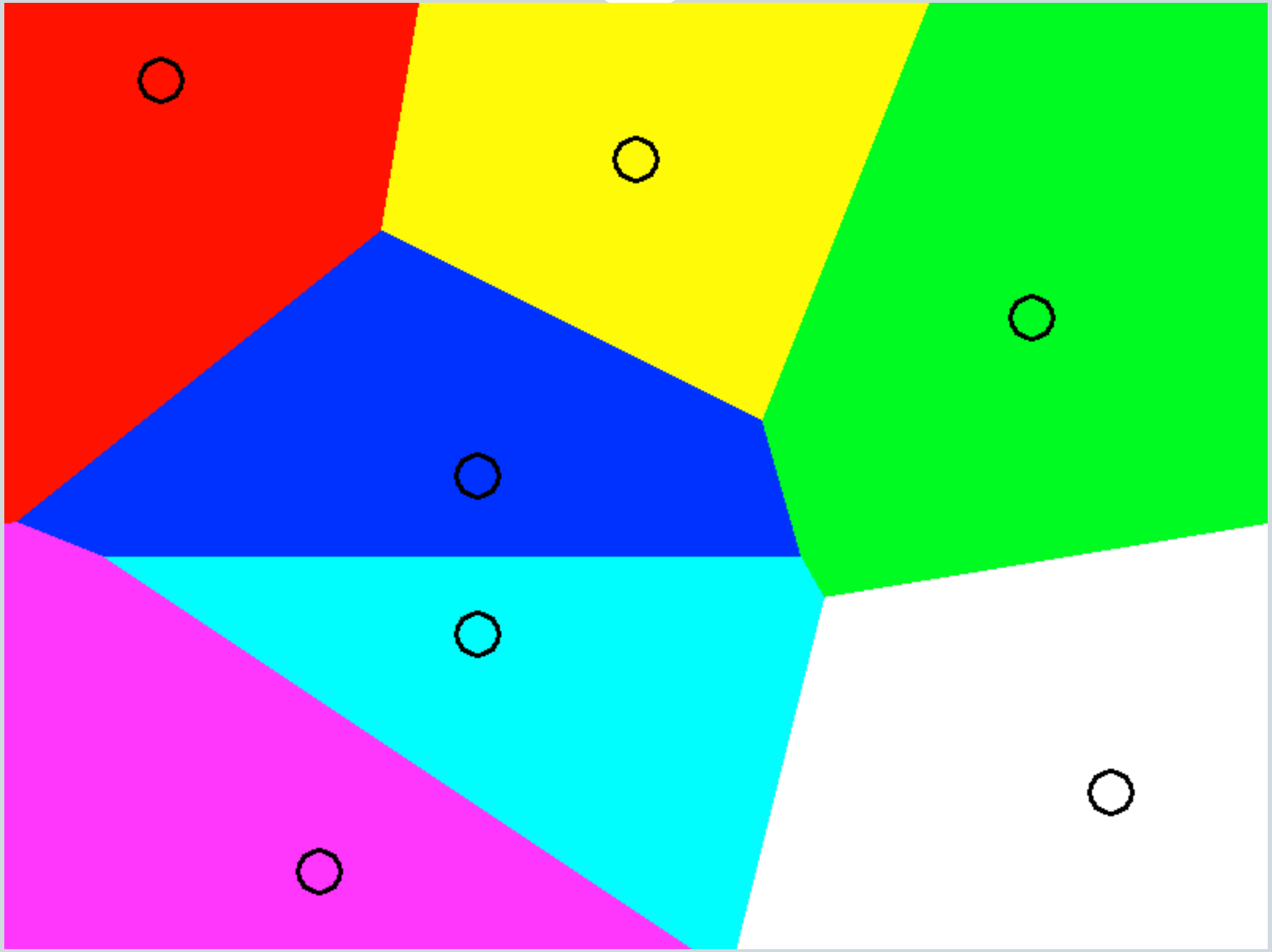
7th order VD



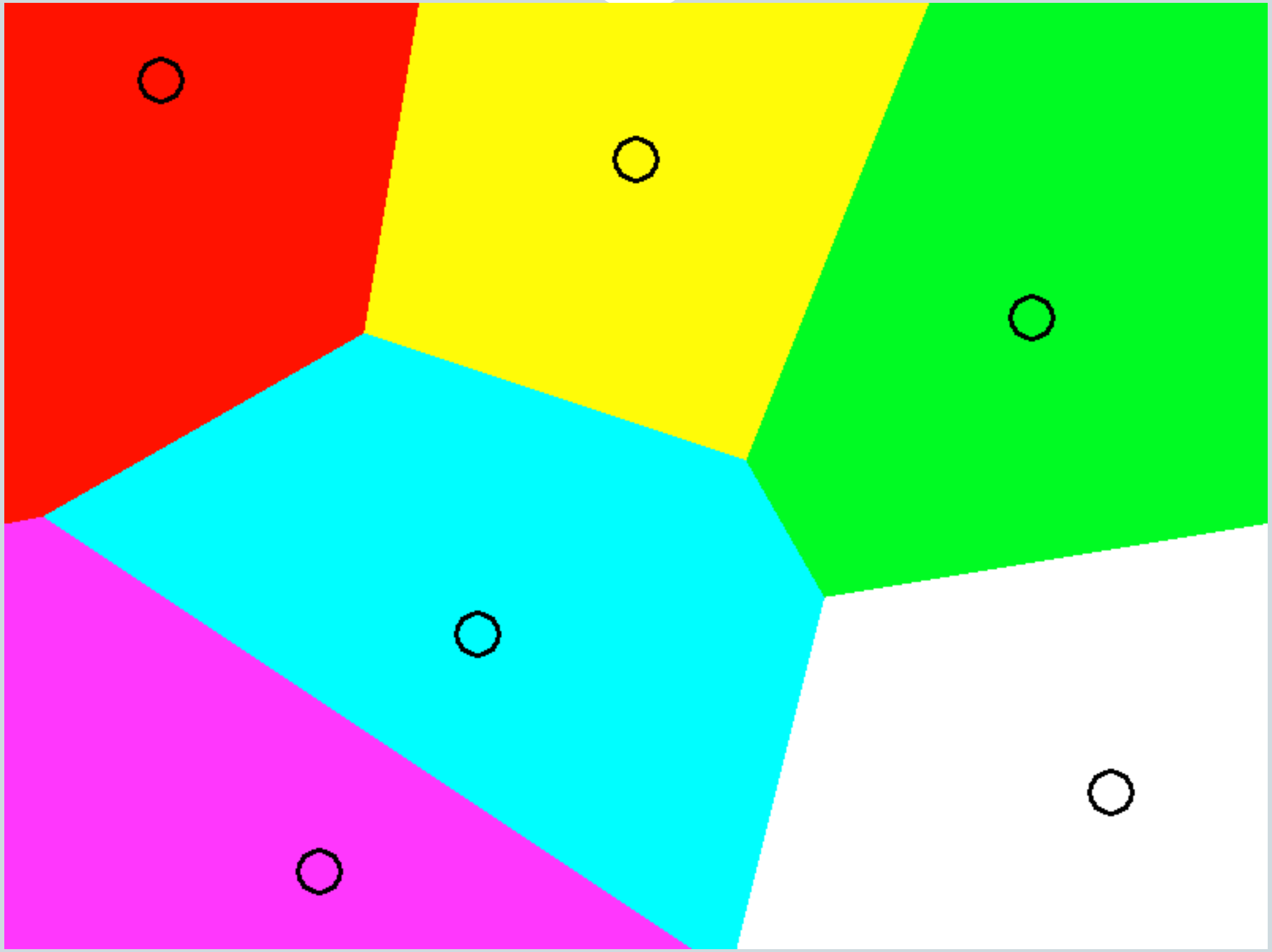
Interesting things

- „Vertex stays for 3 orders“
- „Edge stays for 2 orders“

Take a V(T); |T|=k…



partition it as if Vor_1(S-T)...



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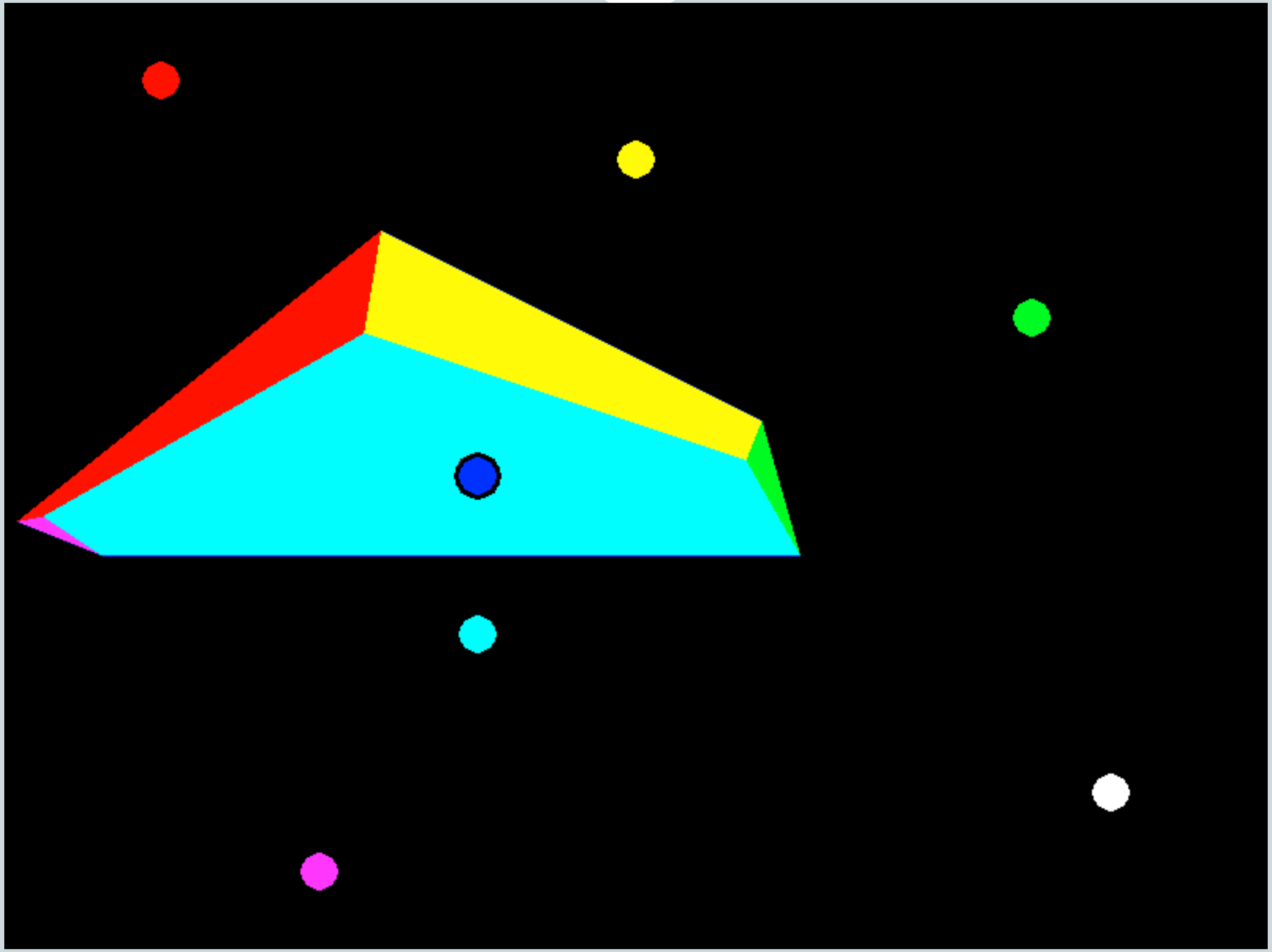
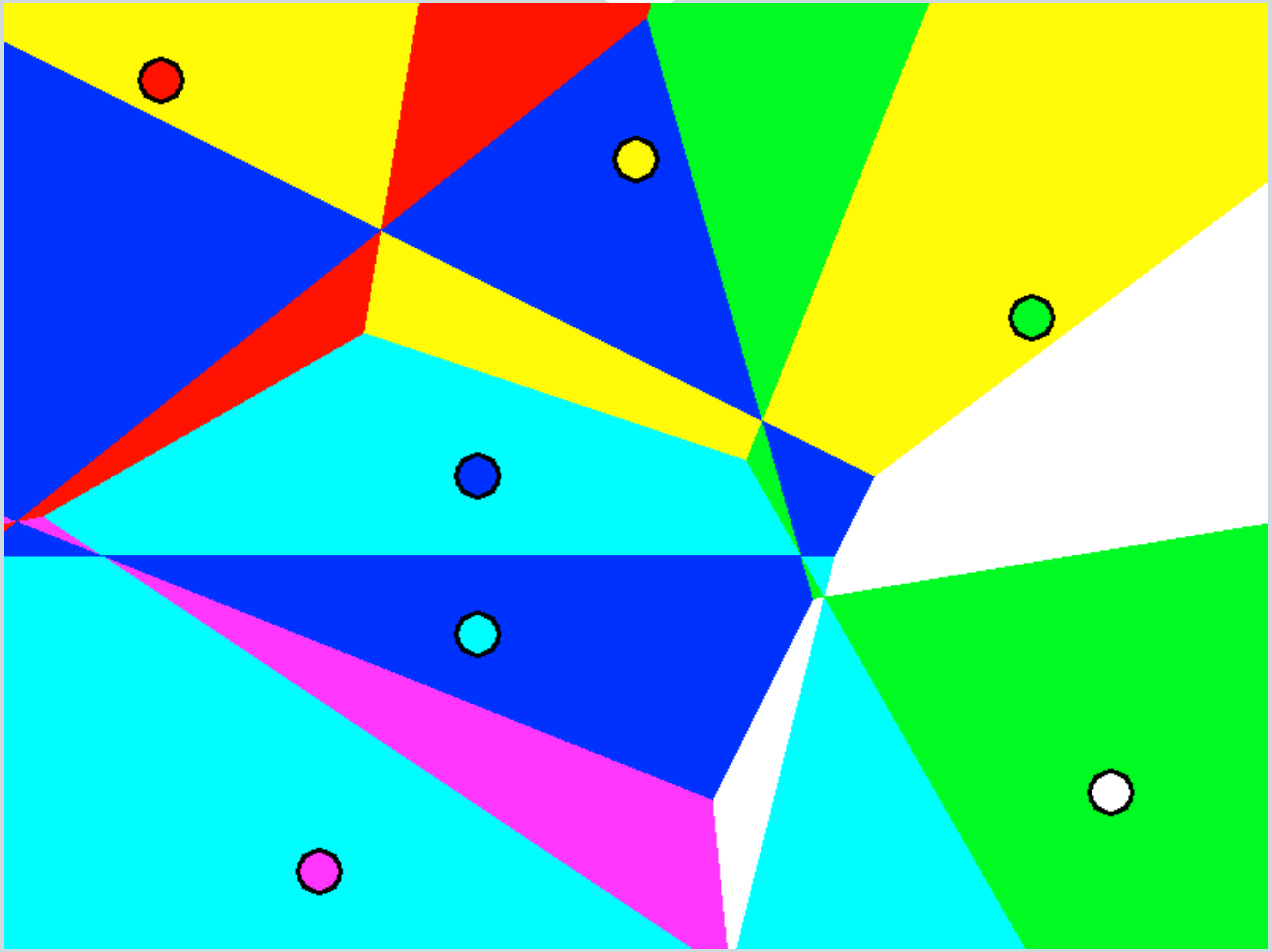


…and now do it for all V(T), |T|=k



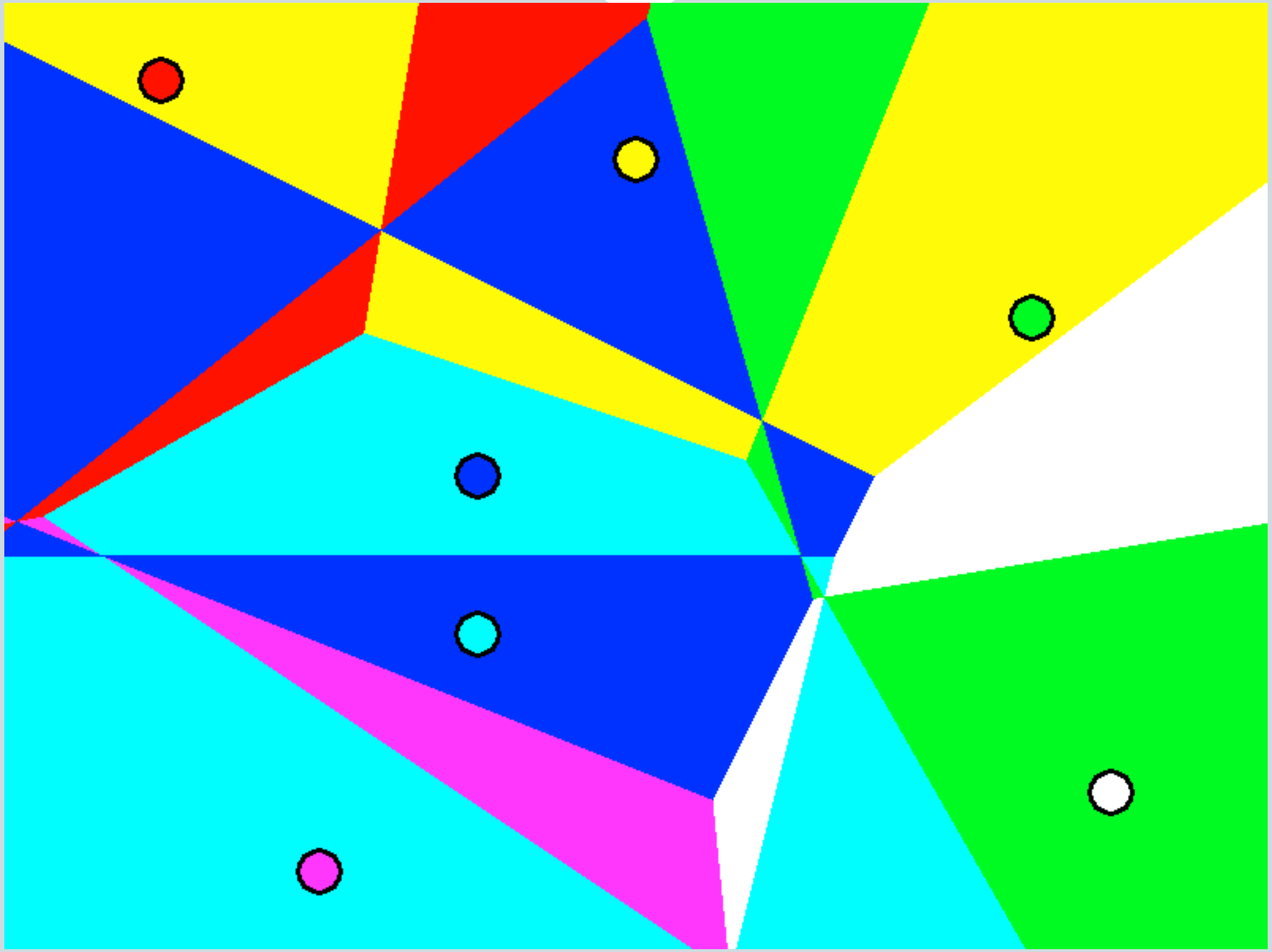
That's all!

...WELL, BASICALLY...

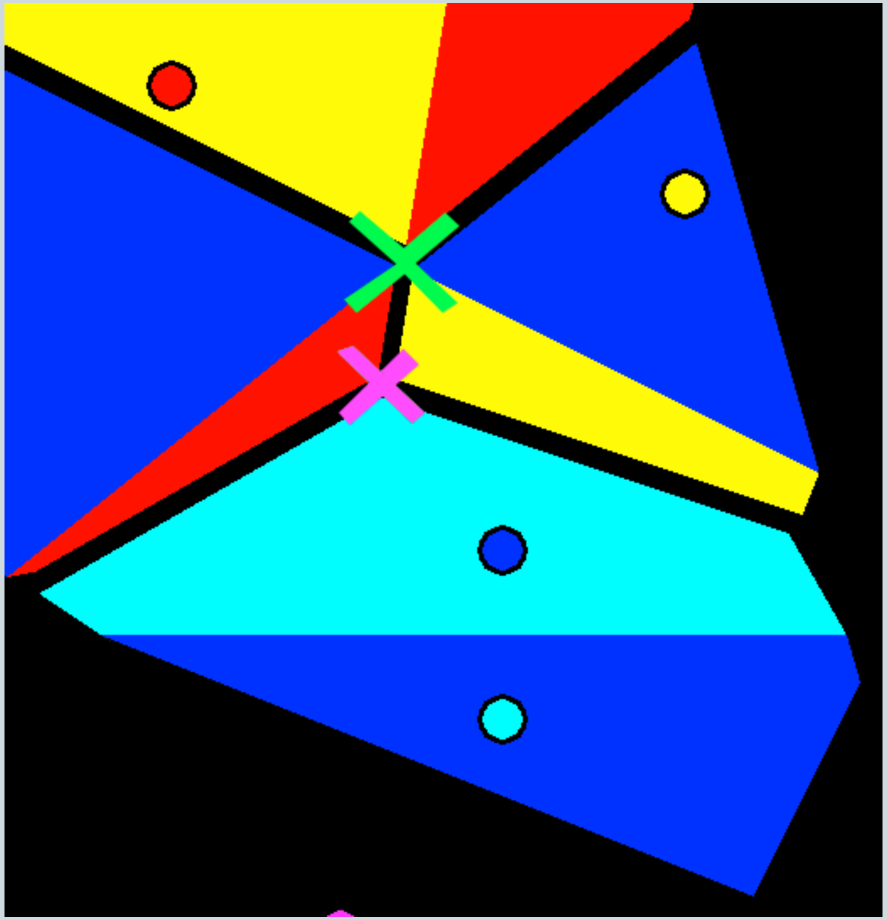
we don‘t need the whole Vor_1(S-T)

- two types of polygon‘s vertices
- vertex v incident with V(T1), V(T2) and V(T3)
- two options (⊙ means symmetric difference)
  - $|T_1 \odot T_2 \odot T_3| = k-2$
  - $|T_1 \odot T_2 \odot T_3| = k+2$
    - first type is called far-type, second is close-type
- k=1 => only close-types
- k=N => only far-types

some examples please?

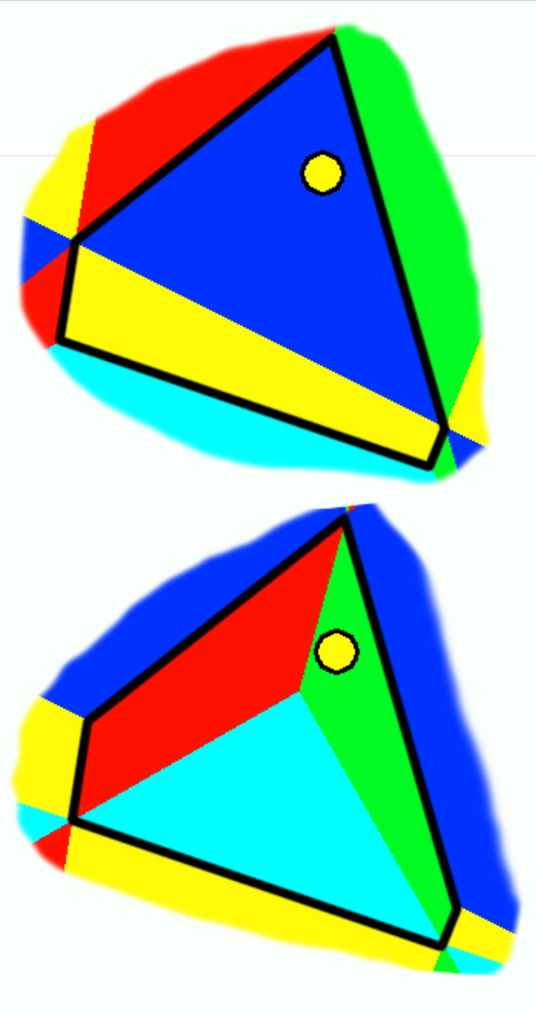


some examples please?



- k = 2
- (r,y) ⊙ (b,y) ⊙ (r,b) = {}
  - 0 = 2-2 = k-2
  - far-type
- (b,y) ⊙ (c,b) ⊙ (r,b) = (r,b,c,y)
  - 4 = 2+2 = k+2
  - close-type
- close-type of Vor_k becomes far-type of Vor_k+1

Step by step



- delete previous internal edges
  - edges incident with far-types only
  - „edges stay only for two orders“
- extend edges from close-types
  - see triangle example
- compute the rest of the partitioning if necessary
  - *x* close-type vertices -> *x* partitions (*x*-1 if unbounded)

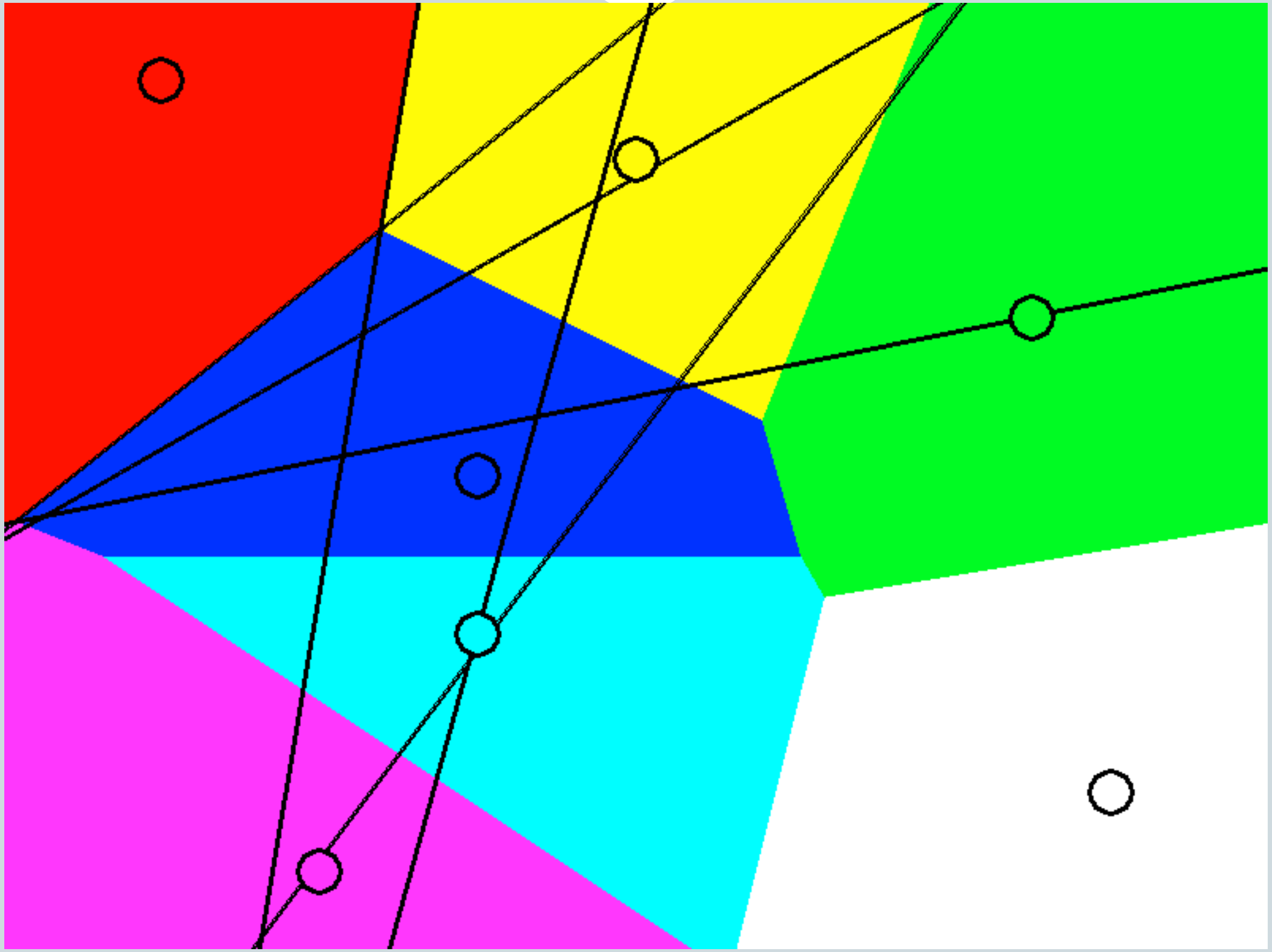
Literature

- [1] Computational Geometry; An Introductions F. PREPARATA (pages 242-246)

Questions?

THE TIME HAS COME!

Bonus round!



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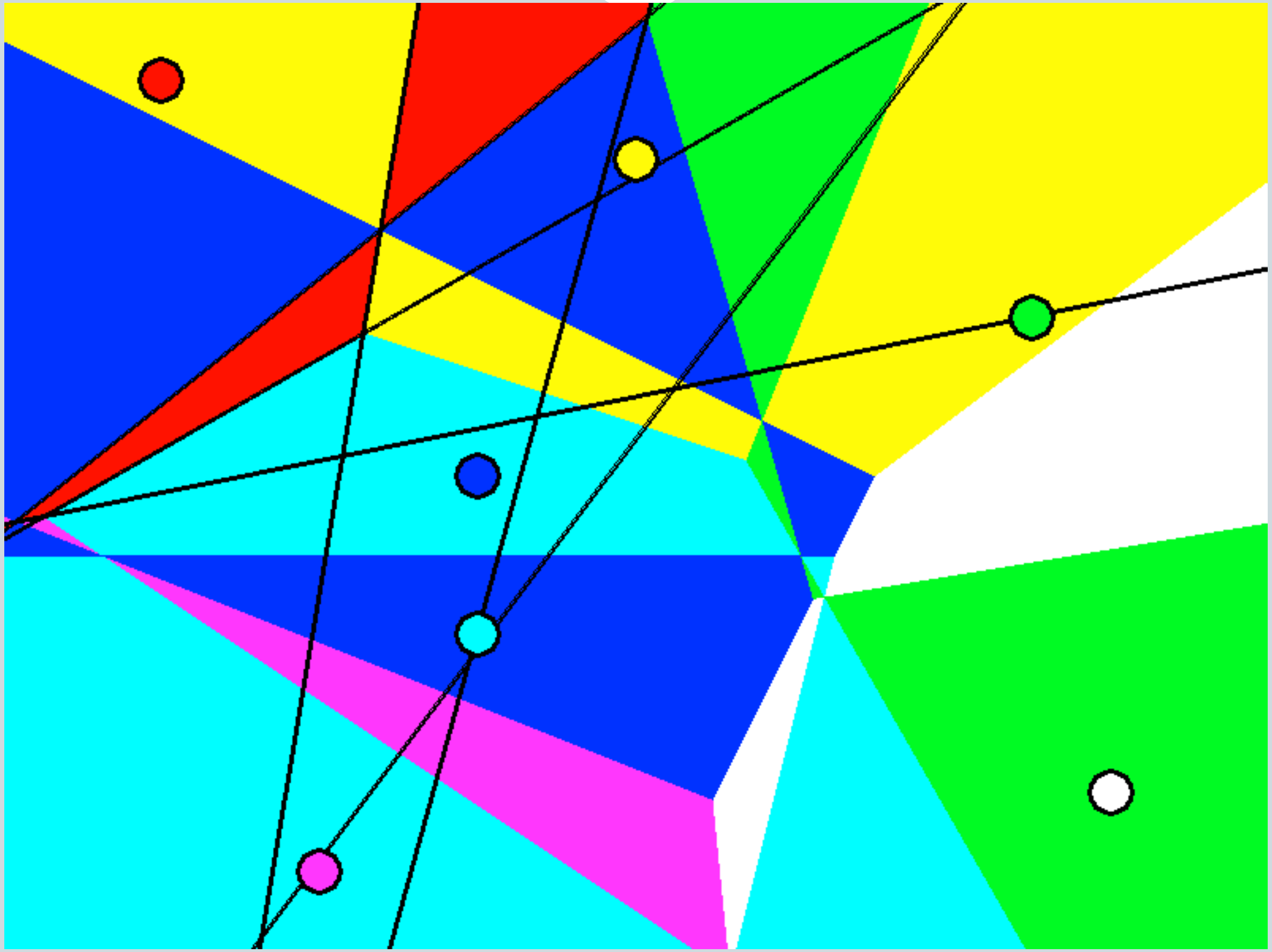

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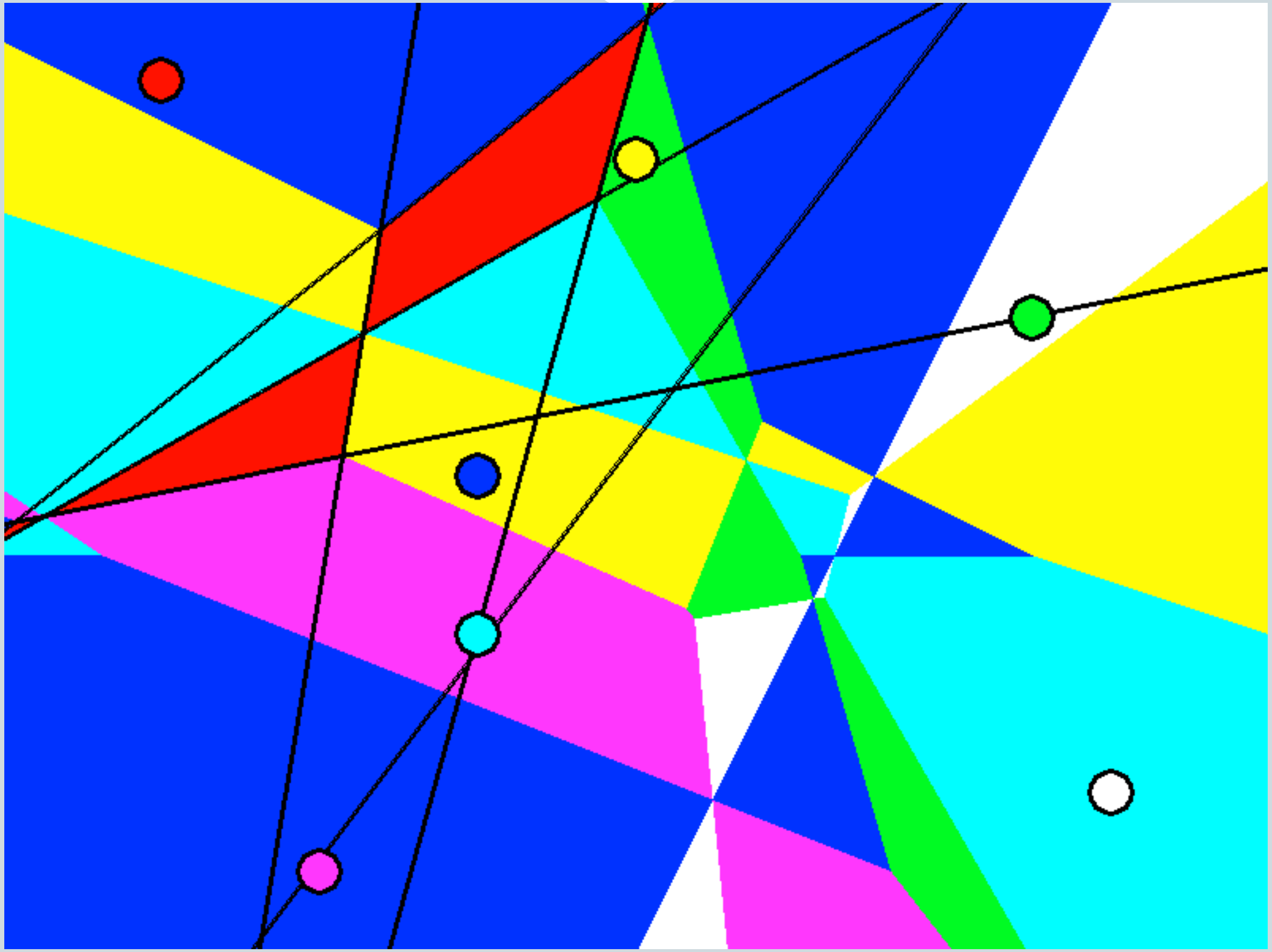

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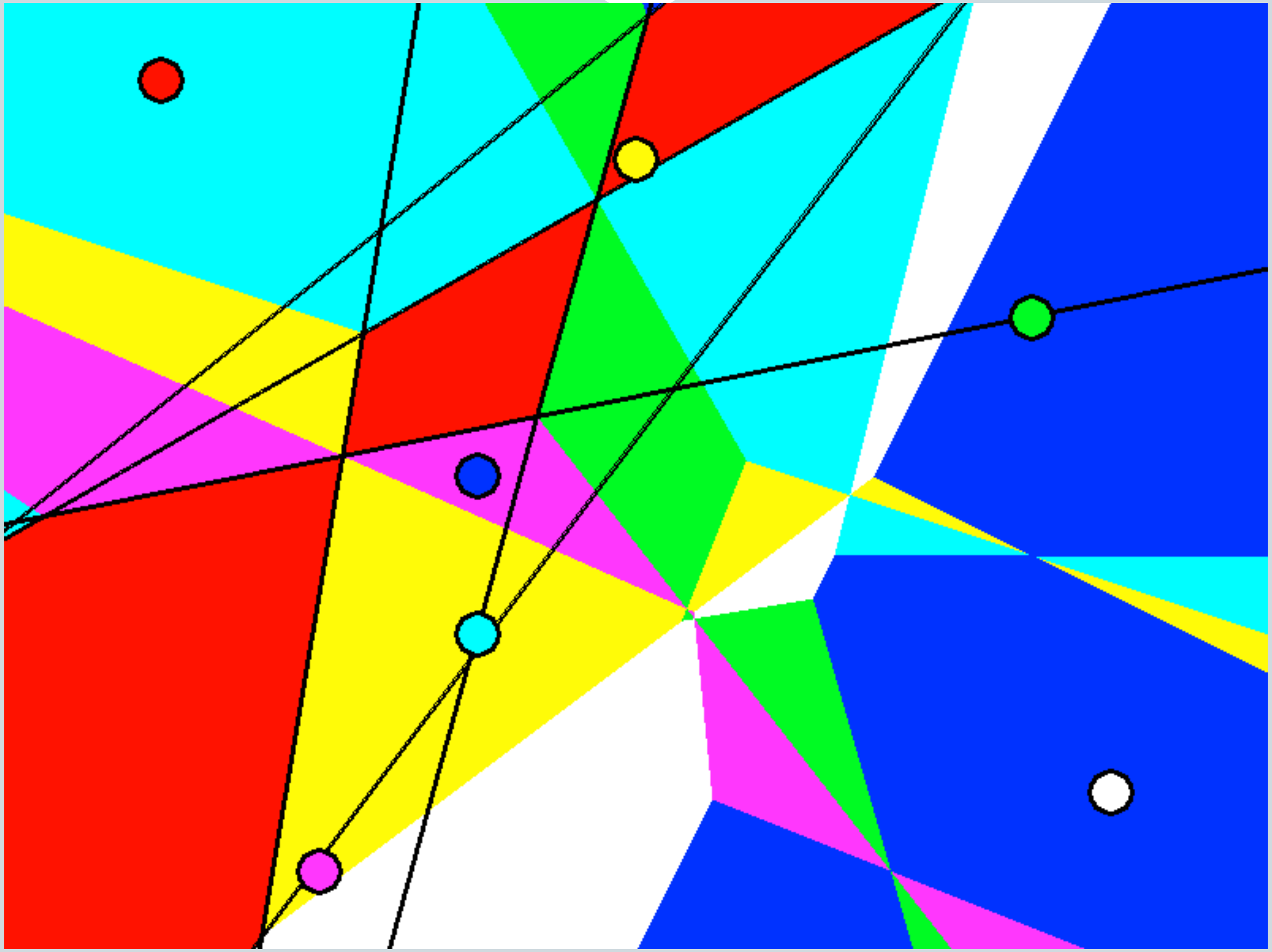

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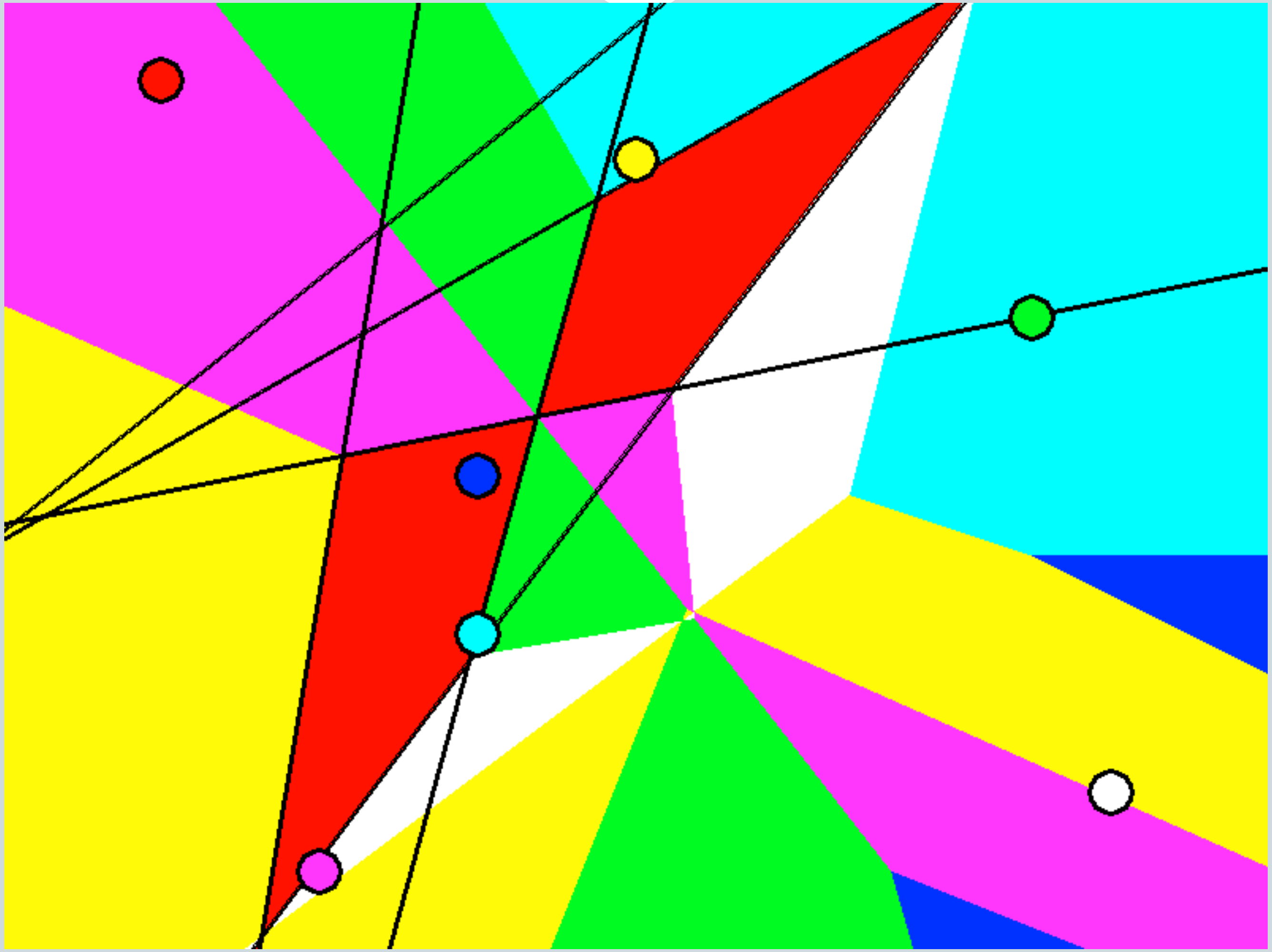

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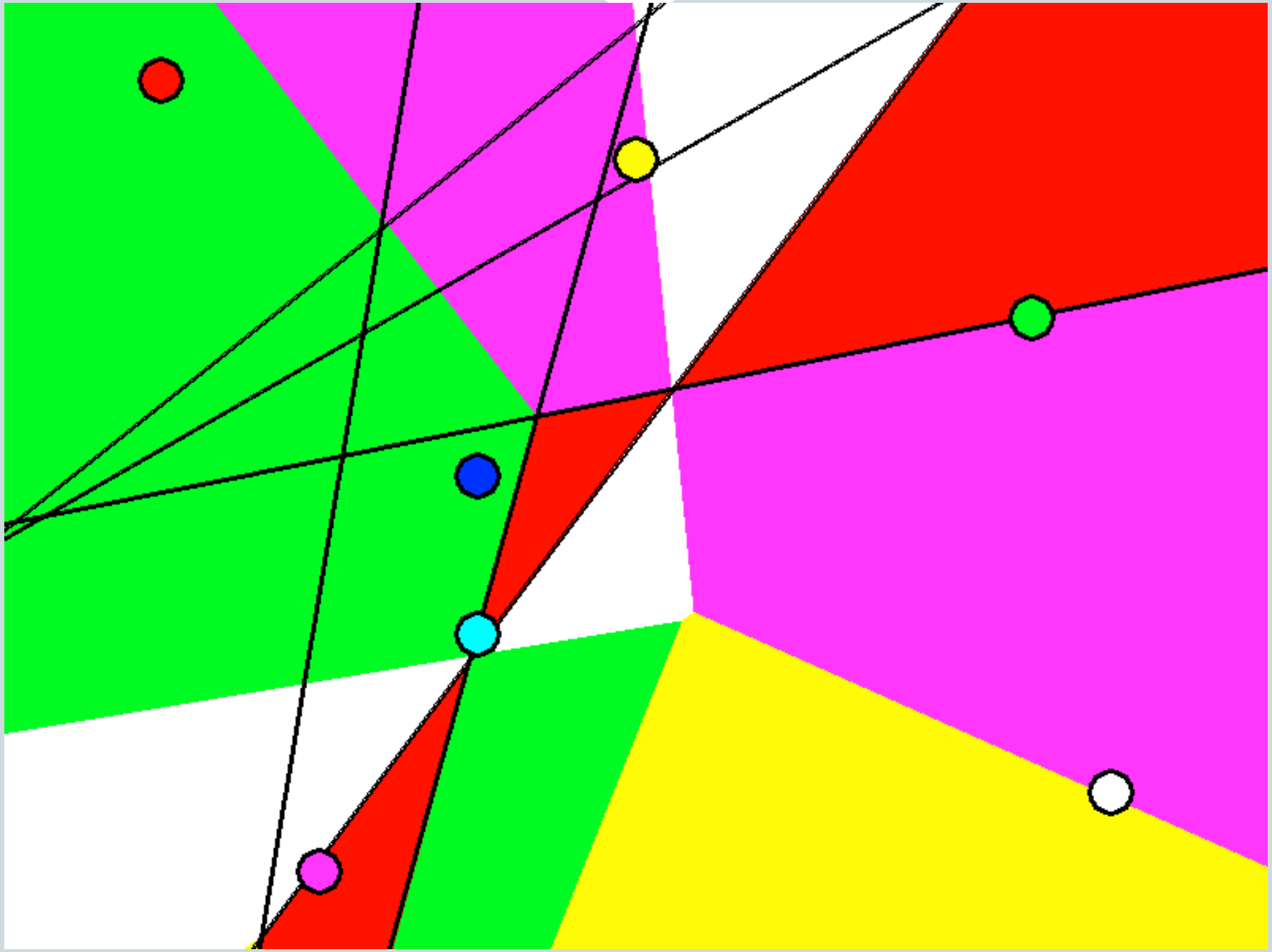

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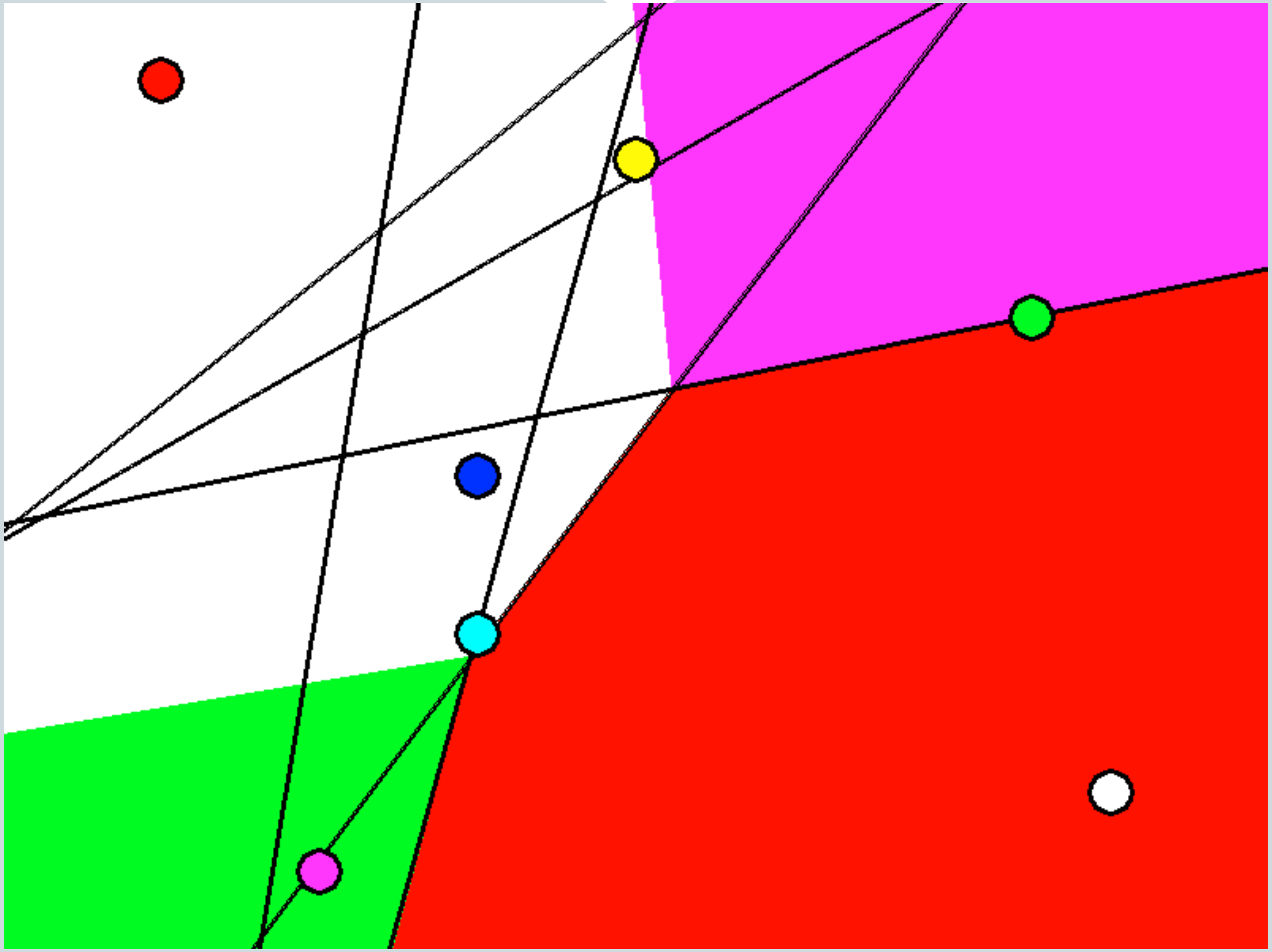

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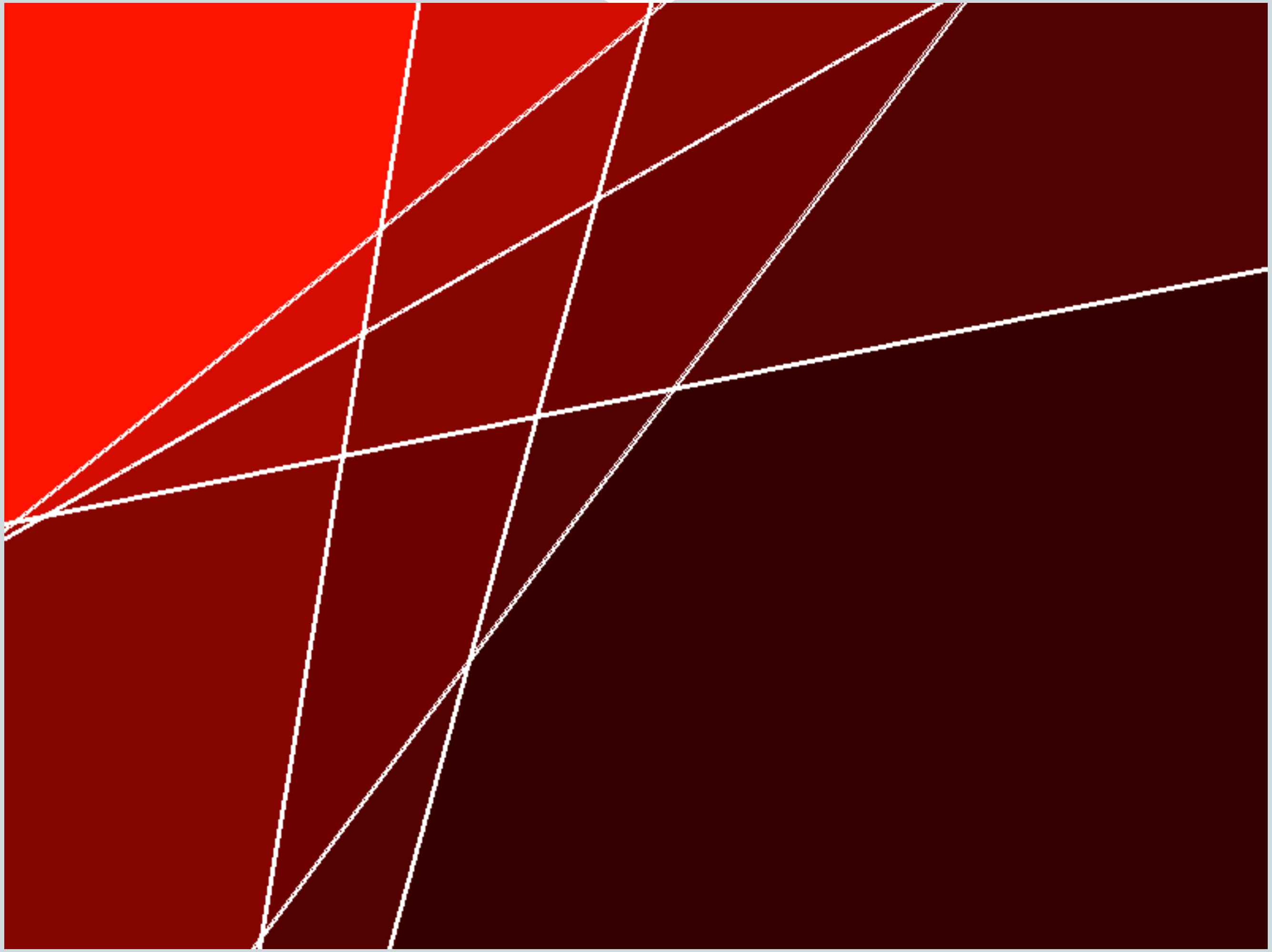

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