DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

INTERSECTIONS OF LINE SEGMENTS AND POLYGONS

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https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start

Based on [Berg], [Mount], [Kukral], and [Drtina]

Version from 14.11.2013

Talk overview

- Intersections of line segments (Bentley-Ottmann)
  - Motivation
  - Sweep line algorithm recapitulation
  - Sweep line intersections of line segments
- Intersection of polygons or planar subdivisions
  - See assignment [21] or [Berg, Section 2.3]
- Intersection of axis parallel rectangles
  - See assignment [26]

Geometric intersections – what are they for?

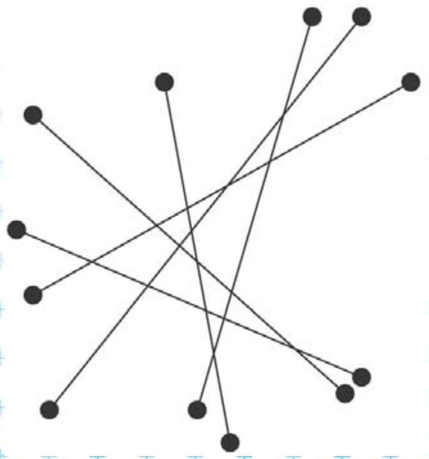
One of the most basic problems in computational geometry

- Solid modeling
  - Intersection of object boundaries in CSG
- Overlay of subdivisions, e.g. layers in GIS
  - Bridges on intersections of roads and rivers
  - Maintenance responsibilities (road network X county boundaries)
- Robotics
  - Collision detection and collision avoidance
- Computer graphics
  - Rendering via ray shooting (intersection of the ray with objects)
- …

Line segment intersection

- Intersection of complex shapes is often reduced to simpler and simpler intersection problems
- Line segment intersection is the most basic intersection algorithm
- Problem statement:

  Given *n* line segments in the plane, report all points where a pair of line segments intersect.
- Problem complexity
  - Worst case – $I = O(n^2)$ intersections
  - Practical case – only some intersections
  - Use an output sensitive algorithm
    - $O(n \log n + I)$ optimal randomized algorithm
    - $O(n \log n + I \log n)$ sweep line algorithm - %



[Berg]

Plane sweep line algorithm recapitulation

- Horizontal line (sweep line, *scan line*) ℓ moves top-down (or vertical line: left to right) over the set of objects
- The move is not continuous, but ℓ jumps from one event point to another
  - Event points are in priority queue or sorted list
  - The left-most event point is removed first
  - New event points may be created (usually as interaction of neighbors on the sweep line) and inserted in the queue
- Scan-line status
  - Stores information about the objects intersected by SL
  - It is updated while stopping on event point

Line segment intersection - Sweep line alg.

- Avoid testing of pairs of segments far apart
- Compute intersections of neighbors on the sweep line only
- O(n log n + I log n) time in O(n) memory

  2n steps for end points, I steps for intersections, log n search the tree

- Ignore “nasty cases” (most of them will be solved later on)
  - No segment is parallel to a sweep line
  - Segments intersect in one point and do not overlap
  - No three segments meet in a common point

Line segment intersections

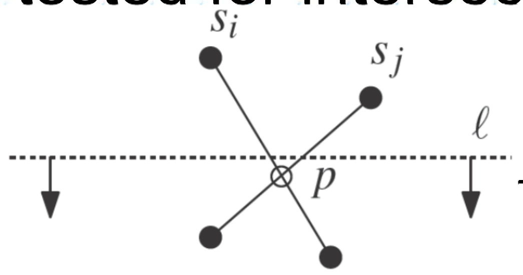
- *Status* = ordered sequence of segments
  intersecting the sweep line ℓ

- *Events* (waiting in the priority queue)
  - = points, where the algorithm actually does something
  - – Segment *end-points*
    - • known at algorithm start
  - – Segment *intersections* between neighboring segments along SL
    - • Discovered as the sweep executes

Detecting intersections

- Intersection events must be detected and inserted to the event queue before they occur
- Given two segments *a, b* intersecting in a point *p*, there must be a placement of sweep line ℓ prior to *p*, such that segments *a, b* are adjacent along ℓ (only adjacent will be tested for intersection)
  - segments *a, b* are not adjacent when the alg. starts
  - segments *a, b* are adjacent just before *p*

  => there must be an event point when *a,b* become adjacent and therefore are tested for intersection

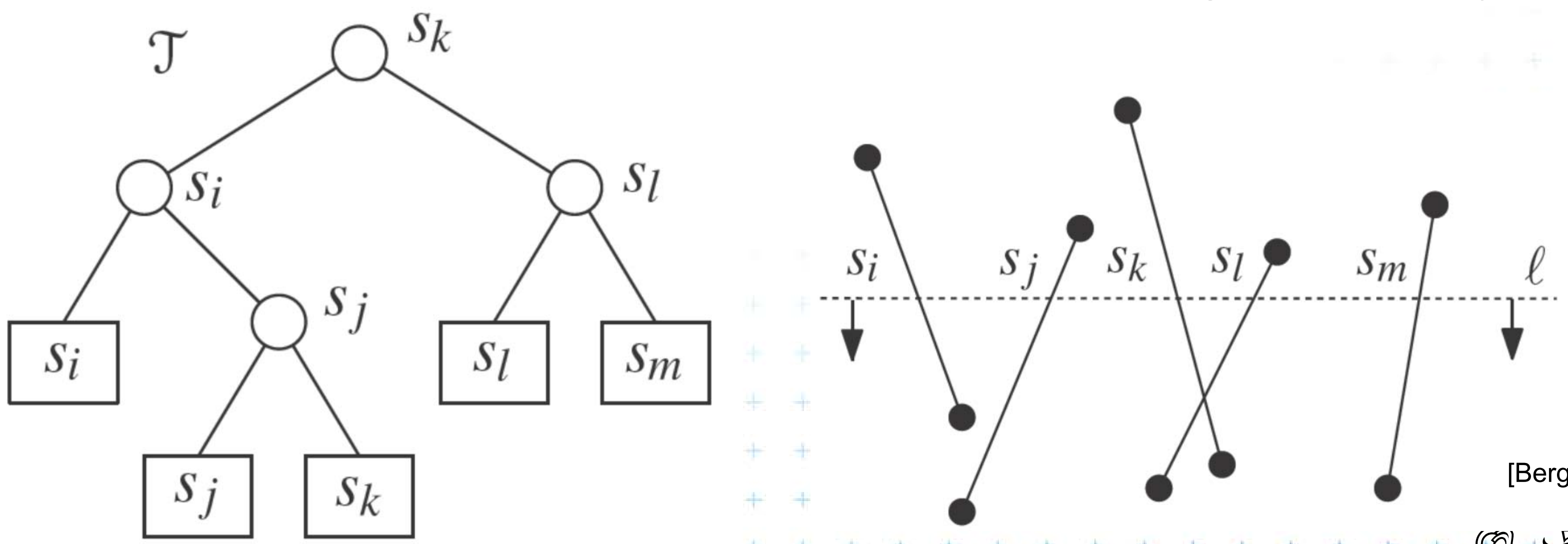


[Berg]

Data structures

Sweep line ℓ status = order of segments along ℓ

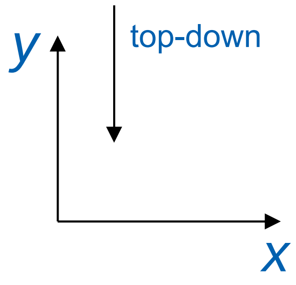
- Balanced binary search tree of segments
- Coords of intersections with ℓ vary as ℓ moves
  => store pointers to line segments in tree nodes
  - Position of ℓ is plugged in the $y=mx+b$ to get the key



[Berg]

Data structures

Event queue (postupový plán, časový plán)



- Define: Order $\diagup$ (top-down, lexicographic)

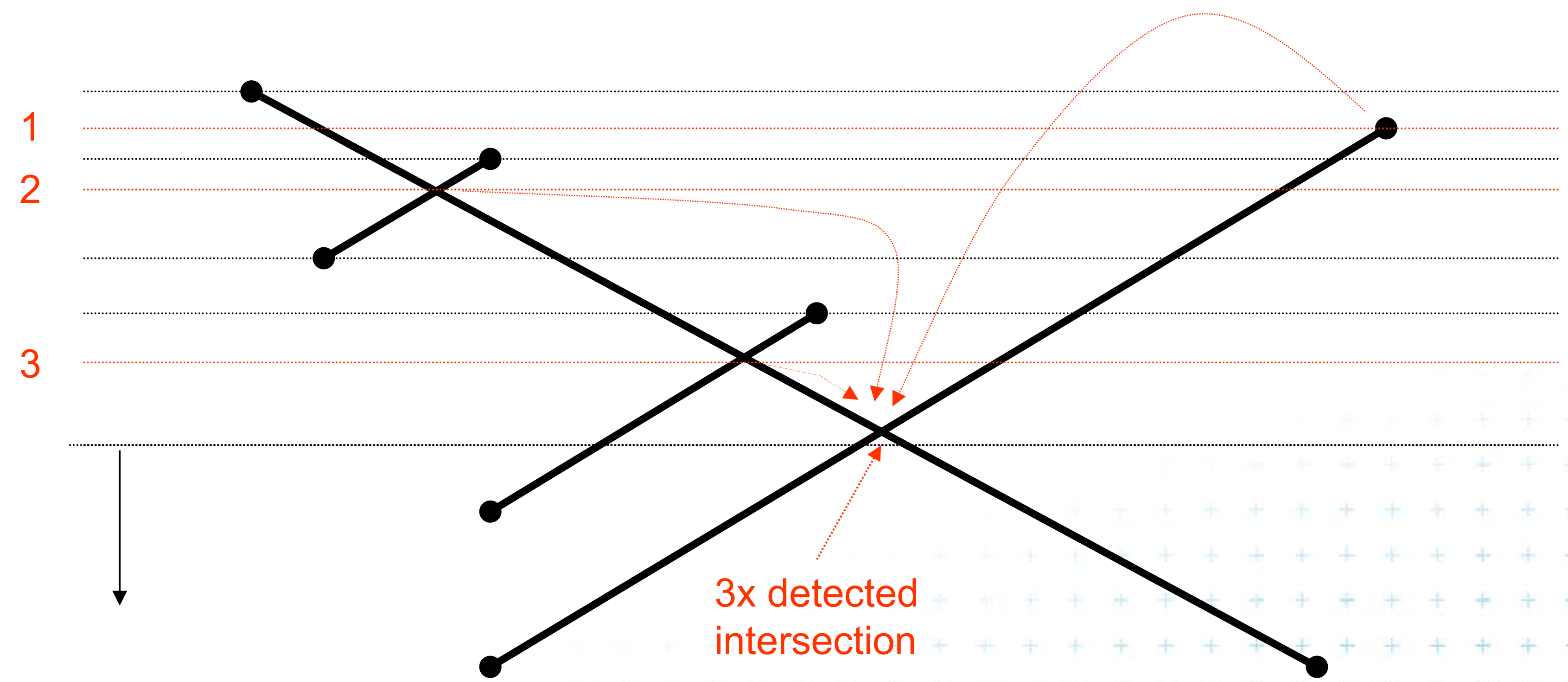
  $p \diagup q$ **iff** $p_y > q_y$ **or** $p_y = q_y$ and $p_x < q_x$

  top-down, left-right approach

  (points on ℓ treated left to right)

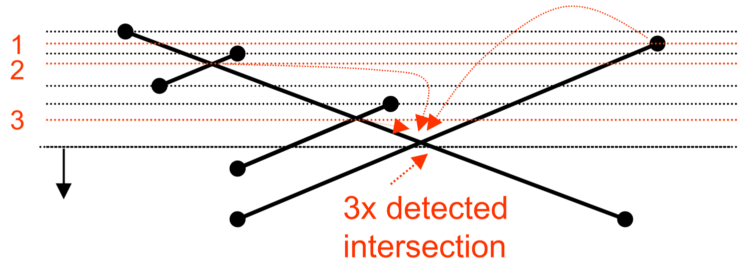
- Operations
  - Insertion of computed intersection points
  - Fetching the next event (highest *y* below ℓ)
  - Test, if the segment is already present in the queue
  - (Delete intersection event in the queue)

Problem with duplicities of intersections



Data structures

Event queue data structure



- Heap
  - Problem: can not check duplicated intersection events (reinvented more than once)
  - Intersections processed twice or even more
  - Memory complexity up to O(n^2)
- Ordered dictionary (balanced binary tree)
  - Can check duplicated events (adds just constant factor)
  - Nothing inserted twice
  - If non-neighbor intersections are deleted i.e., only intersection of neighbors is stored then memory complexity just O(n)

Line segment intersection algorithm

FindIntersections(*S*)

Input: A set *S* of line segments in the plane

Output: The set of intersection points + pointers to segments in each

```
1.  init an empty event queue Q and insert the segment endpoints
2.  init an empty status structure T
3.  while Q in not empty
4.      remove next event p from Q
5.      handleEventPoint(p)
```

Upper endpoint

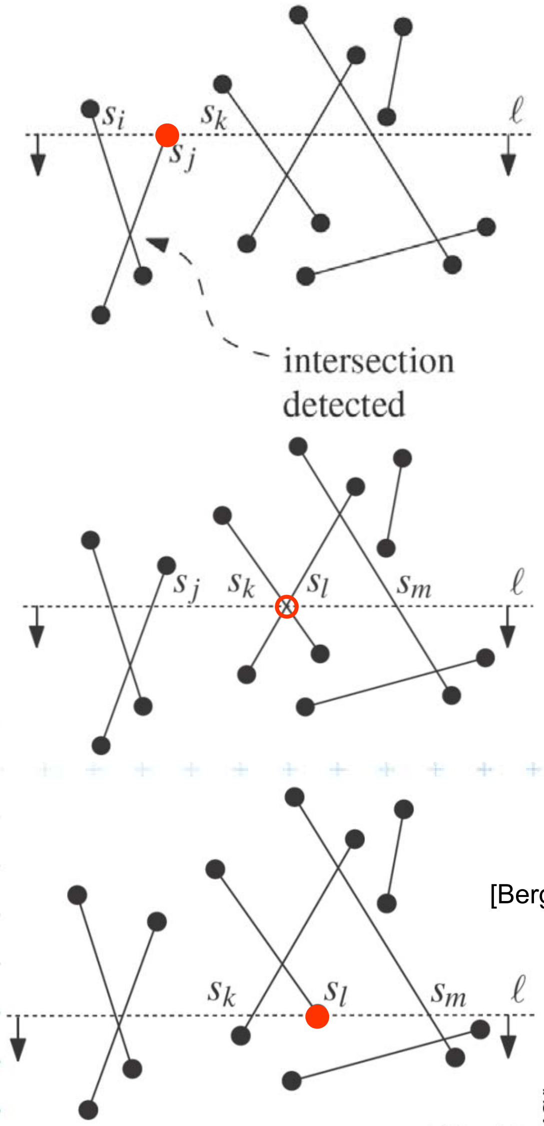
Intersection

Lower endpoint

Note: Upper-end-point events store info about the segment

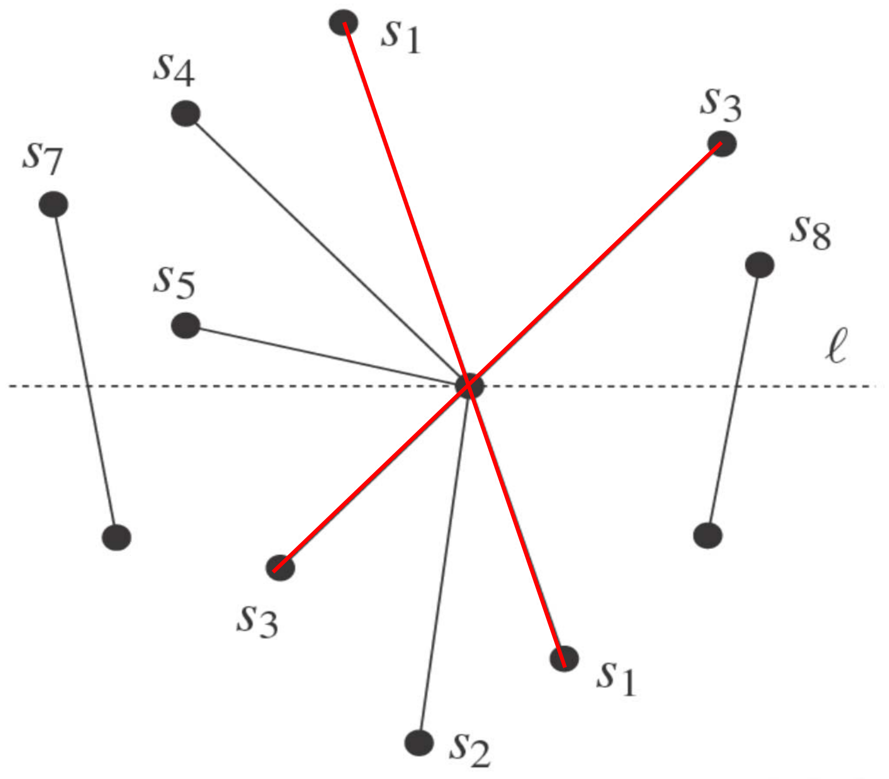
handleEventPoint principle

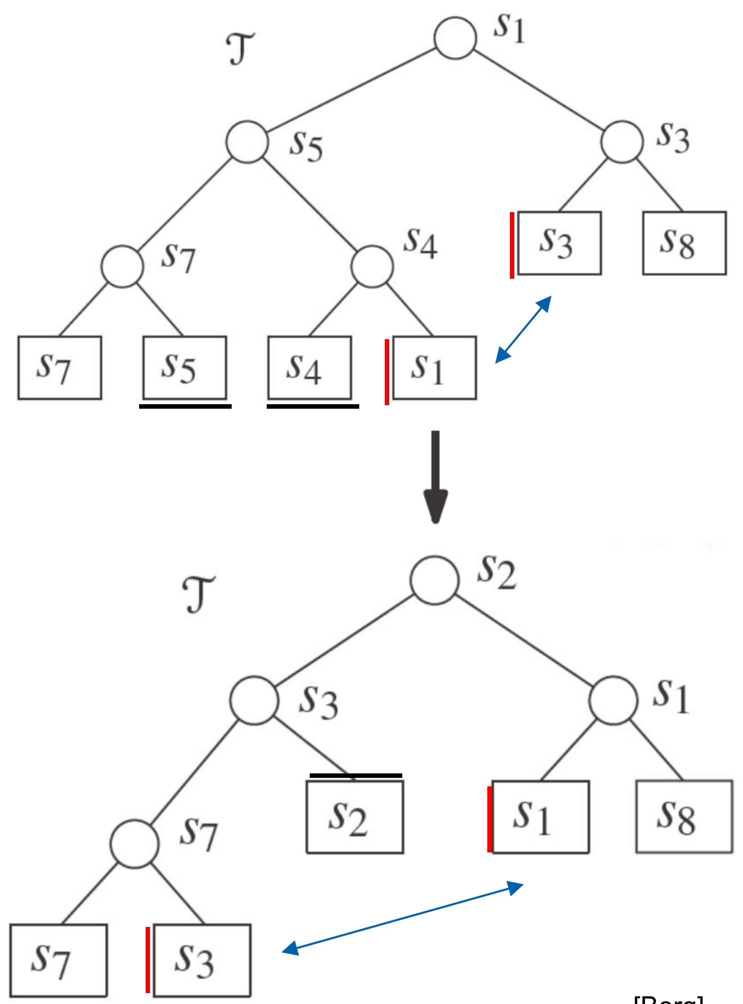
- Upper endpoint $U(p)$
  - insert p (on s_j) to status T
  - add intersections with left and right neighbors to Q
- Intersection $C(p)$
  - switch order of segments in T
  - add intersections of left and right neighbors to Q
- Lower endpoint $L(p)$
  - remove p (on s_l) from T
  - add intersections of left and right neighbors to Q



[Berg]

More than two segments incident





$U(p) = \{s_2\}$ — start here

$C(p) = \{s_1, s_3\}$ — cross on ℓ

$L(p) = \{s_4, s_5\}$ — end here

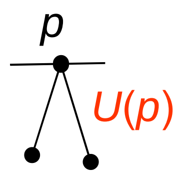
[Berg]

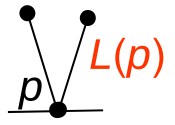
Handle Events [Berg, page 25]

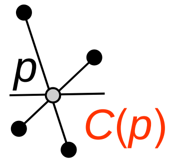
handleEventPoint(p)

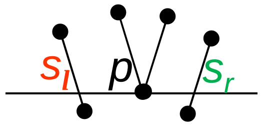
```
1.  Let $U(p)$ = set of segments whose Upper endpoint is $p$.
    These segmets are stored with the event point $p$ (will be added to $T$)
2.  Search $T$ for all segments $S(p)$ that contain $p$ (are adjacent in $T$):
        Let $L(p)$ ð $S(p)$ = segments whose Lower endpoint is $p$
        Let $C(p)$ ð $S(p)$ = segments that Contains $p$ in interior
3.  if( $L(p)$ Ã $U(p)$ Ã $C(p)$ contains more than one segment )
4.          report $p$ as intersection together with $L(p)$, $U(p)$, $C(p)$
5.  Delete the segments in $L(p)$ Ã $C(p)$ from $T$
6.  Insert  the segments in $U(p)$ Ã $C(p)$ into $T$
    (order as below $\ell$, horizontal segment as the last)
7.  if( $U(p)$ Ã $C(p)$ = þ ) then findNewEvent($s_l$, $s_r$, $p$) // left & right neighbors
8.  else  s'  = leftmost segment of $U(p)$ Ã $C(p)$;   findNewEvent($s_l$ , s', $p$)
          s'' = rightmost segment of $U(p)$ Ã $C(p)$; findNewEvent(s'', $s_r$ , $p$)
```

Reverse order of $C(p)$ in T (steps 5–6)



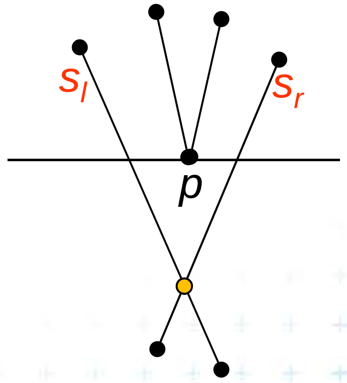




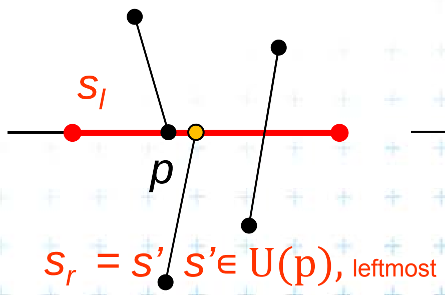


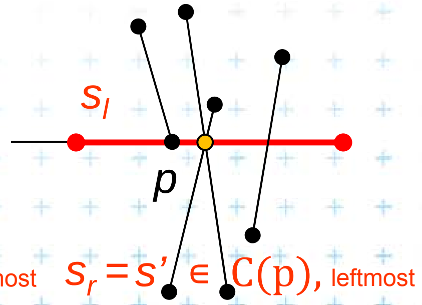
Detection of new intersections

```
findNewEvent($s_l$, $s_r$, $p$)    // with handling of horizontal segments
Input:   two segments (left & right from $p$ in $T$) and a current event point $p$
Output:  updated event queue $Q$ with new intersection
1.  if [ ( $s_l$ and $s_r$ intersect below the sweep line $\ell$ ) or
        ( intersect on $\ell$ and to the right of $p$ ) ] and      // horizontal segments
        ( the intersection is not present in $Q$ )
2.  then
        insert $p$ as an event into $Q$
```



s_l and s_r intersect below





s_l and s_r = s' intersect on ℓ and to the right of p

Line segment intersections

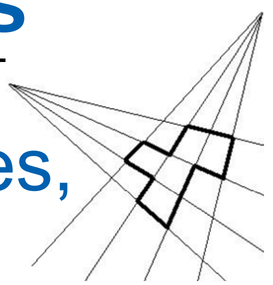
- Memory O(I) = O(n^2) with duplicities in Q
  or O(n) with duplicities in Q deleted
- Operational complexity
  – $n + I$ stops
  – log n each
  => O($I + n$) log n total
- The algorithm is by Bentley-Ottmann

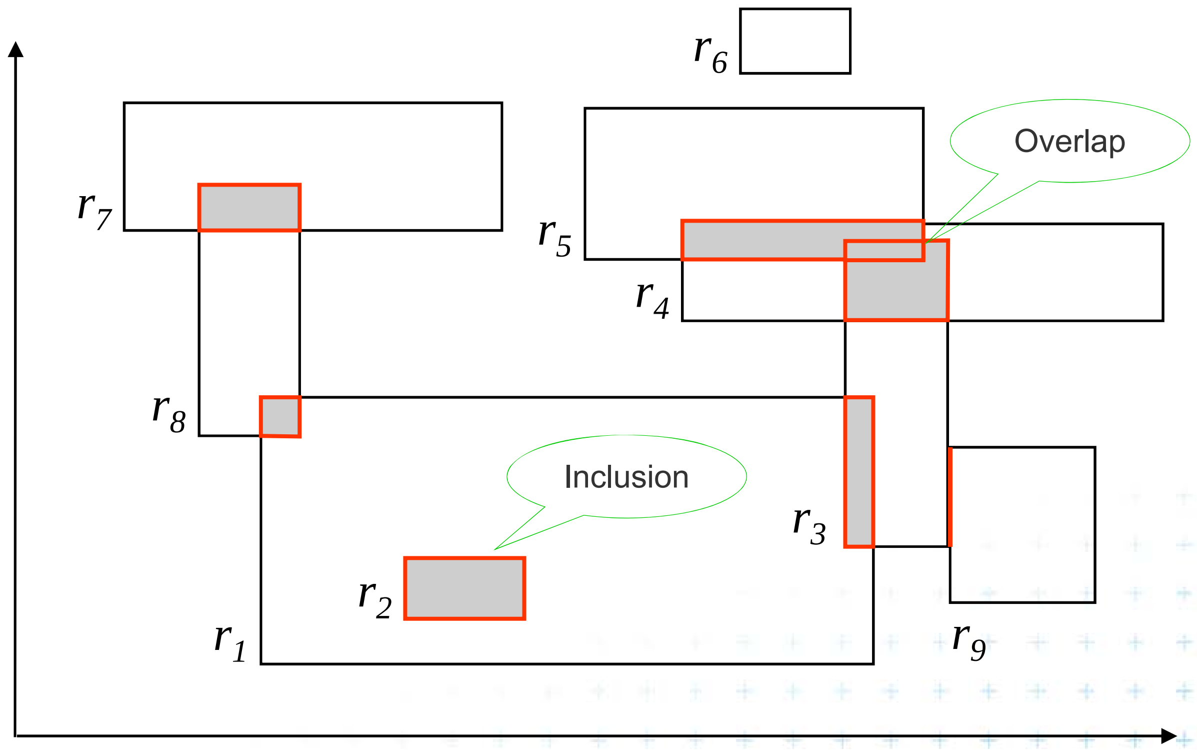
Bentley, J. L.; Ottmann, T. A. (1979), "Algorithms for reporting and counting geometric intersections", *IEEE Transactions on Computers* **C-28** (9): 643-647, doi:10.1109/TC.1979.1675432 .

See also http://wapedia.mobi/en/Bentley%E2%80%93Ottmann_algorithm

Intersection of axis parallel rectangles

- Given the collection of *n isothetic* rectangles, report all intersecting parts





Alternate sides belong to two pencils of lines
(trsy přímek)
(often used with points in infinity = axis parallel)

[?]

Answer: (r_1, r_2) (r_1, r_3) (r_1, r_8) (r_3, r_4) (r_3, r_5) (r_3, r_9) (r_4, r_5) (r_7, r_8)

Brute force intersection

Brute force algorithm

Input: set *S* of axis parallel rectangles

Output: pairs of intersected rectangles

```
1. For every pair $(r_i, r_j)$ of rectangles $\in S, i \neq j$
2.     if $(r_i \cap r_j \neq \emptyset)$ then
3.         report $(r_i, r_j)$
```

Analysis

Preprocessing: None.

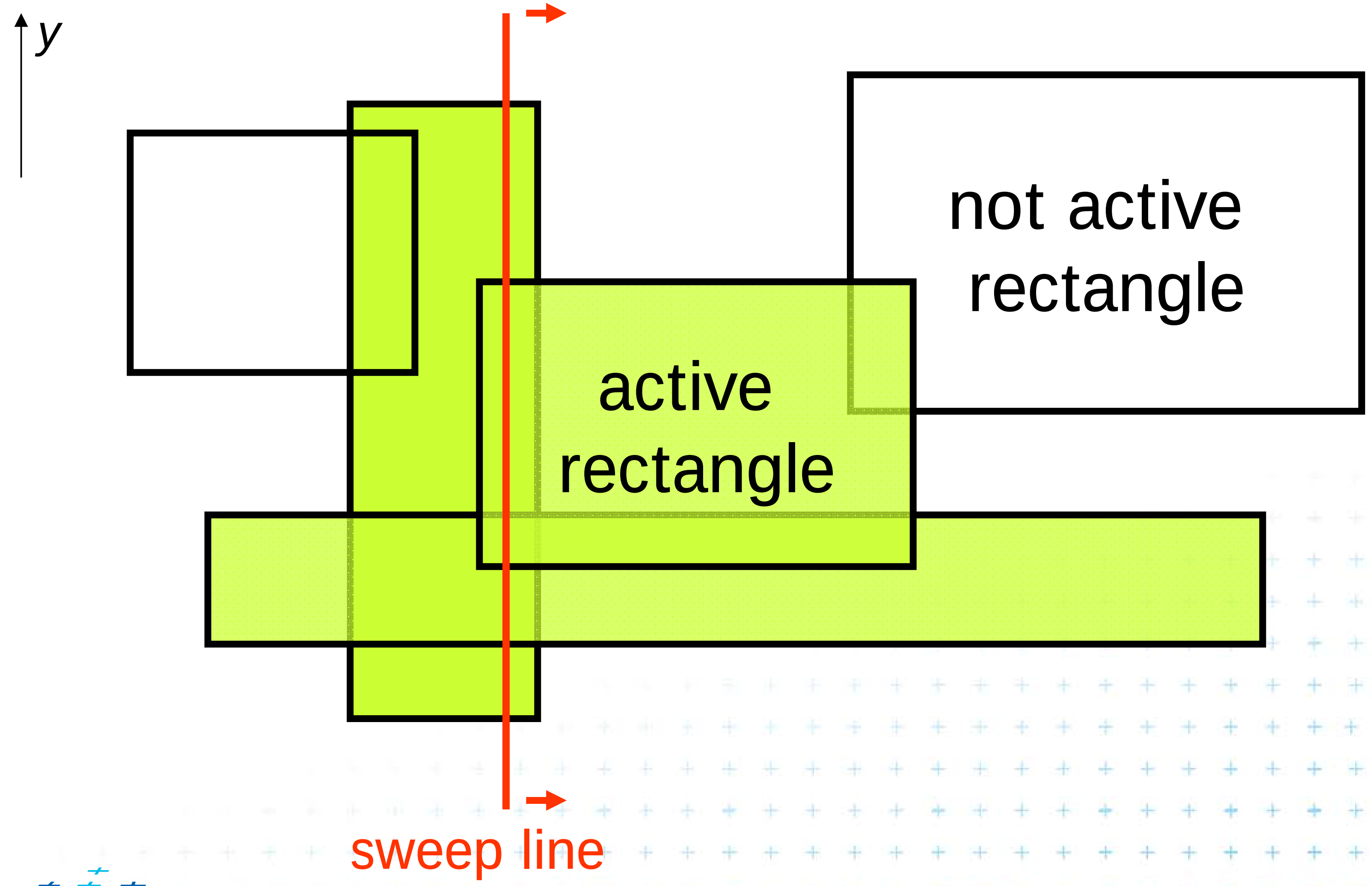
Query: $O(N^2)$ $\binom{N}{2} = \frac{N(N-1)}{2} \in O(N^2)$.

Storage: $O(N)$

Plane sweep intersection algorithm

- Vertical sweep line moves from left to right
- Stops at every x-coordinate of a rectangle (either its left side or its right side).
- active rectangles – a set
  = rectangles currently intersecting the sweep line
  – left side event of a rectangle
    => the rectangle is added to the active set.
  – right side
    => the rectangle is deleted from the active set.
- The active set used to detect rectangle intersection

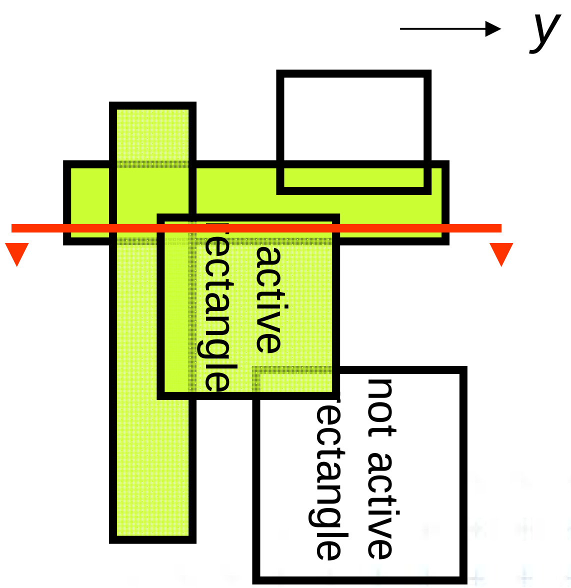
Example rectangles and sweep line



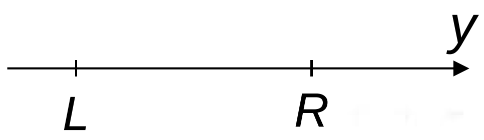
[Drtina]

Interval tree as sweep line status structure

- Vertical sweep-line => Only *y*-coordinates along it
- Turn our view in slides 90° right
- Sweep line (*y*-axis) will be drawn as horizontal



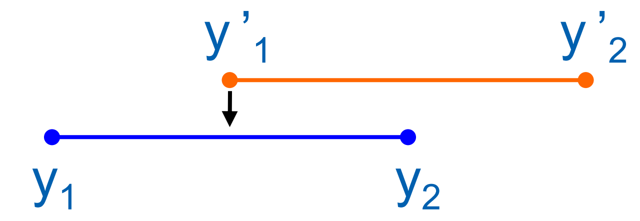
sweep line [Drtina]



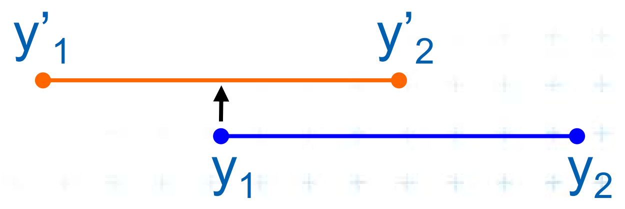
Intersection test – between pair of intervals

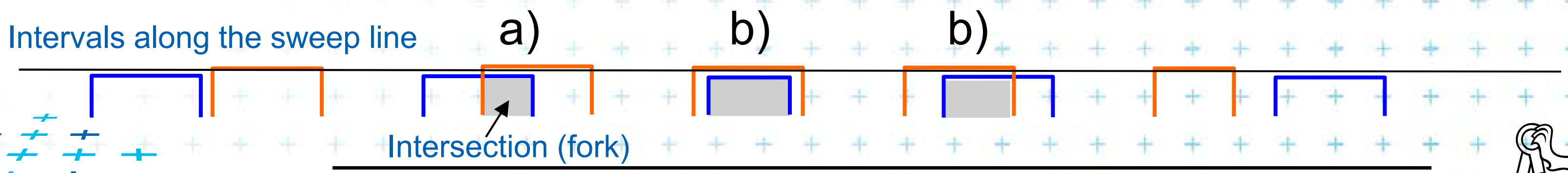
- Given two intervals R = [y_1, y_2] and R’ = [y'_1, y'_2] the condition R ∩ R’ is equivalent to one of these mutually exclusive conditions:

a) y_1 ¸ y'_1 ¸ y_2



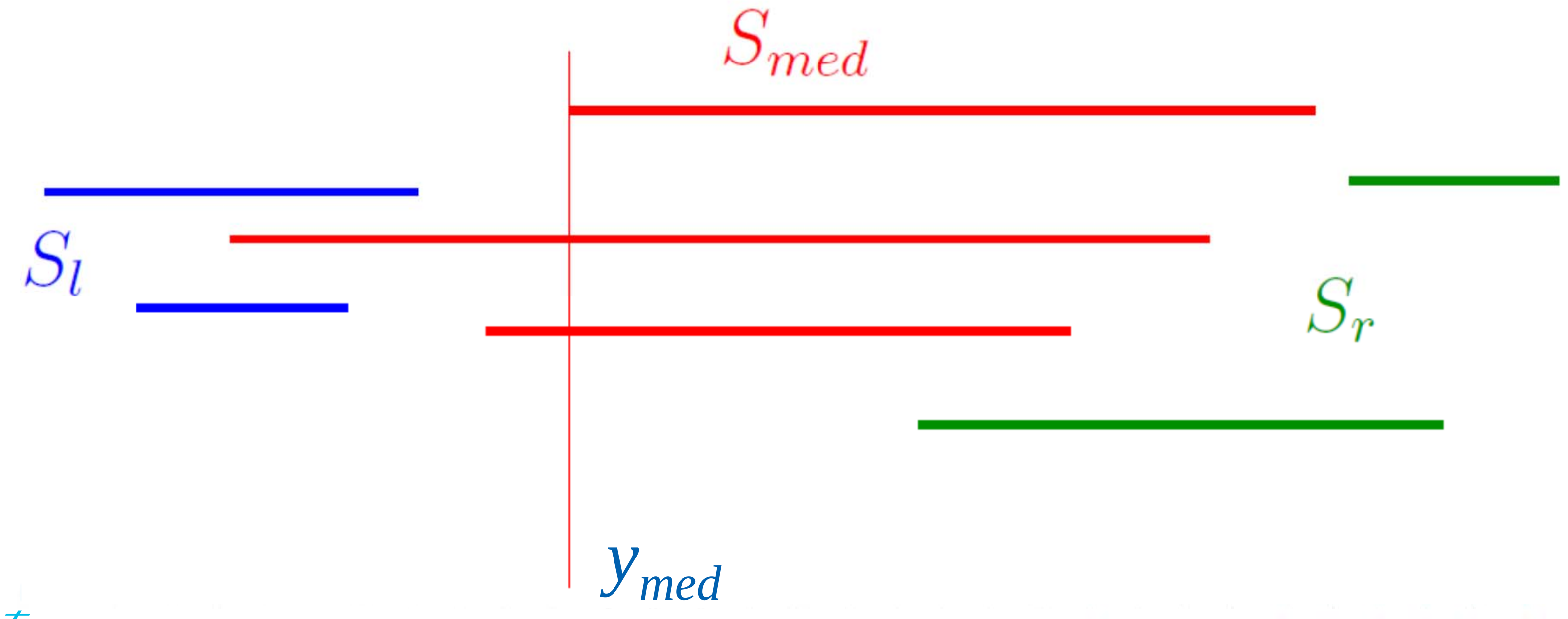
b) y'_1 ¸ y_1 ¸ y'_2





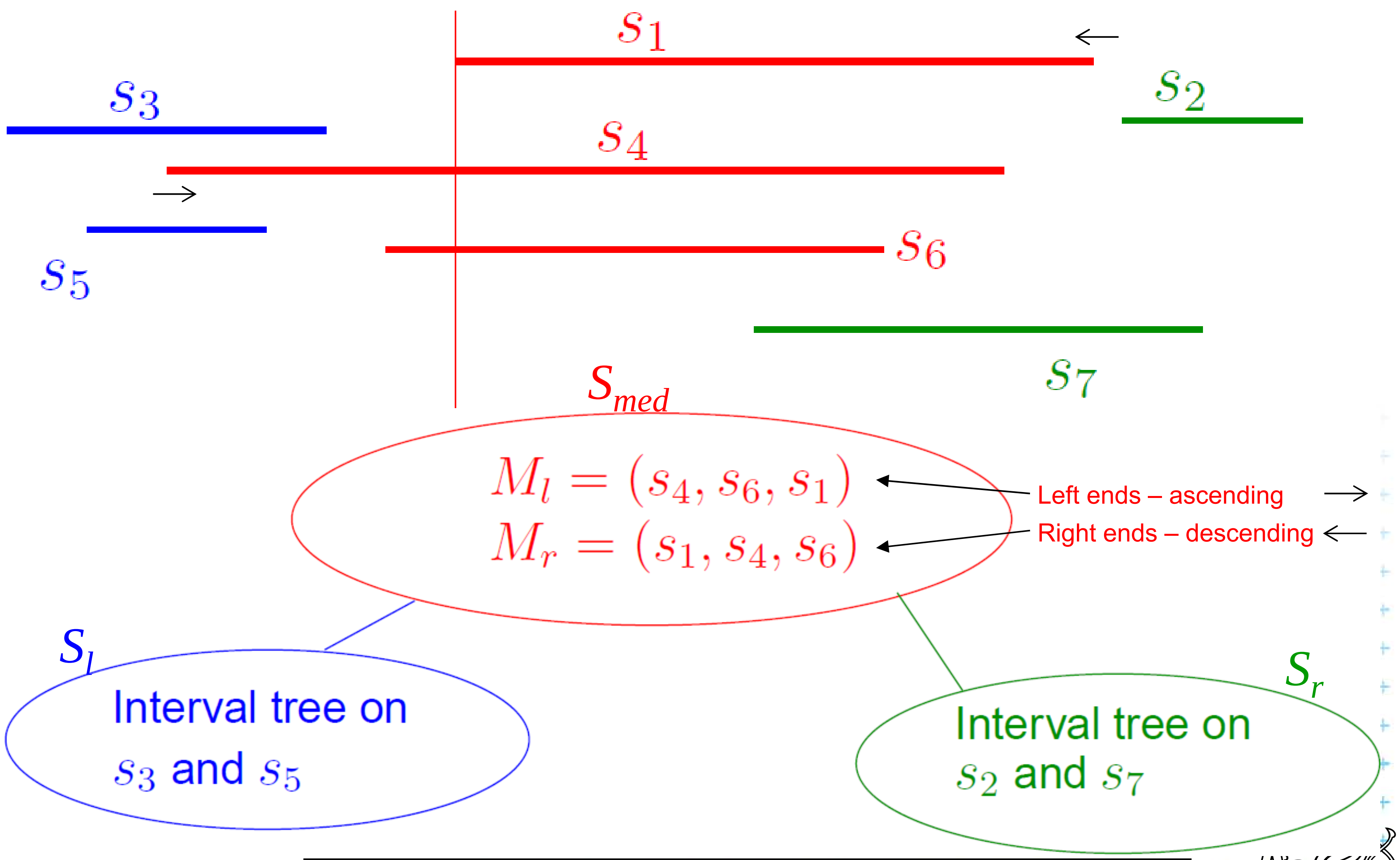
Static interval tree – stores all end points

- Let $v = y_{med}$ be the median of end-points of segments
- S_l : segments of S that are completely to the left of y_{med}
- S_{med} : segments of S that contain y_{med}
- S_r : segments of S that are completely to the right of y_{med}



[Vigneron]

Static interval tree – Example

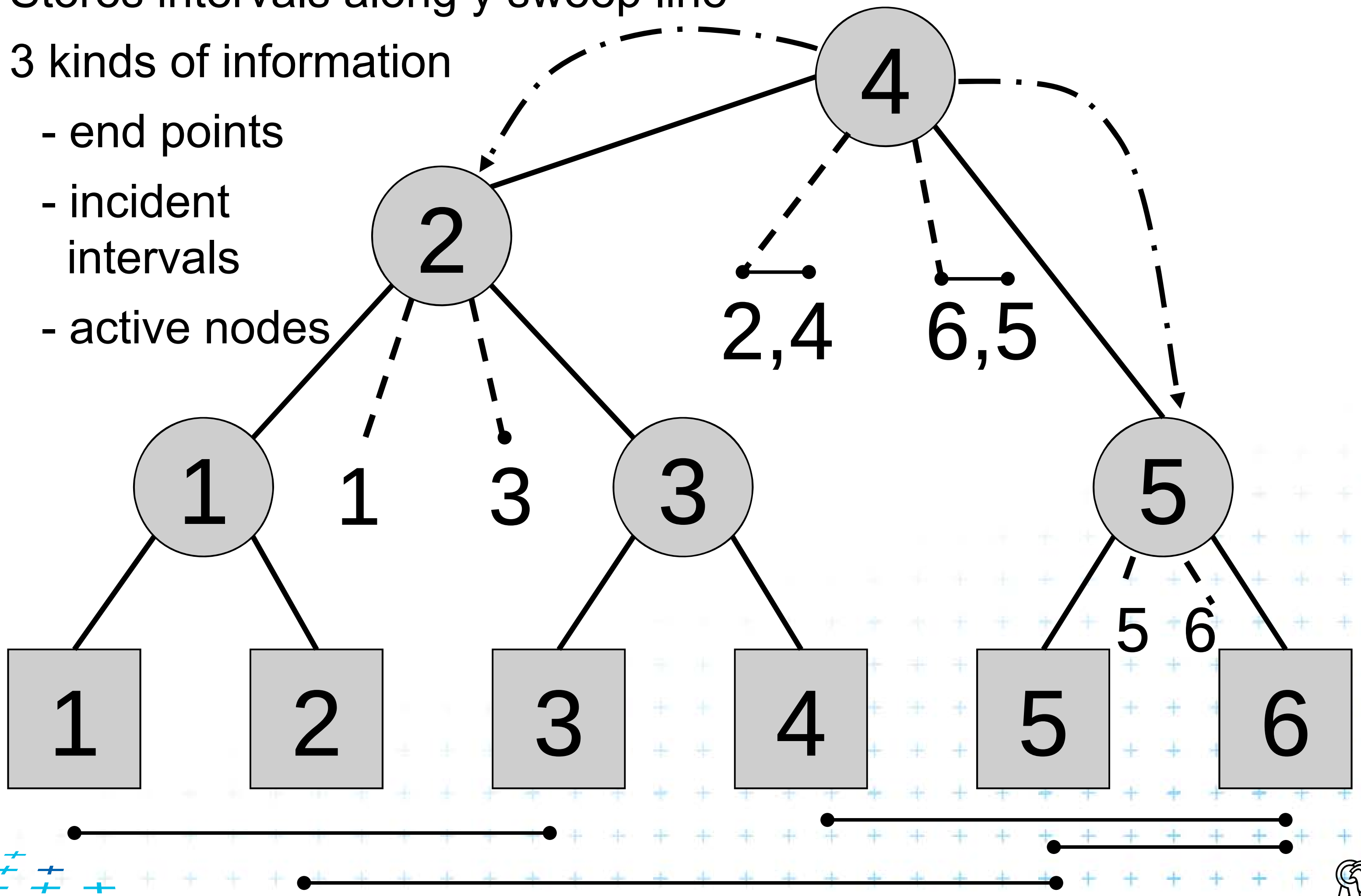


s_1
s_2
s_3
s_4
s_5
s_6
s_7
S_{med}
$M_l = (s_4, s_6, s_1)$
$M_r = (s_1, s_4, s_6)$
Left ends – ascending $\rightarrow$
Right ends – descending $\leftarrow$
S_l
Interval tree on s_3 and s_5
S_r
Interval tree on s_2 and s_7

[Vigneron]

Static interval tree [Edelsbrunner80]

- Stores intervals along y sweep line
- 3 kinds of information
  - end points
  - incident intervals
  - active nodes

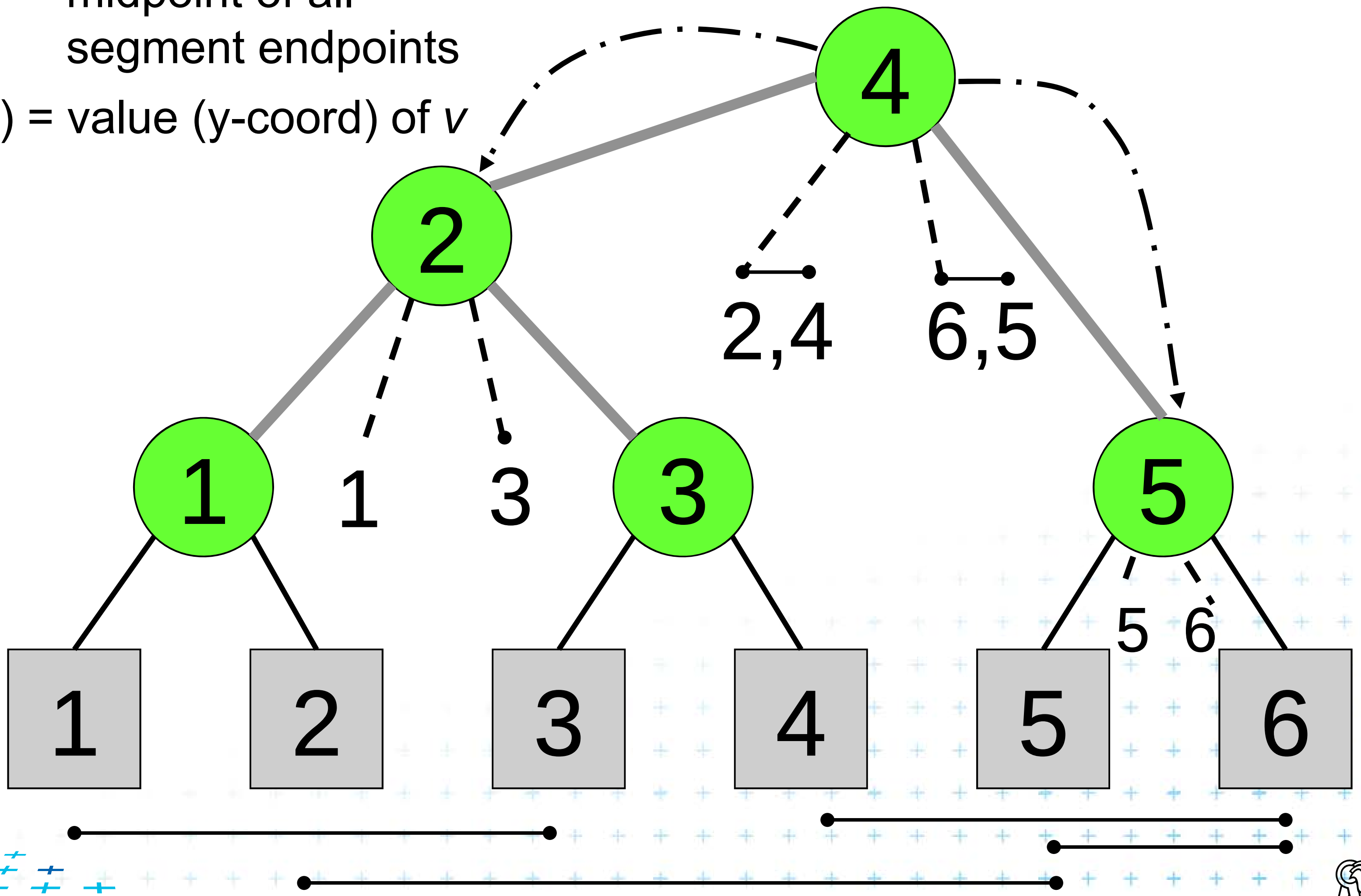


[Kukral]

Primary structure – static tree for endpoints

v = midpoint of all segment endpoints

H(v) = value (y-coord) of *v*

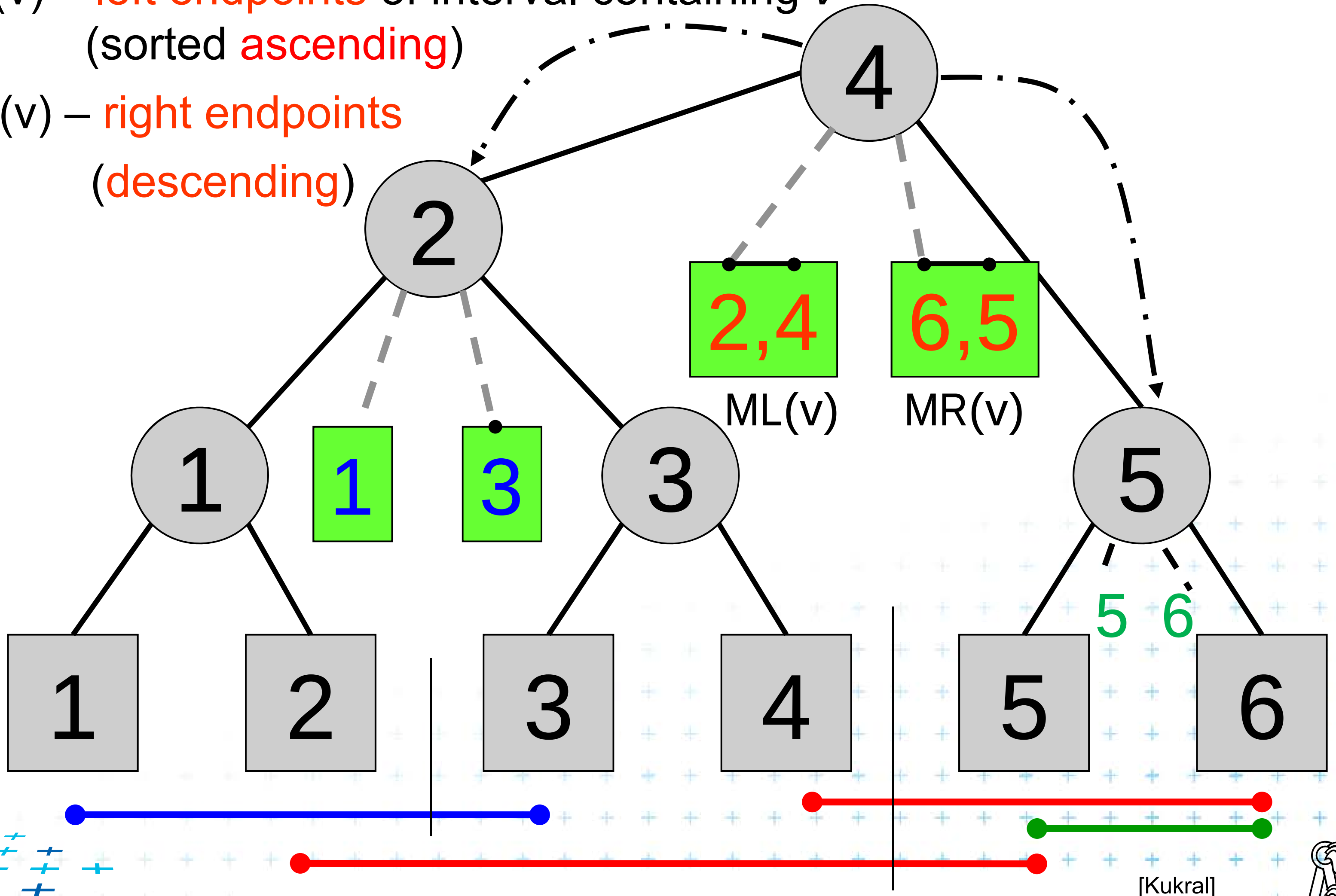


[Kukral]

Secondary lists of incident interval end-pts.

ML(v) – left endpoints of interval containing *v* (sorted ascending)

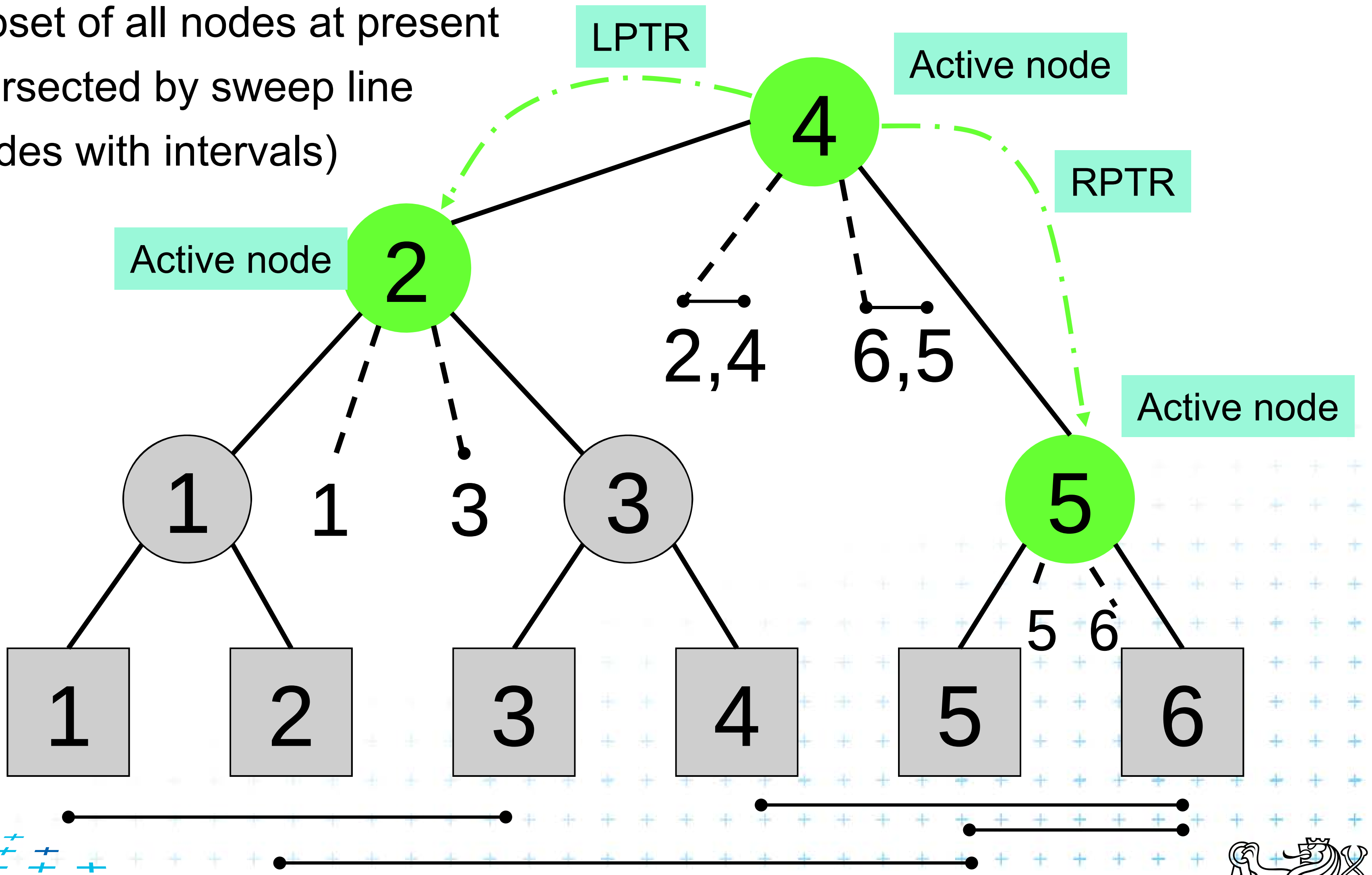
MR(v) – right endpoints (descending)



[Kukral]

Active nodes – intersected by the sweep line

Subset of all nodes at present intersected by sweep line (nodes with intervals)

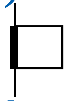


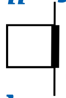
[Kukral]

Query = sweep and report intersections

RectangleIntersections(*S*)
Input: Set *S* of rectangles
Output: Intersected rectangle pairs

```
1.  Preprocess( S )          // create the interval tree T (for y-coords)
                             // and event queue Q         (for x-coords)
2.  while ( Q û þ ) do
3.      Get next entry ($x_i , y_{il} , y_{ir} , t$) from Q        // t @ { left | right }
4.      if ( $t$ = left )    // left edge
5.              a) QueryInterval ( $y_{il} , y_{ir}$, root(T))   // report intersections
6.              b) InsertInterval ( $y_{il} , y_{ir}$, root(T))  // insert new interval
7.      else                 // right edge
8.              c) DeleteInterval ( $y_{il} , y_{ir}$, root(T) )
```





Preprocessing

Preprocess(S)

Input: Set *S* of rectangles

Output: Primary structure of the interval tree *T* and the event queue *Q*

```
1.  T = PrimaryTree(S)          // Construct the static primary structure
                                // of the interval tree -> sweep line STATUS T

2.  // Init event queue Q with vertical rectangle edges in ascending order.
    // Put the left edges with the same x ahead of right ones.
3.  for i = 1 to n
4.      insert( ( $x_{il}$ , $y_{il}$ , $y_{ir}$ , left ), Q)          // left edges of i-th rectangle
5.      insert( ( $x_{ir}$, $y_{il}$ , $y_{ir}$ , right ), Q)         // right edges
```

Interval tree – primary structure construction

```
PrimaryTree(S)          // only the y-tree structure, without intervals
Input:    Set S of rectangles
Output:   Primary structure of an interval tree T
1.  $S_y$ = Sort endpoints of all segments in S according to y-coordinate
2.  T = BST( $S_y$ )
3.  return T
```

```
BST( $S_y$ )
1.  if( $|S_y| = 0$ ) return null
2.  yMed = median of $S_y$
3.  L = endpoints $p_y$ ¸ yMed
4.  R = endpoints $p_y$ A#yMed
5.  t = new IntervalTreeNode( yMed )
6.  t.left  = BST(L)
7.  t.right = BST(R)
8.  return t
```

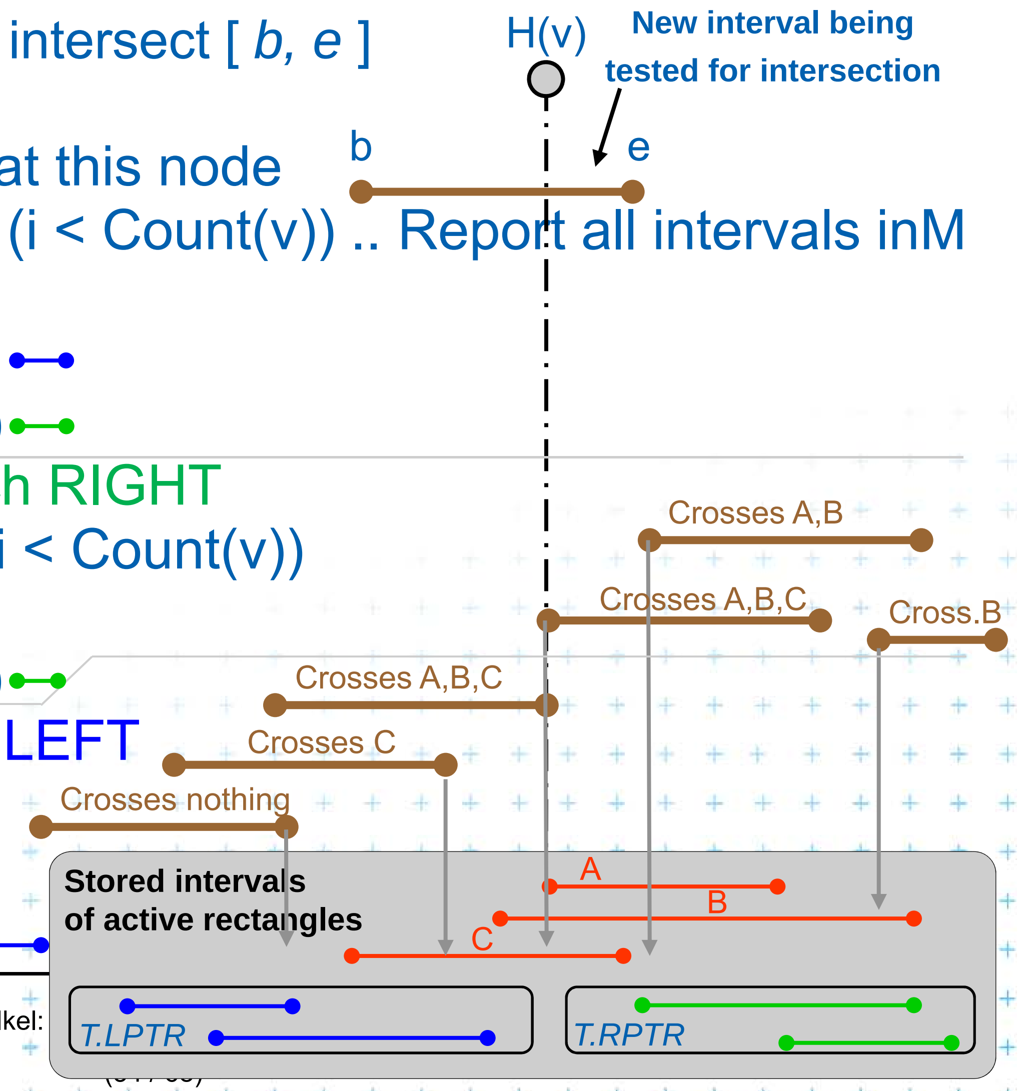
Interval tree – search the intersections

QueryInterval (*b, e, T*)

Input: Interval of the edge and current tree *T*

Output: Report the rectangles that intersect [*b, e*]

```
1.  if( T =  null ) return
2.  i=0; if( b < H(v) < e )  // forks at this node
3.      while ~~( MR(v).[i] >= b ) &&~~ (i < Count(v)) .. Report all intervals inM
4.          ReportIntersection; i++
5.      QueryInterval( b,e,T.LPTR )
6.      QueryInterval( b,e,T.RPTR )
7.  else if (H(v) ¸ b < e)  // search RIGHT
8.      while (MR(v).[i] >= b) && (i < Count(v))
9.          ReportIntersection; i++
10.     QueryInterval( b,e,T.RPTR )
11. else  // b < e ¸ H(v)  //search LEFT
12.     while (ML(v).[i] <= e)
13.         ReportIntersection; i++
14.     QueryInterval( b,e,T.LPTR )
```



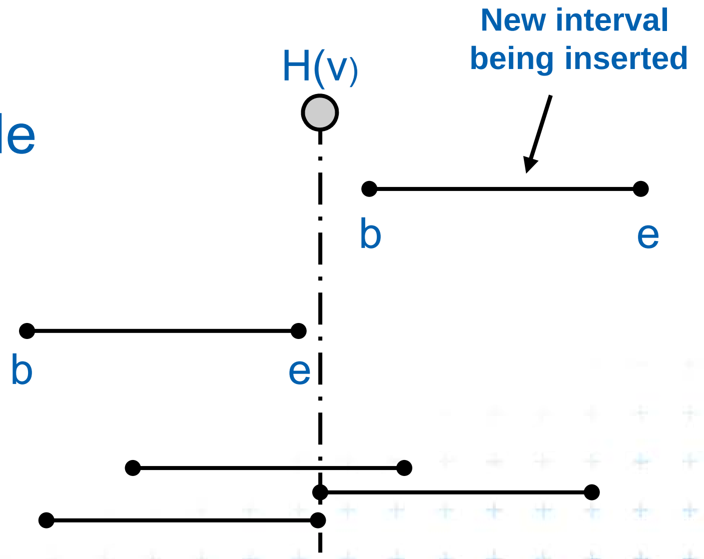
Interval tree - interval insertion

InsertInterval (*b, e, T*)

Input: Interval [*b*,*e*] and interval tree *T*

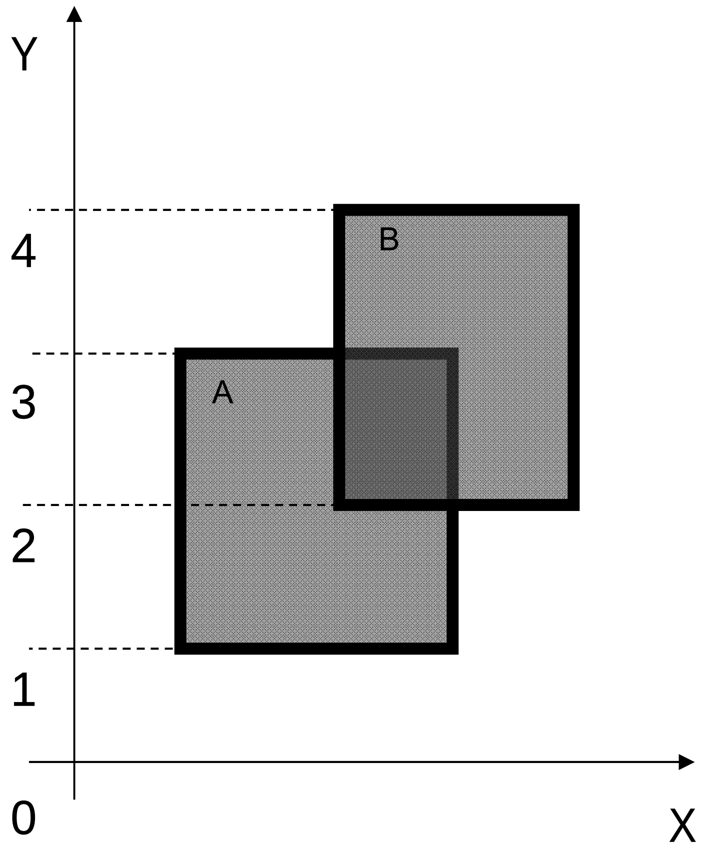
Output: *T* after insertion of the interval

```
1.  v = root(T )
2.  while( v != null )      // find the fork node
3.      if (H(v) < b < e)
4.          v = v.right     // continue right
5.      else if (b < e < H(v))
6.          v = v.left      // continue left
7.      else  // b ¸ H(v) ¸ e  // insert interval
8.          set v node to active
9.          connect LPTR resp. RPTR to its parent
10.         insert [b,e] into list ML(v) – sorted in ascending order of b's
11.         insert [b,e] into list MR(v) – sorted in descending order of e's
12.         break
13. endwhile
14. return T
```

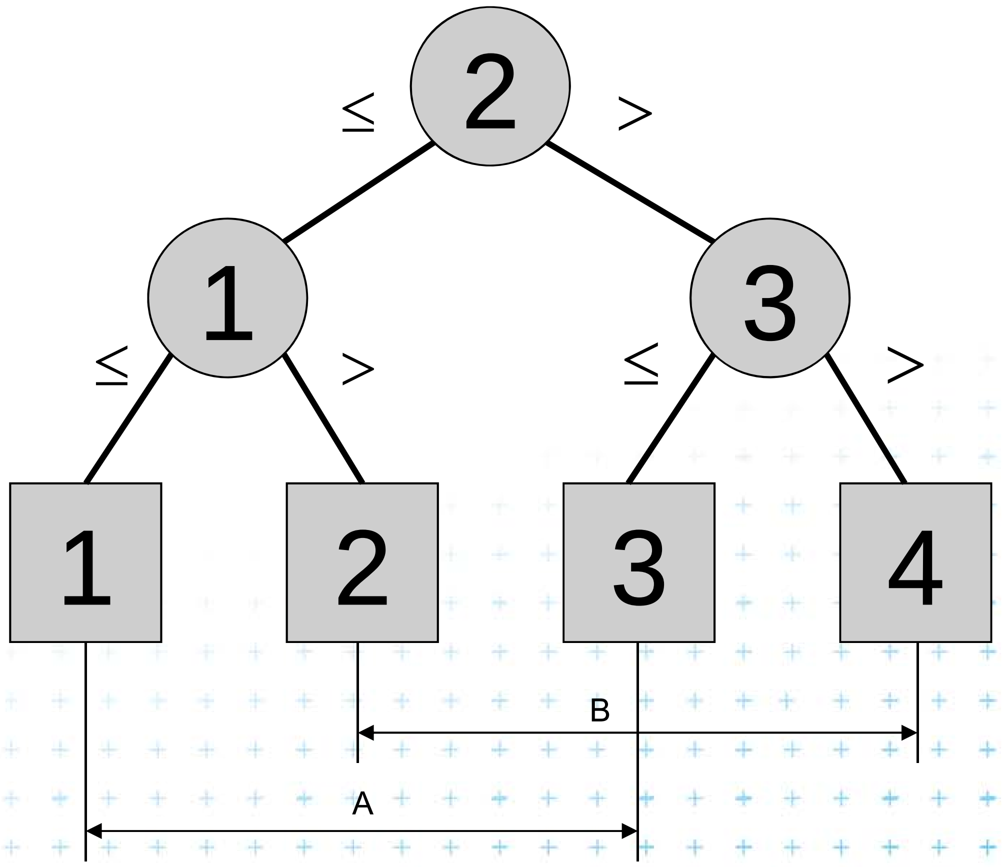


Example 1

Example 1 – static tree on endpoints



H(v) – value of node *v*



[Drtina]

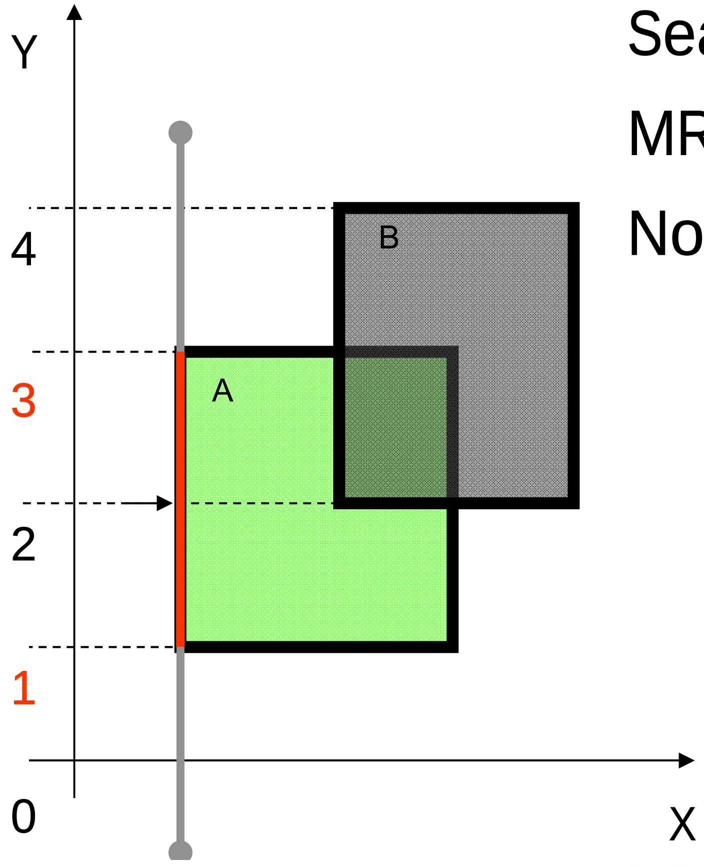
Interval insertion [1,3] a) Query Interval

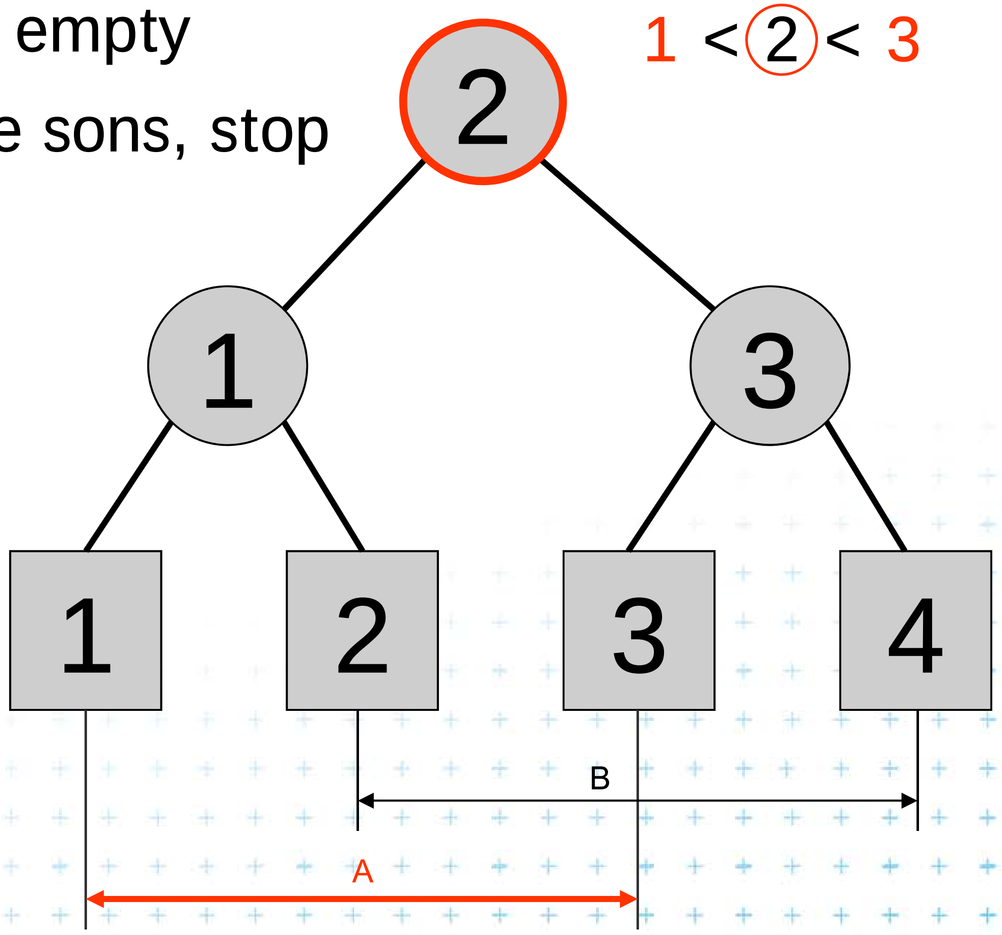
Search MR(v) or ML(v): ← b < H(v) < e

MR(v) is empty

No active sons, stop

$1 < \textcircled{2} < 3$





🟩 Active rectangle

○ Current node

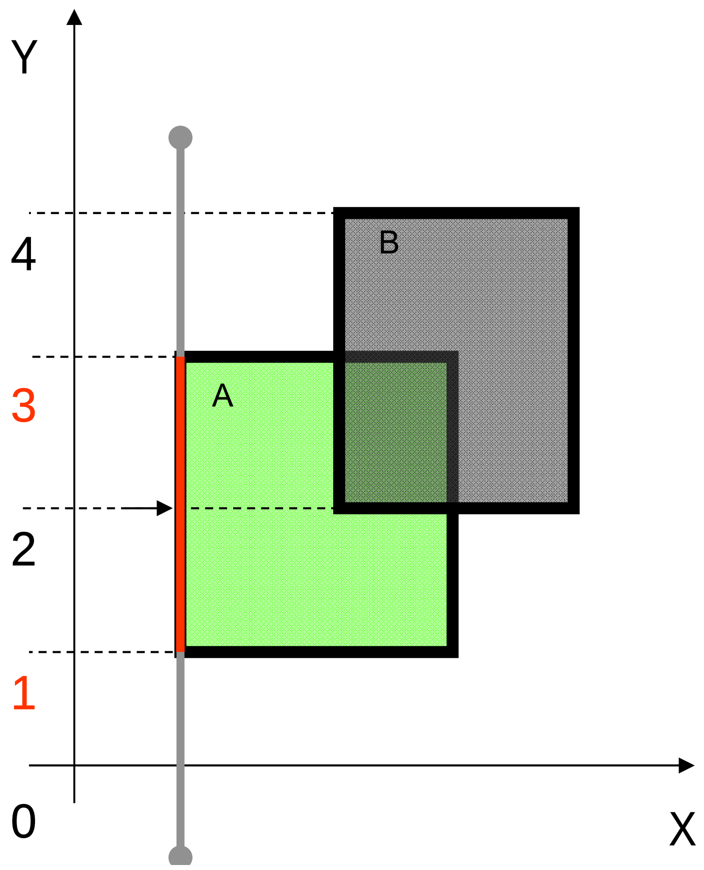
● Active node

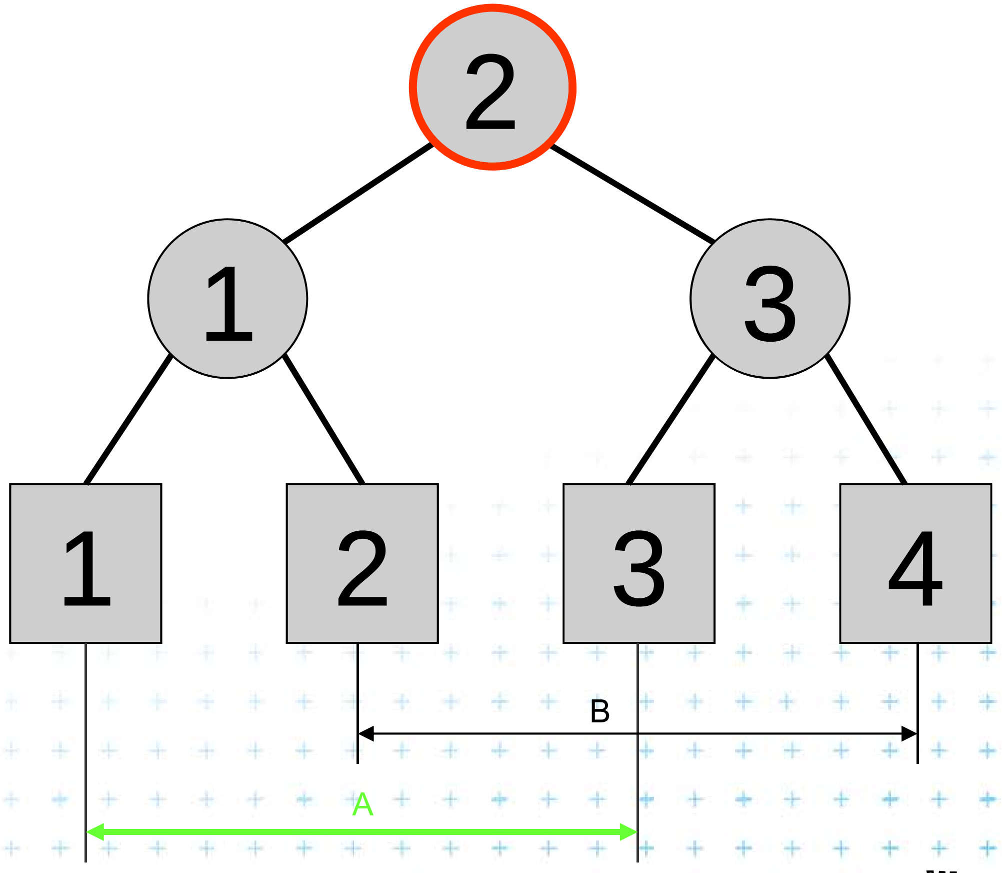
[Drtina]

Interval insertion [1,3]   b) Insert Interval

b ¸ H(v) ¸ e

? 1 ¸ 2 ¸ 3 ?





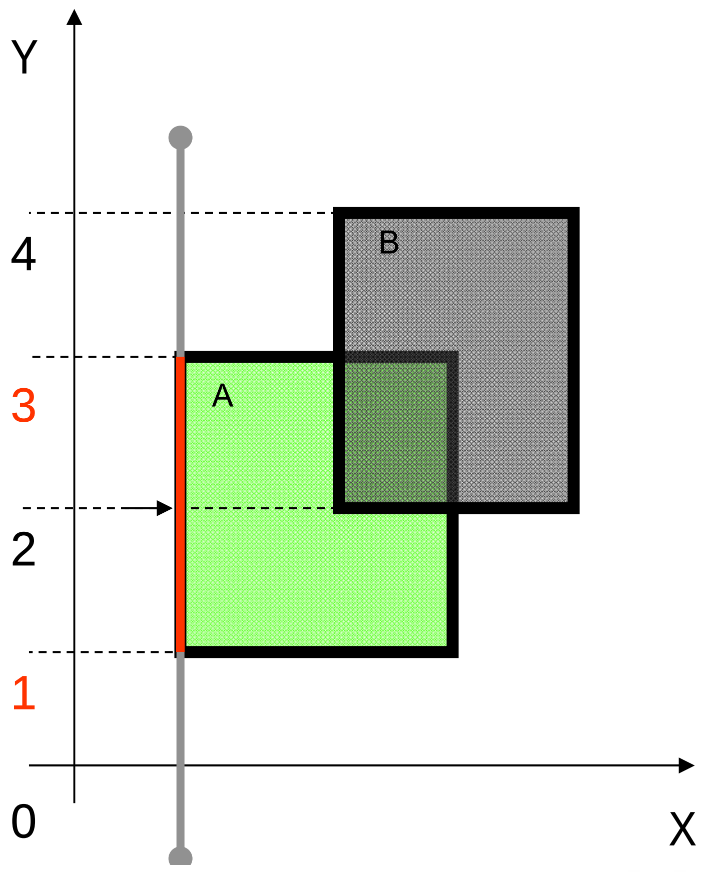
🟩 Active rectangle

⭕ Current node

● Active node

[Drtina]

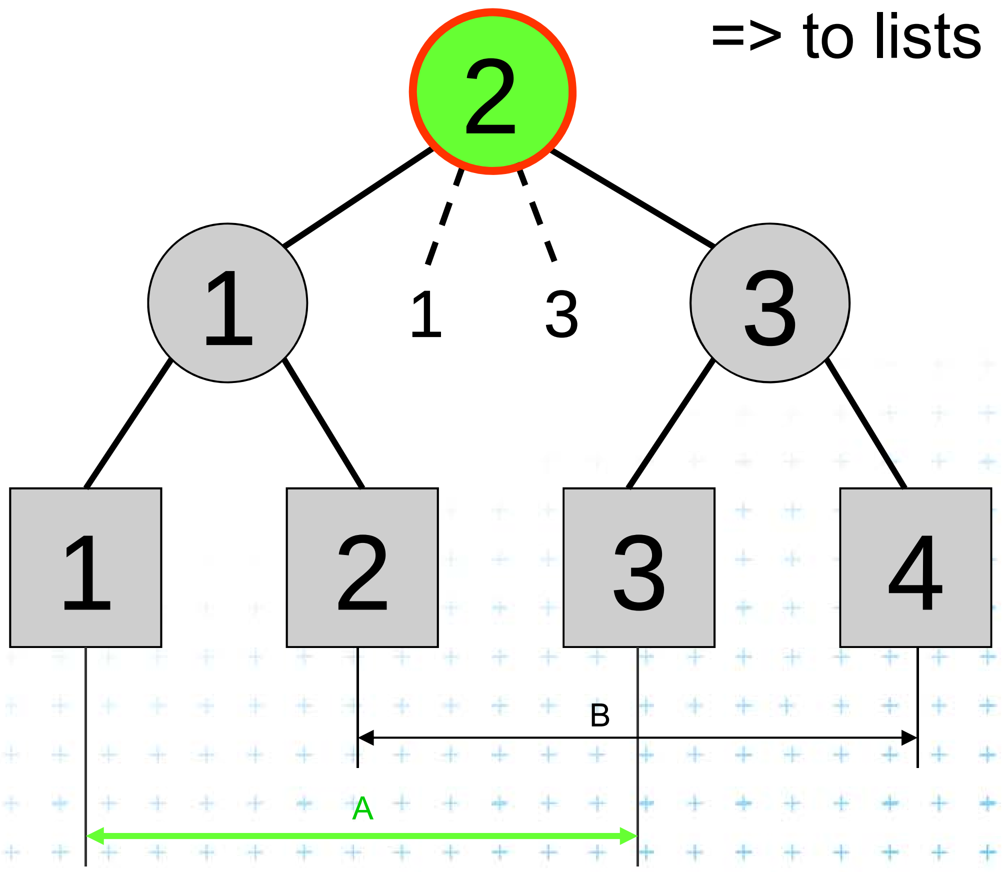
Interval insertion [1,3]   b) Insert Interval



$b \le H(v) \le e$

$1 \le 2 \le 3$

fork
=> to lists



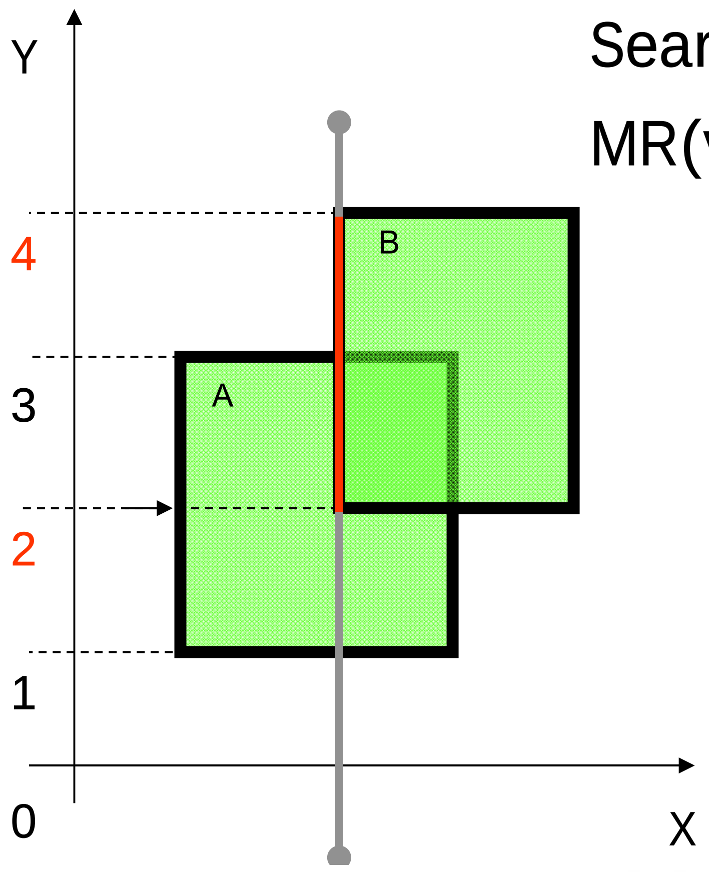
[Drtina]

🟩 Active rectangle

○ Current node

● Active node

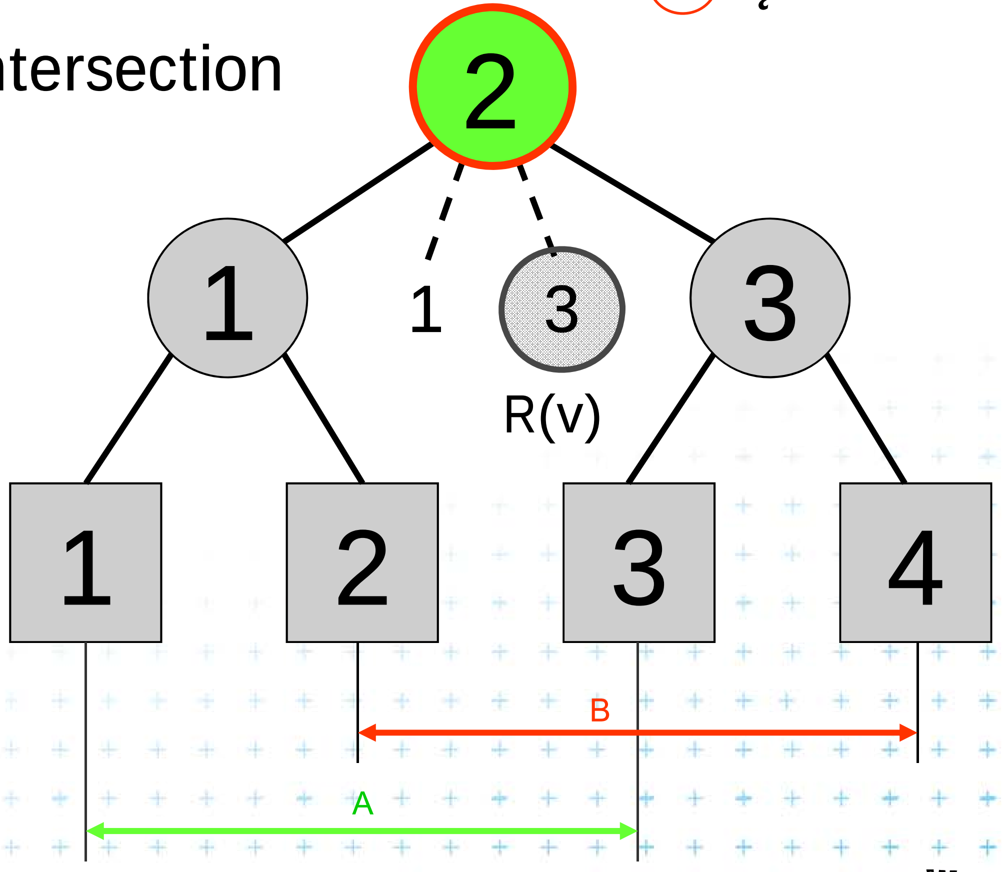
Interval insertion [2,4] a) Query Interval



Search MR(v) only: ← H(v) ¸ b < e

MR(v)[1] = 3 – 2? ② ¸ 2 < 4

=> intersection



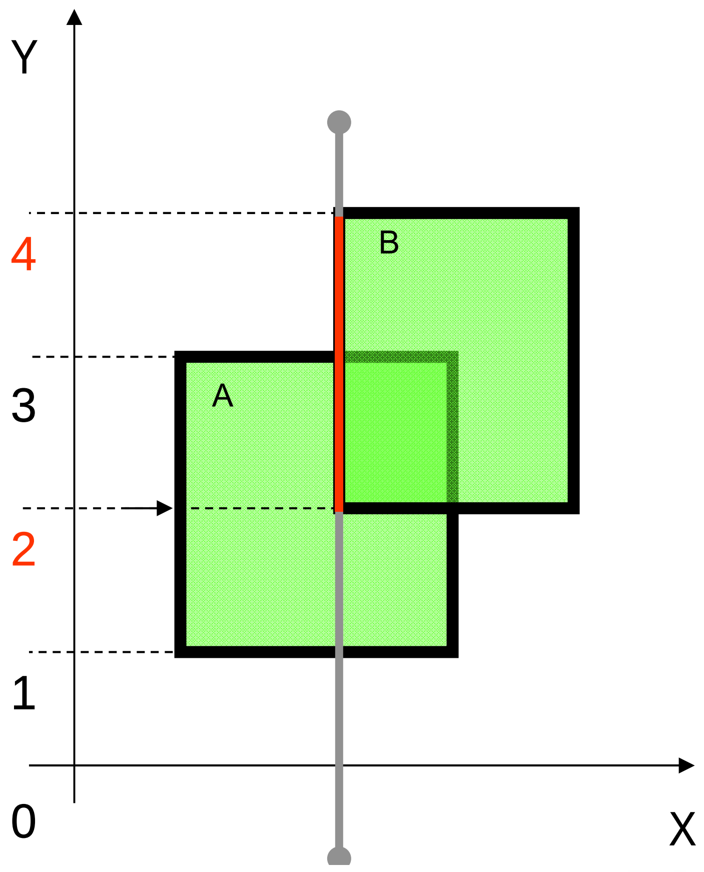
🟩 Active rectangle

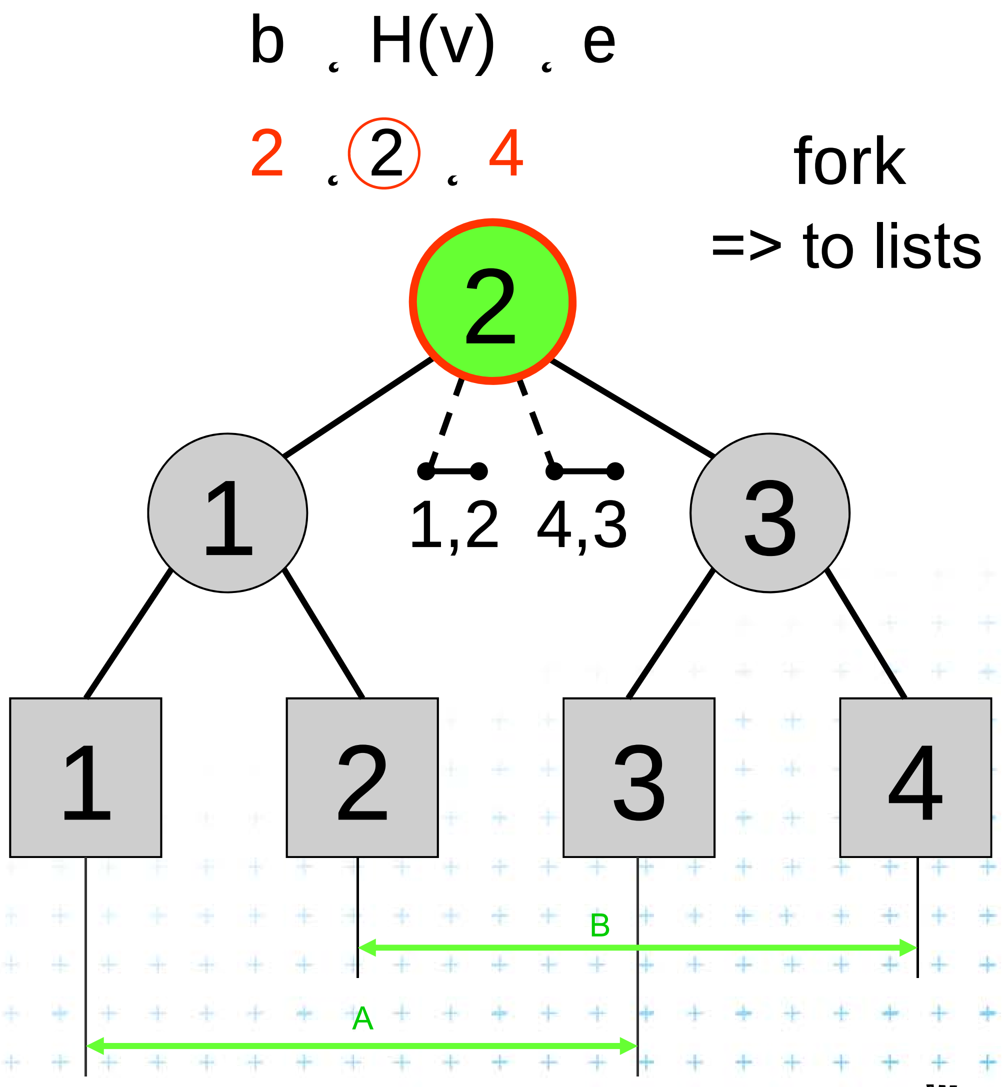
⭕ Current node

● Active node

[Drtina]

Interval insertion [2,4]   b) Insert Interval





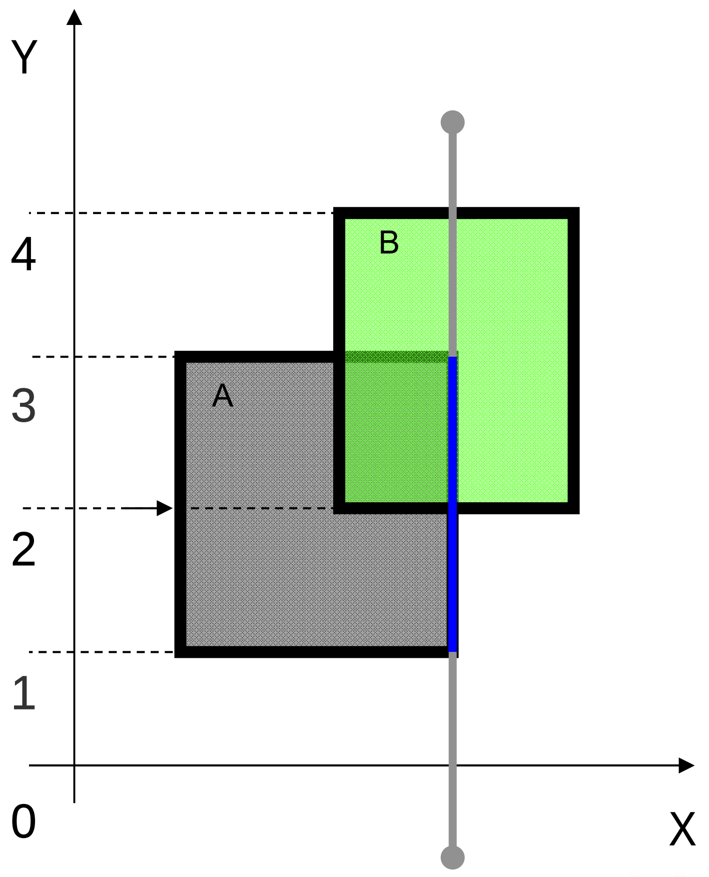
🟩 Active rectangle

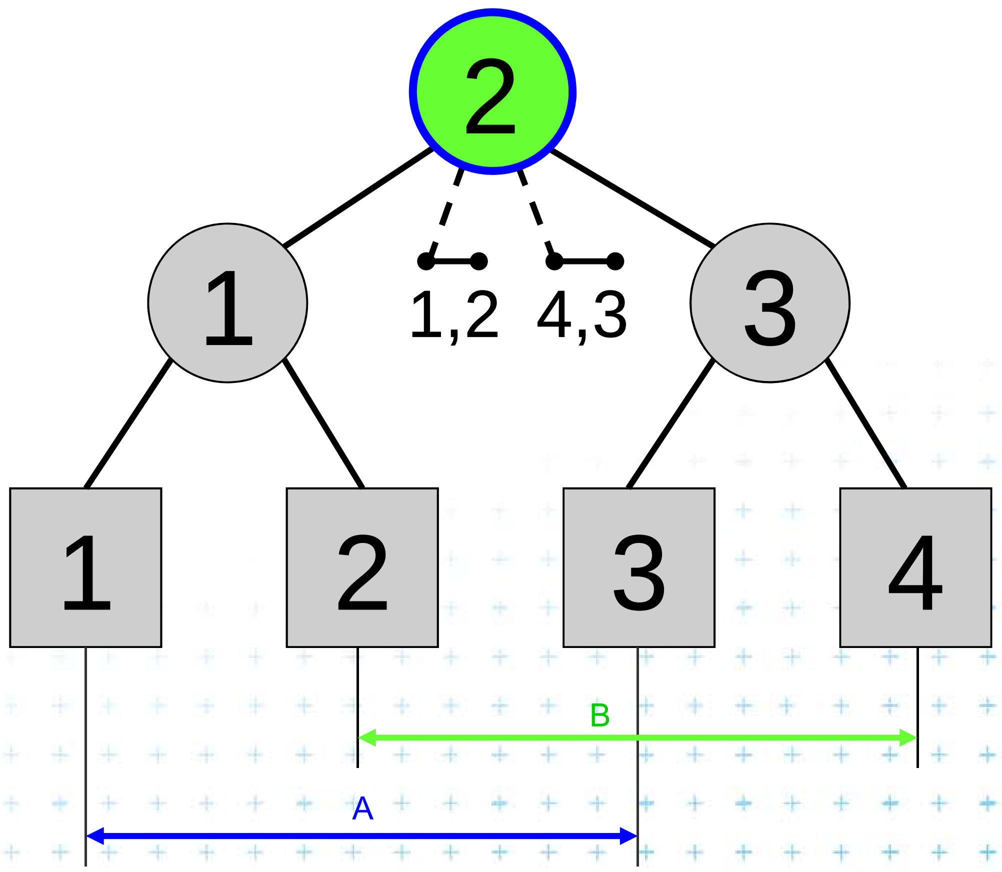
⭕ Current node

● Active node

[Drtina]

Interval delete [1,3]





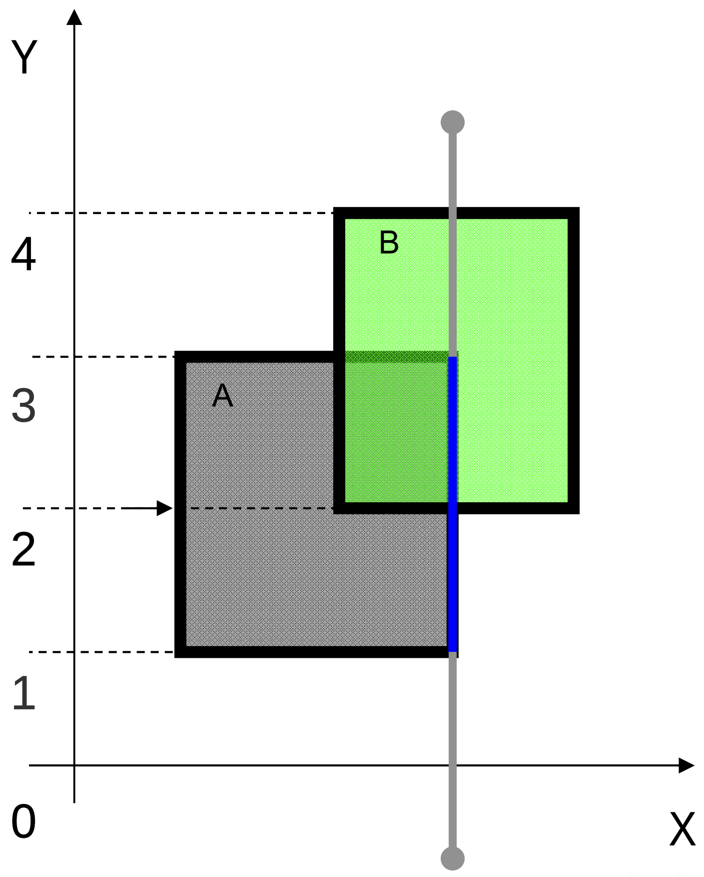
🟩 Active rectangle

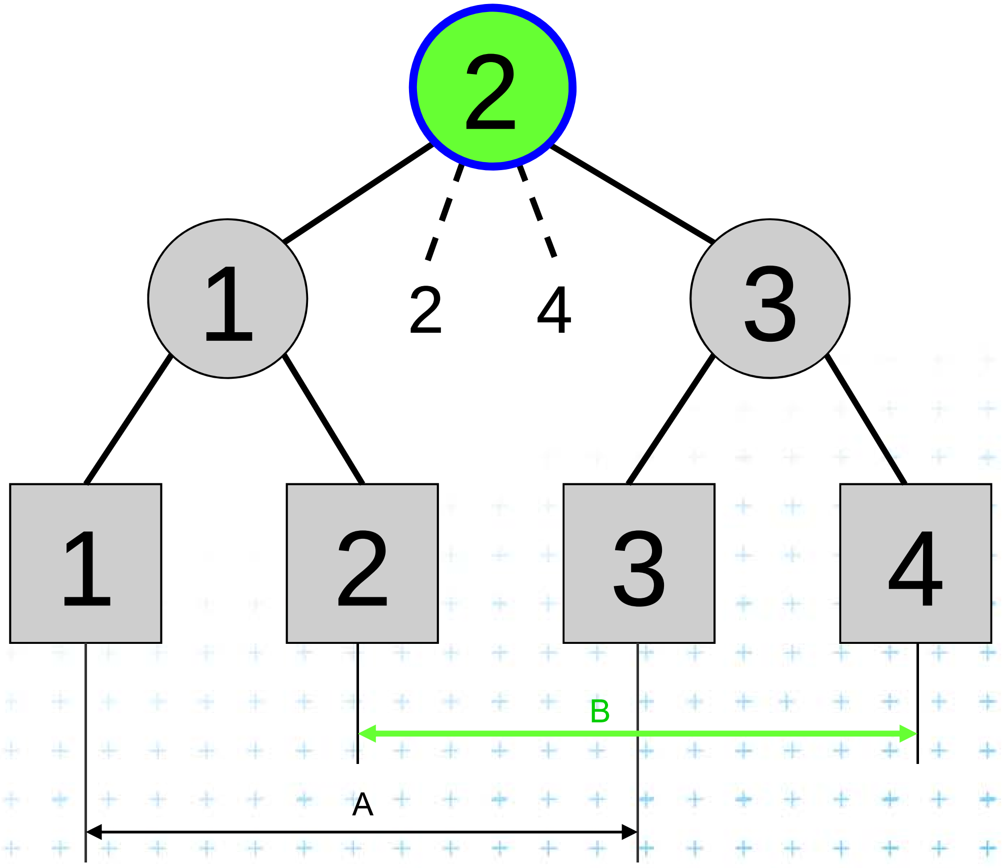
○ Current node

● Active node

[Drtina]

Interval delete [1,3]





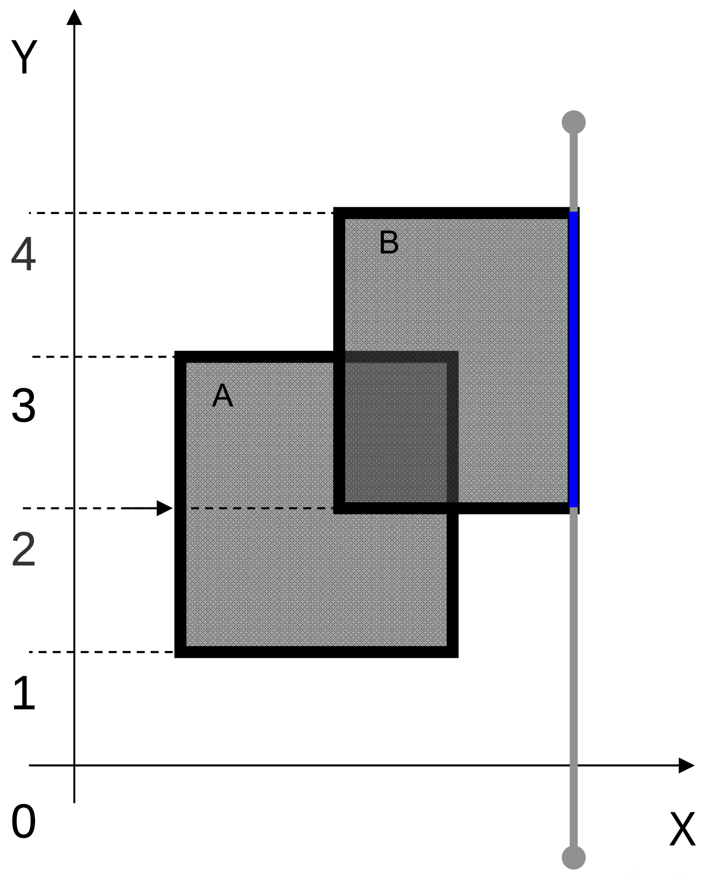
🟩 Active rectangle

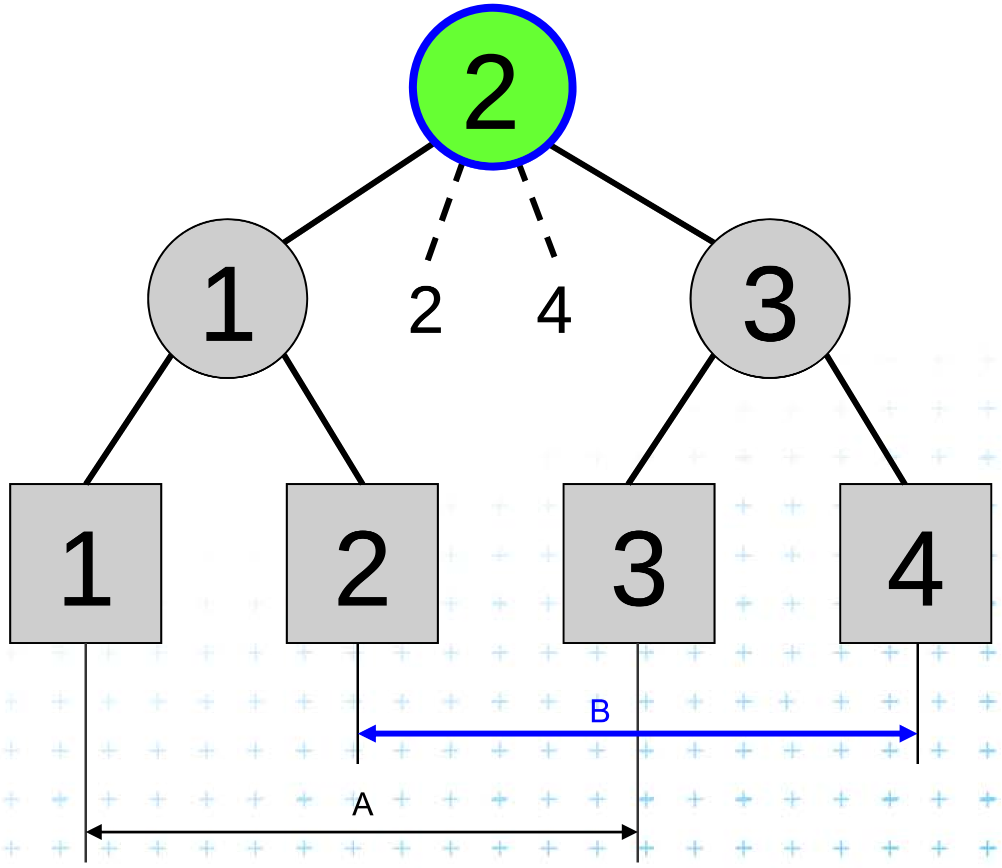
⭕ Current node

● Active node

[Drtina]

Interval delete [2,4]





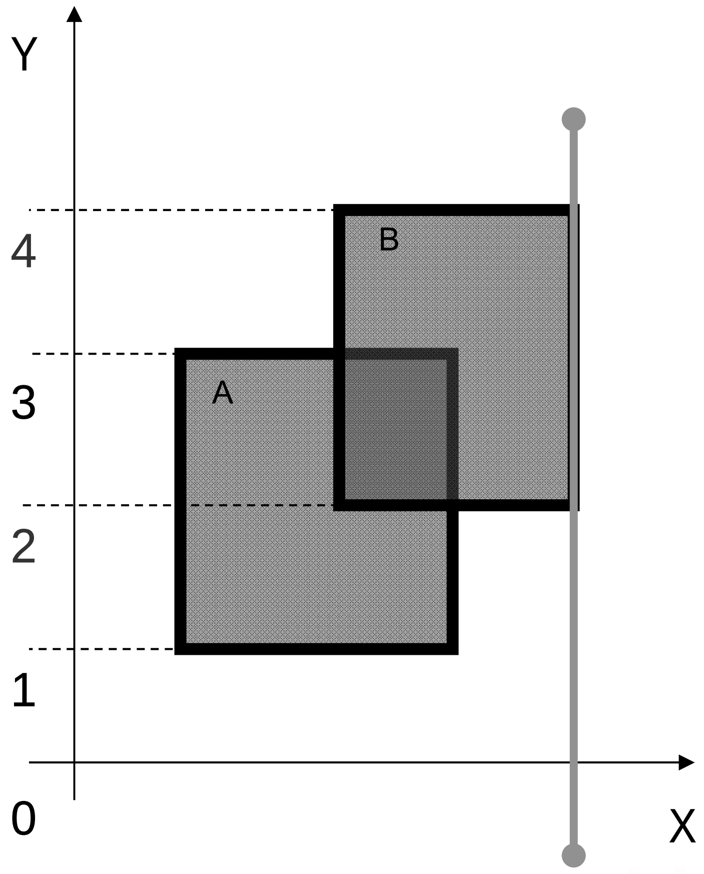
🟩 Active rectangle

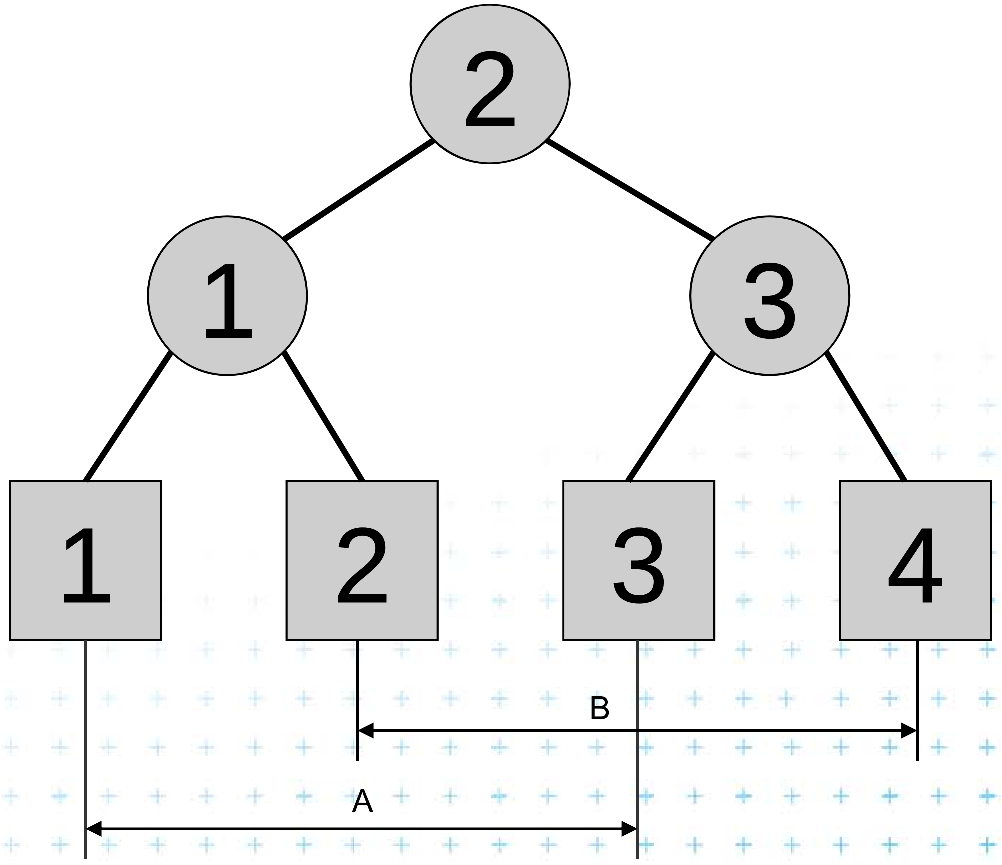
⭕ Current node

● Active node

[Drtina]

Interval delete [2,4]





[Drtina]

Example 2

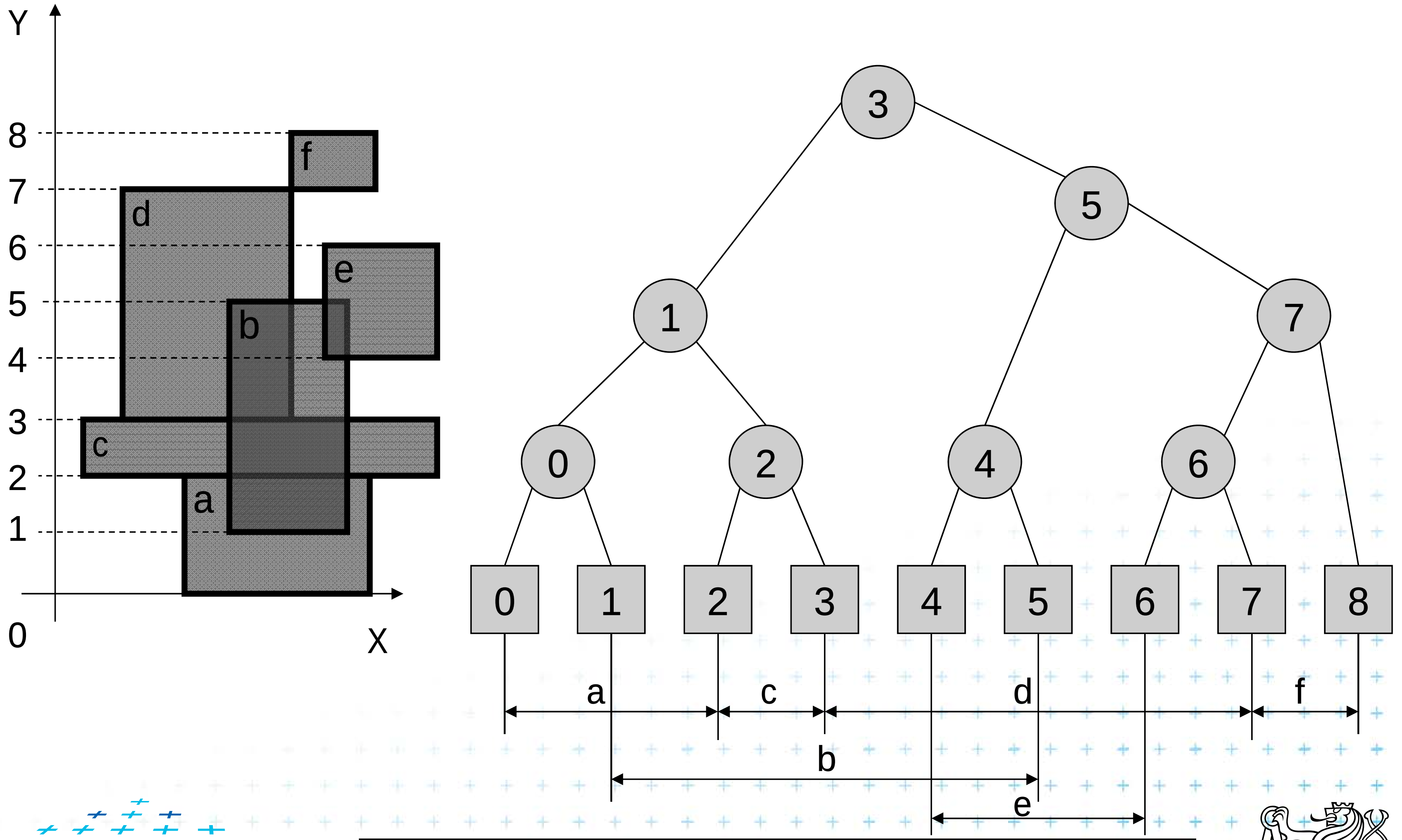
RectangleIntersections(*S*)
Input: Set *S* of rectangles
Output: Intersected rectangle pairs

// this is a copy of the slide before
// just to remember the algorithm

```
1.  Preprocess( S )          // create the interval tree T and event queue Q

2.  while ( Q û þ ) do
3.      Get next entry ($x_{il}$, $y_{il}$, $y_{ir}$, $t$) from Q     // t @ { left | right }
4.      if ( $t$ = left )    // left edge
5.              a) QueryInterval ( $y_{il}$, $y_{ir}$, root($T$))   // report intersections
6.              b) InsertInterval ( $y_{il}$, $y_{ir}$, root($T$))   // insert new interval
7.      else                // right edge
8.              c) DeleteInterval ( $y_{il}$, $y_{ir}$, root($T$) )
```

Example 2



[Drtina]

Insert [2,3] – empty => b) Insert Interval

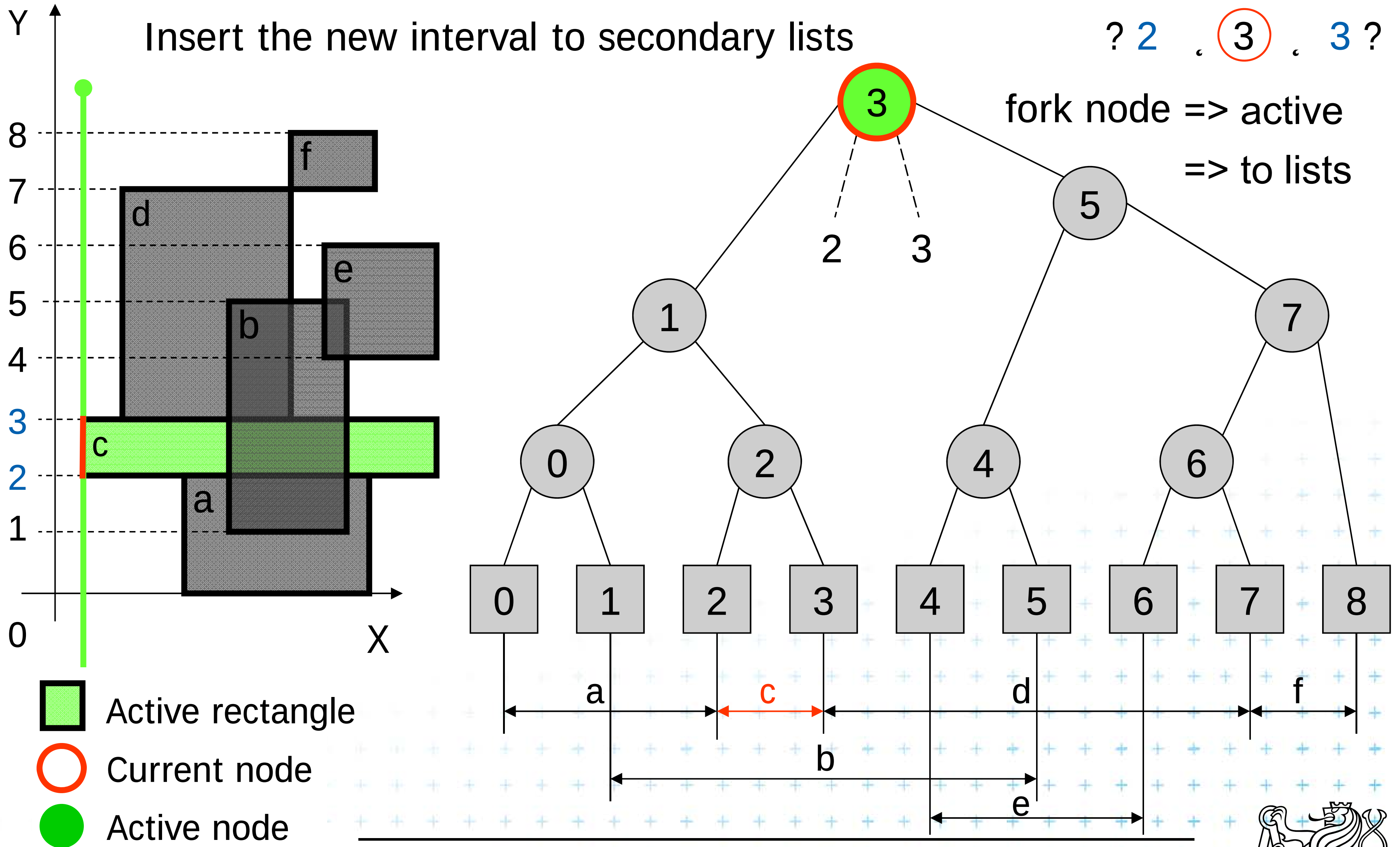
b ˛ H(v) ˛ e

Insert the new interval to secondary lists

? 2 ˛ 3 ˛ 3 ?

fork node => active

=> to lists



🟩 Active rectangle

⭕ Current node

● Active node

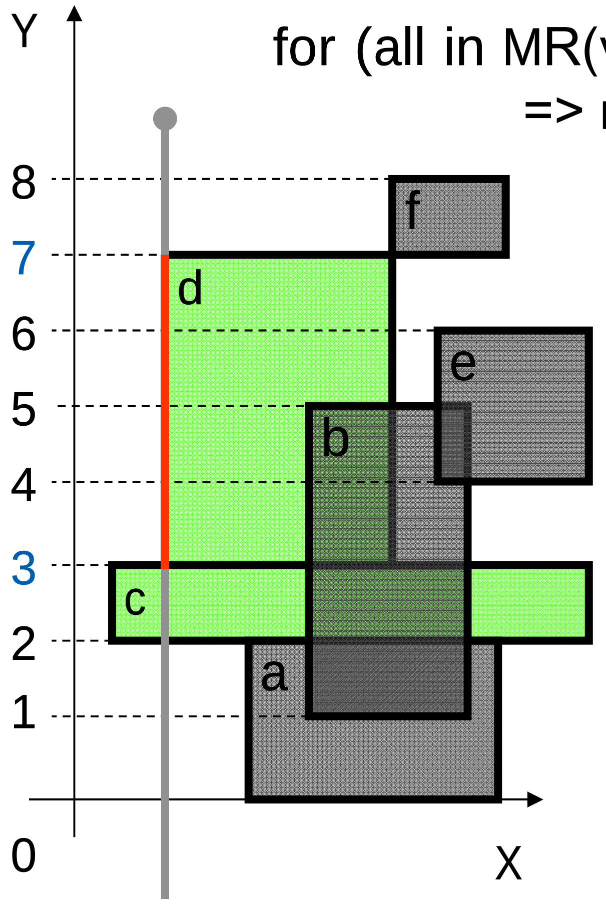
[Drtina]

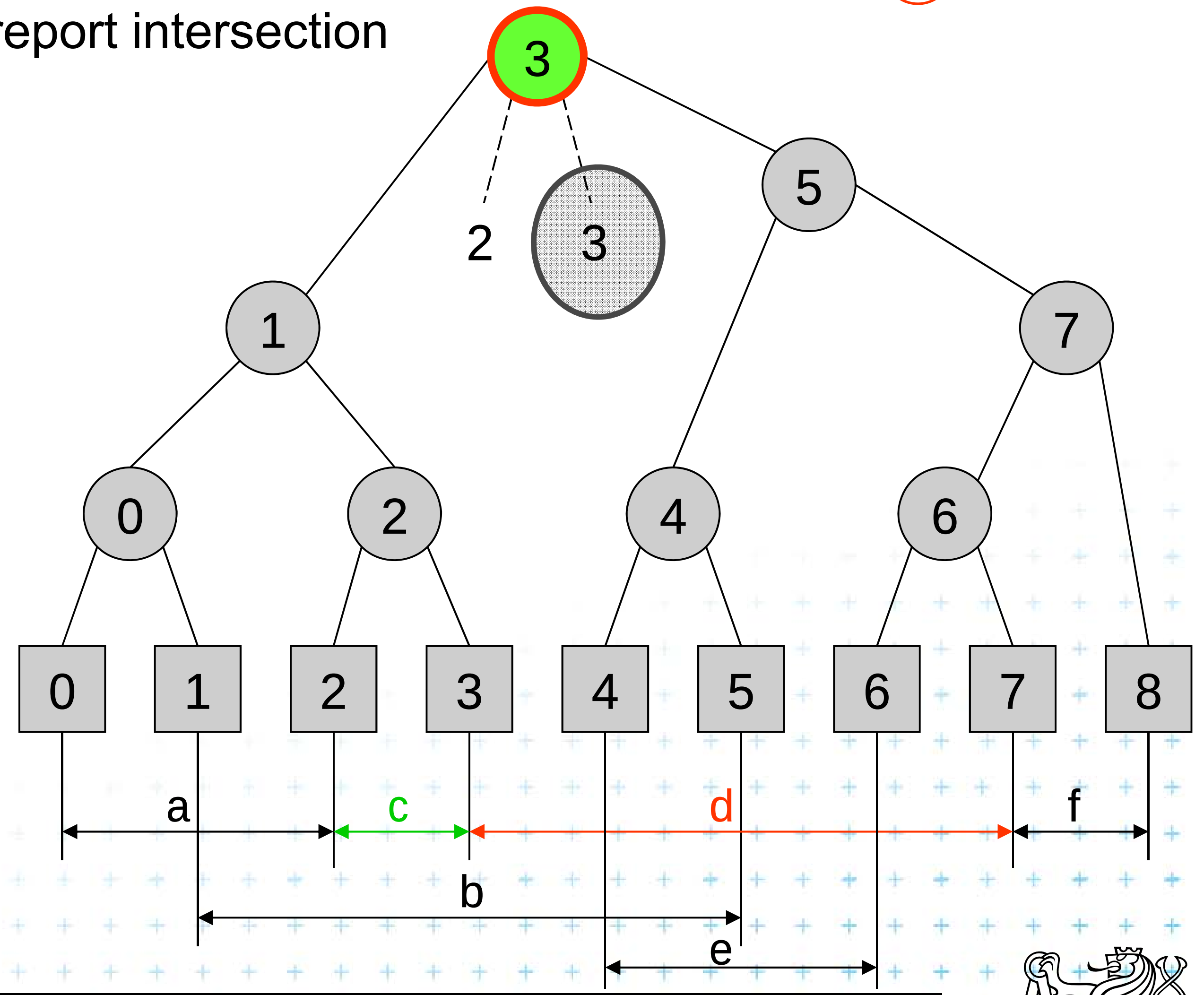
Insert [3,7] a) Query Interval

H(v) ¸ b ? ≠e

? 3 ¸ 3 ? 7 ?

for (all in MR(v)) test MR(v)[i] >= 3
=> report intersection





- 🟩 Active rectangle
- ○ Current node
- ● Active node

[Drtina]

Insert [3,7] b) Insert Interval

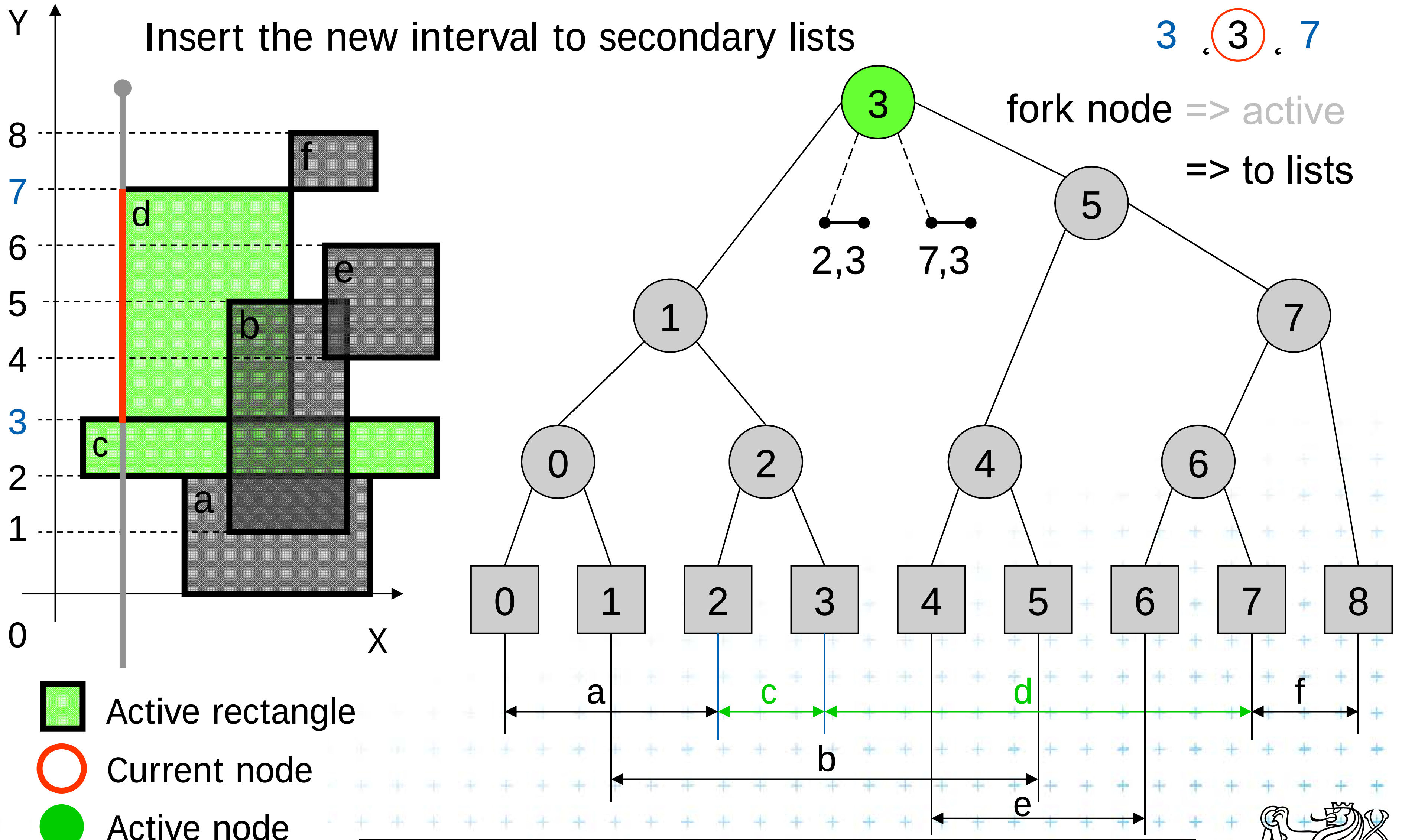
$b \le H(v) \le e$

Insert the new interval to secondary lists

$3 \le \boxed{3} \le 7$

fork node => active

=> to lists



🟩 Active rectangle

⭕ Current node

● Active node

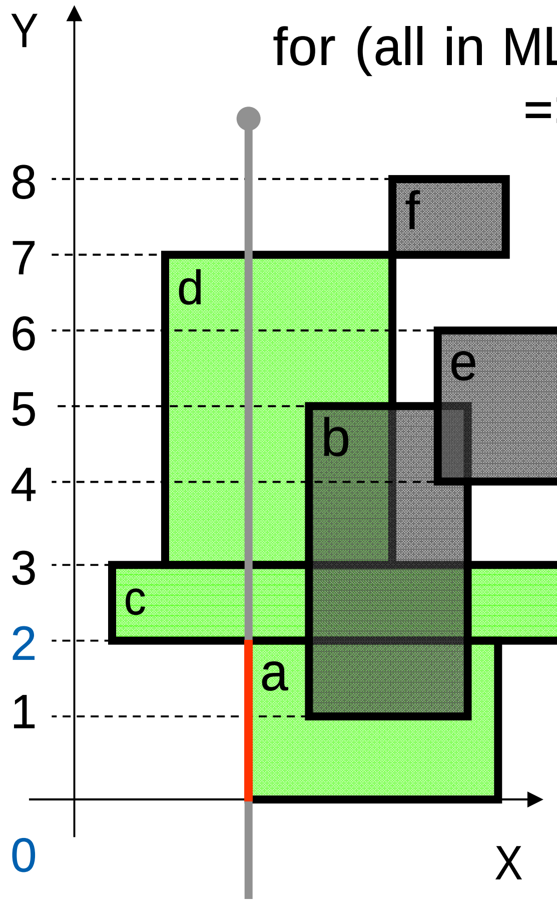
[Drtina]

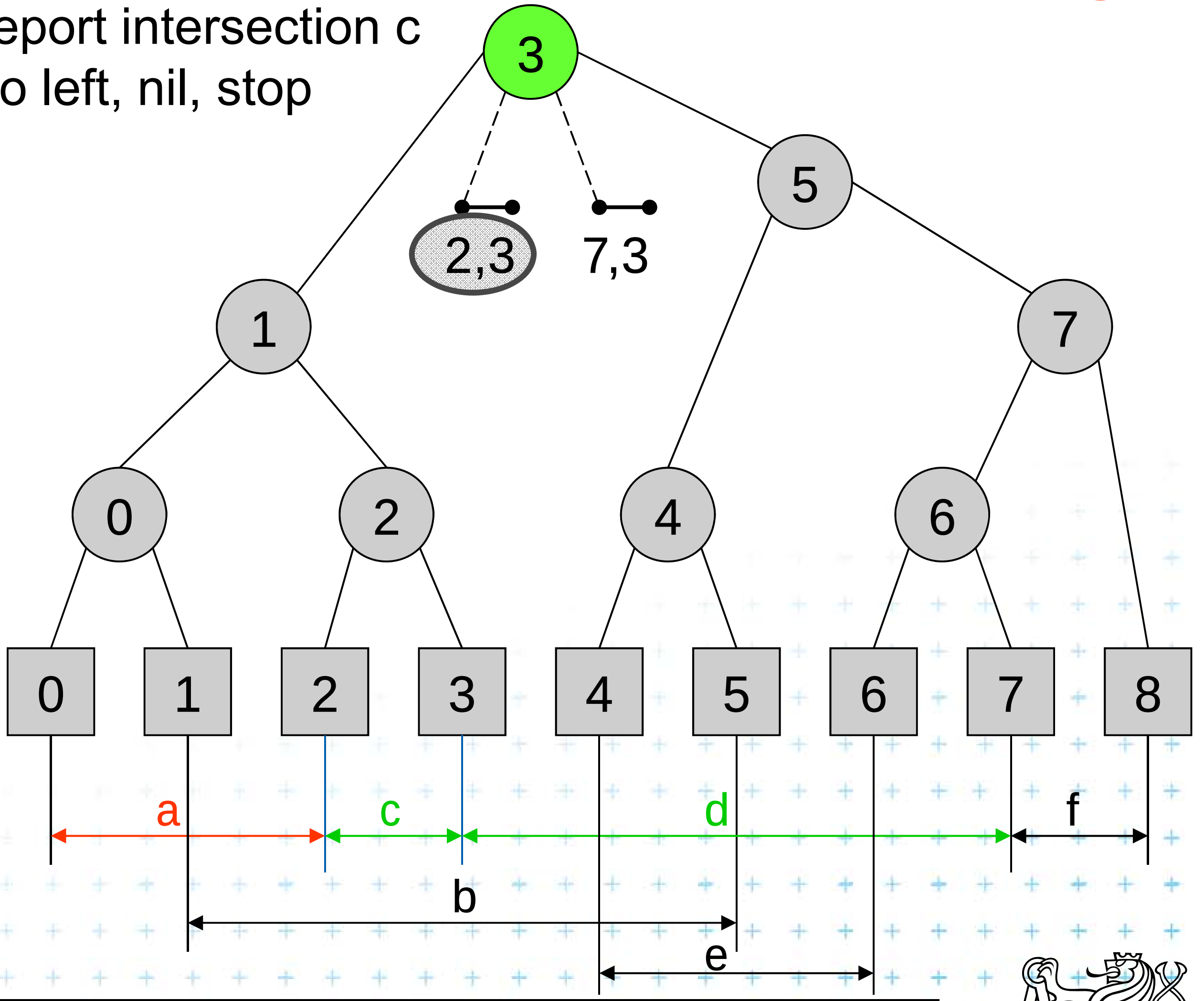
Insert [0,2] a) Query Interval

b ?∉ ¸ H(v)

? 0 ? 2 ¸ 3 ?

for (all in ML(v)) test ML(v).[i] ¸ 2
=> report intersection c
go left, nil, stop





🟩 Active rectangle

○ Current node

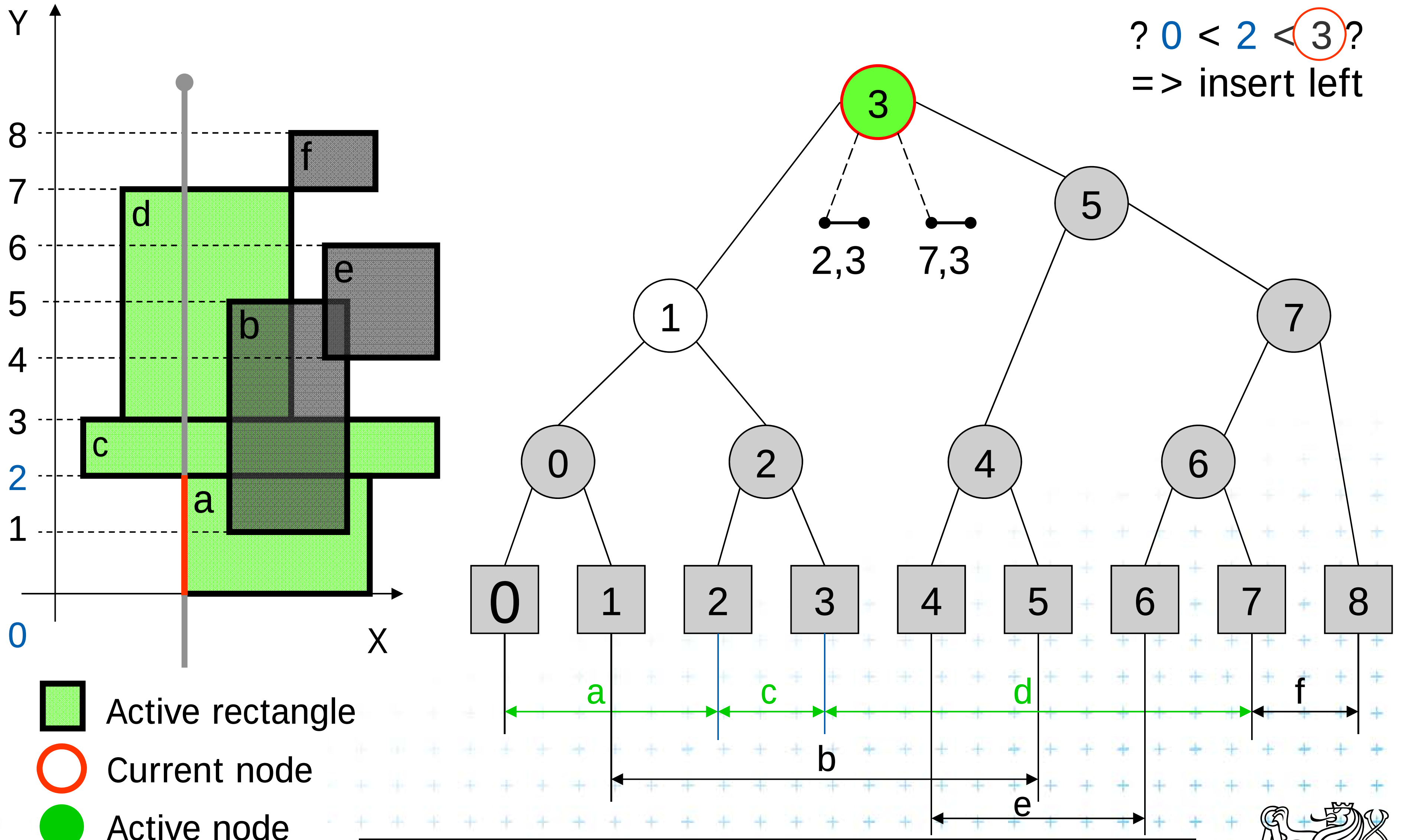
● Active node

[Drtina]

Insert [0,2] b) Insert Interval 1/2

b ? e <H(v)

? 0 < 2 < 3 ?
=> insert left



🟩 Active rectangle

○ Current node

● Active node

[Drtina]

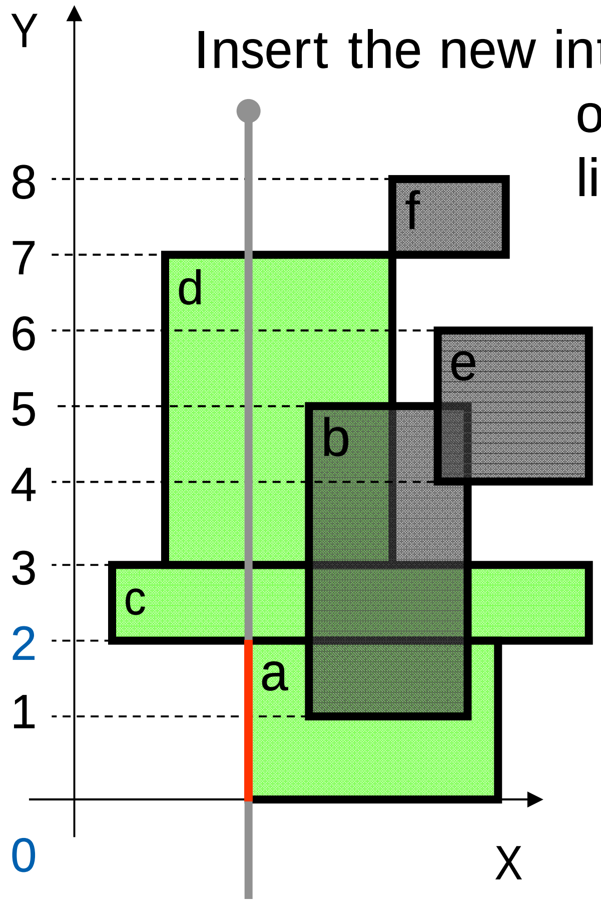
Insert [0,2] b) Insert Interval 2/2

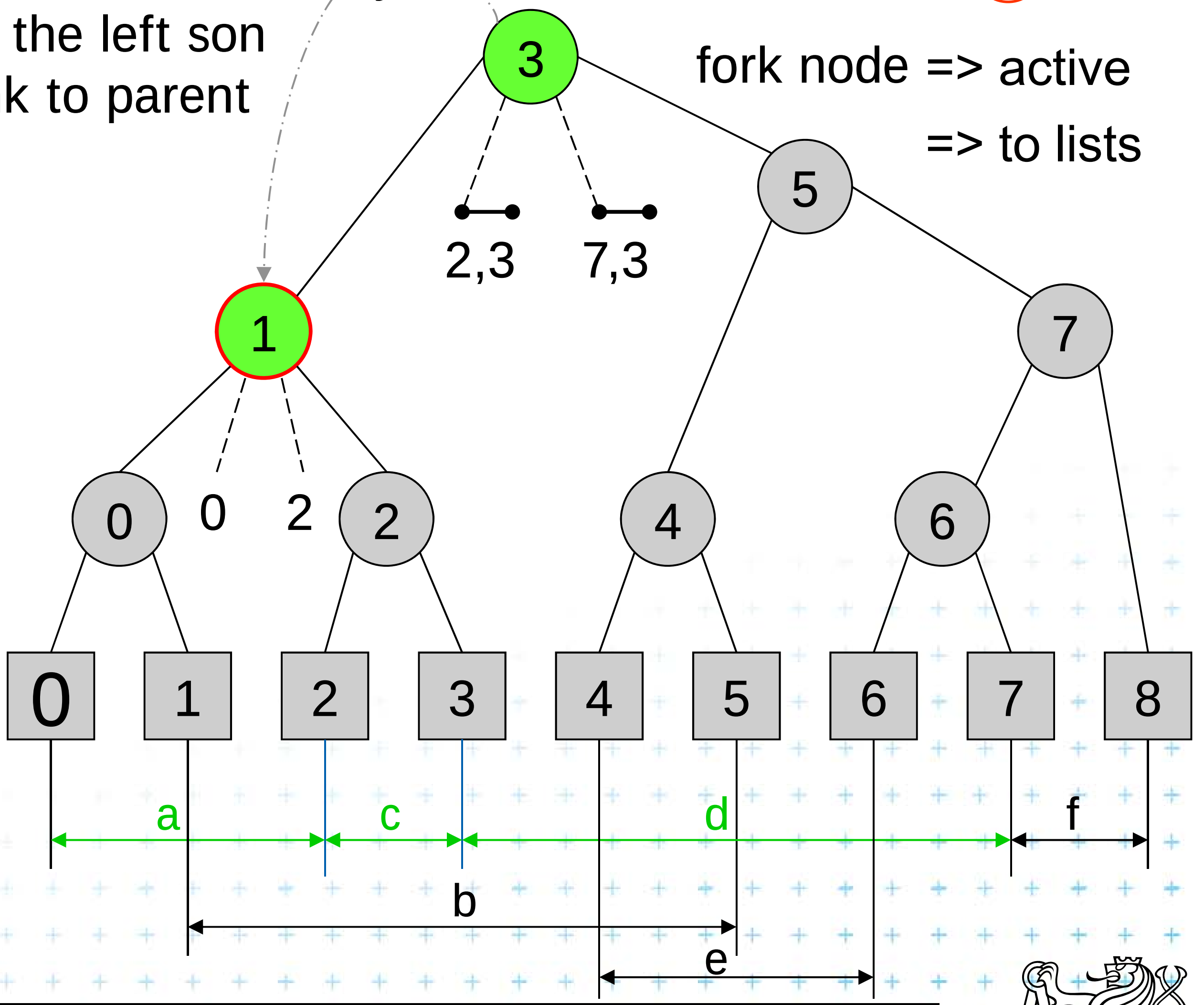
b ˛ H(v) ˛ e

? 0 ˛ 1 ˛ 2 ?

Insert the new interval to secondary lists
of the left son
link to parent

fork node => active
=> to lists





🟩 Active rectangle
⭕ Current node
● Active node

Insert [1,5] a) Query Interval 1/2

b ? H(v) < e

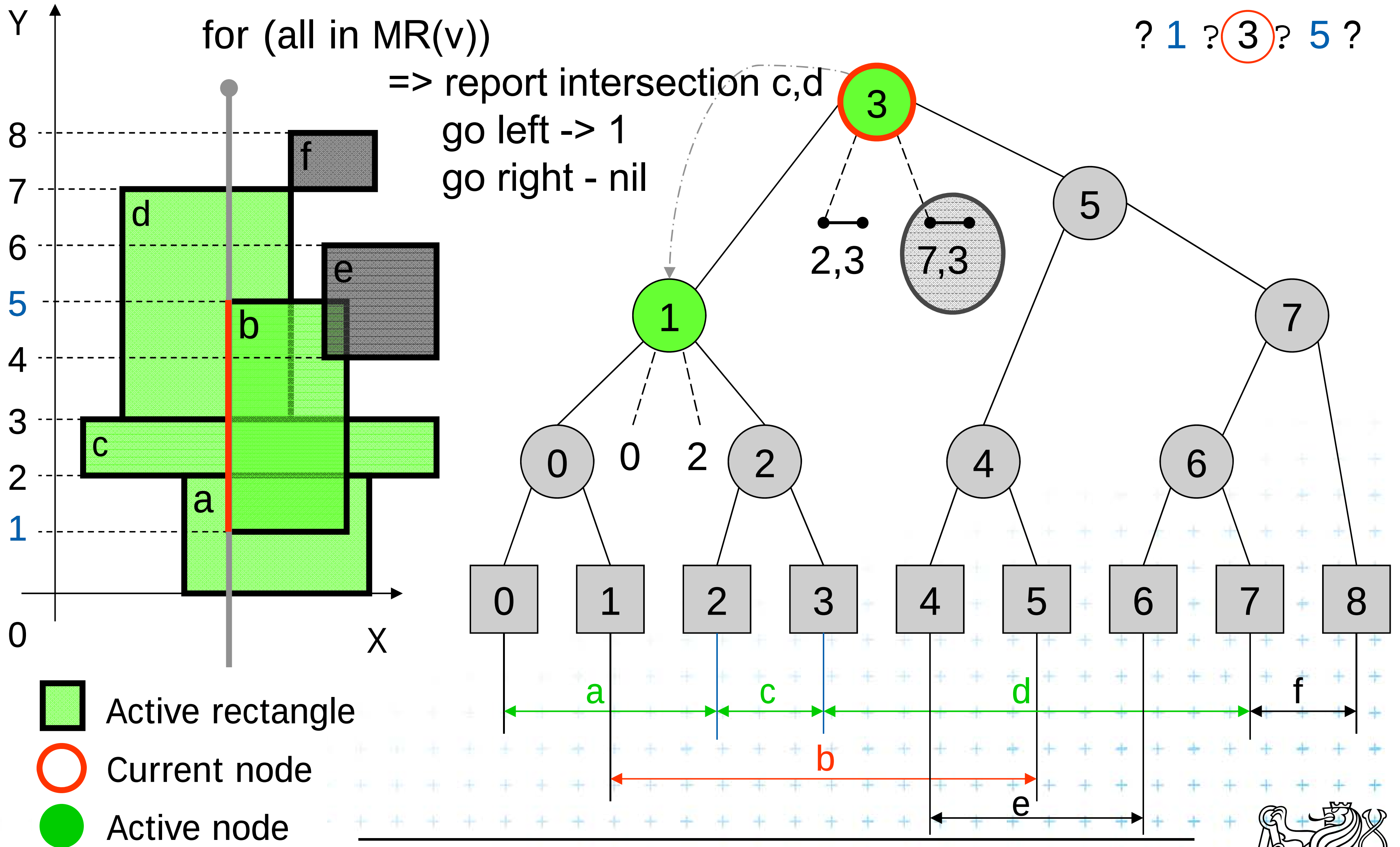
? 1 ? 3 ? 5 ?

for (all in MR(v))

=> report intersection c,d

go left -> 1

go right - nil



🟩 Active rectangle

⭕ Current node

● Active node

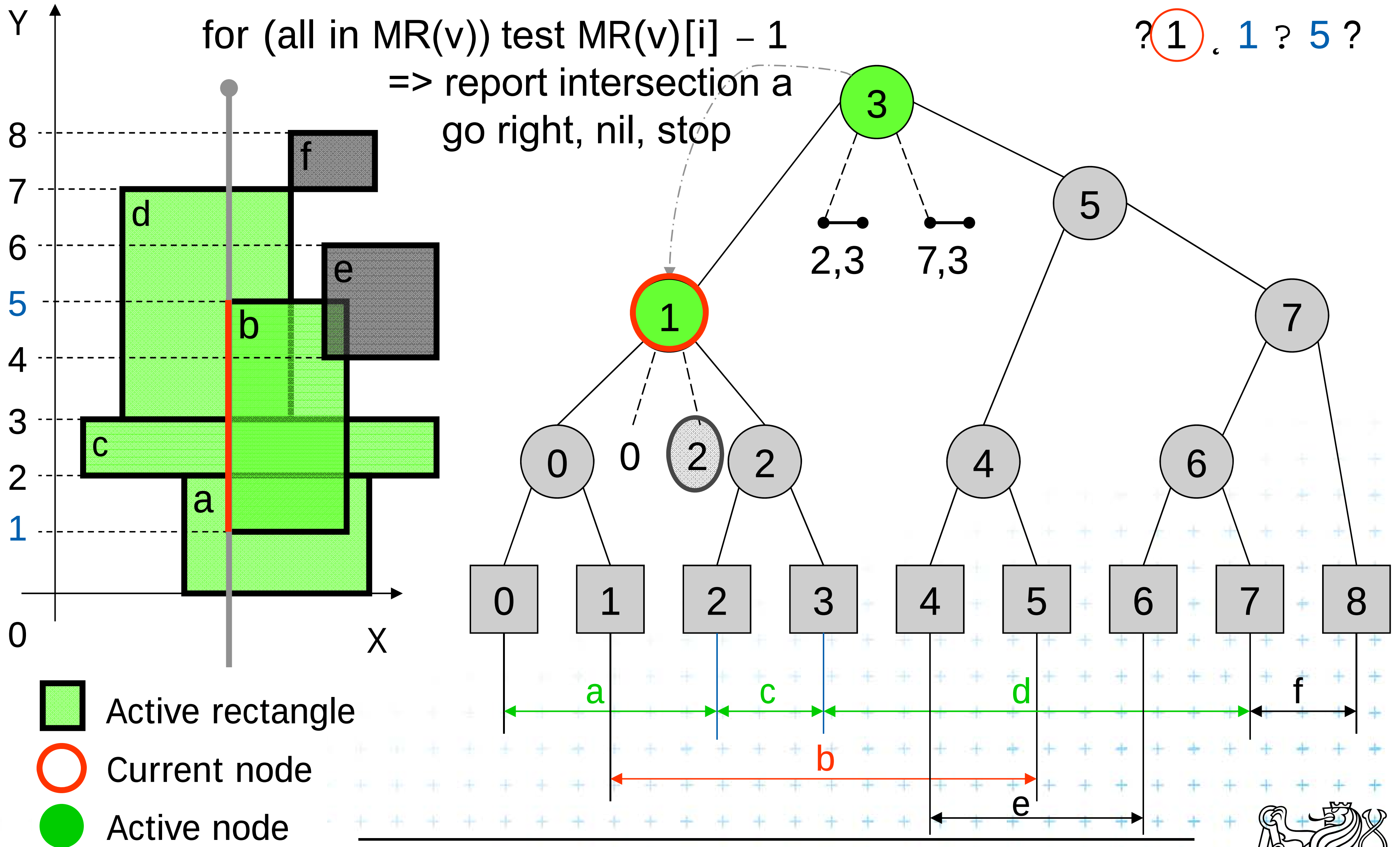
[Drtina]

Insert [1,5] a) Query Interval 2/2

H(v) ¸ b ? e

? 1 ¸ 1 ? 5 ?

for (all in MR(v)) test MR(v)[i] – 1
=> report intersection a
go right, nil, stop



🟩 Active rectangle

⭕ Current node

● Active node

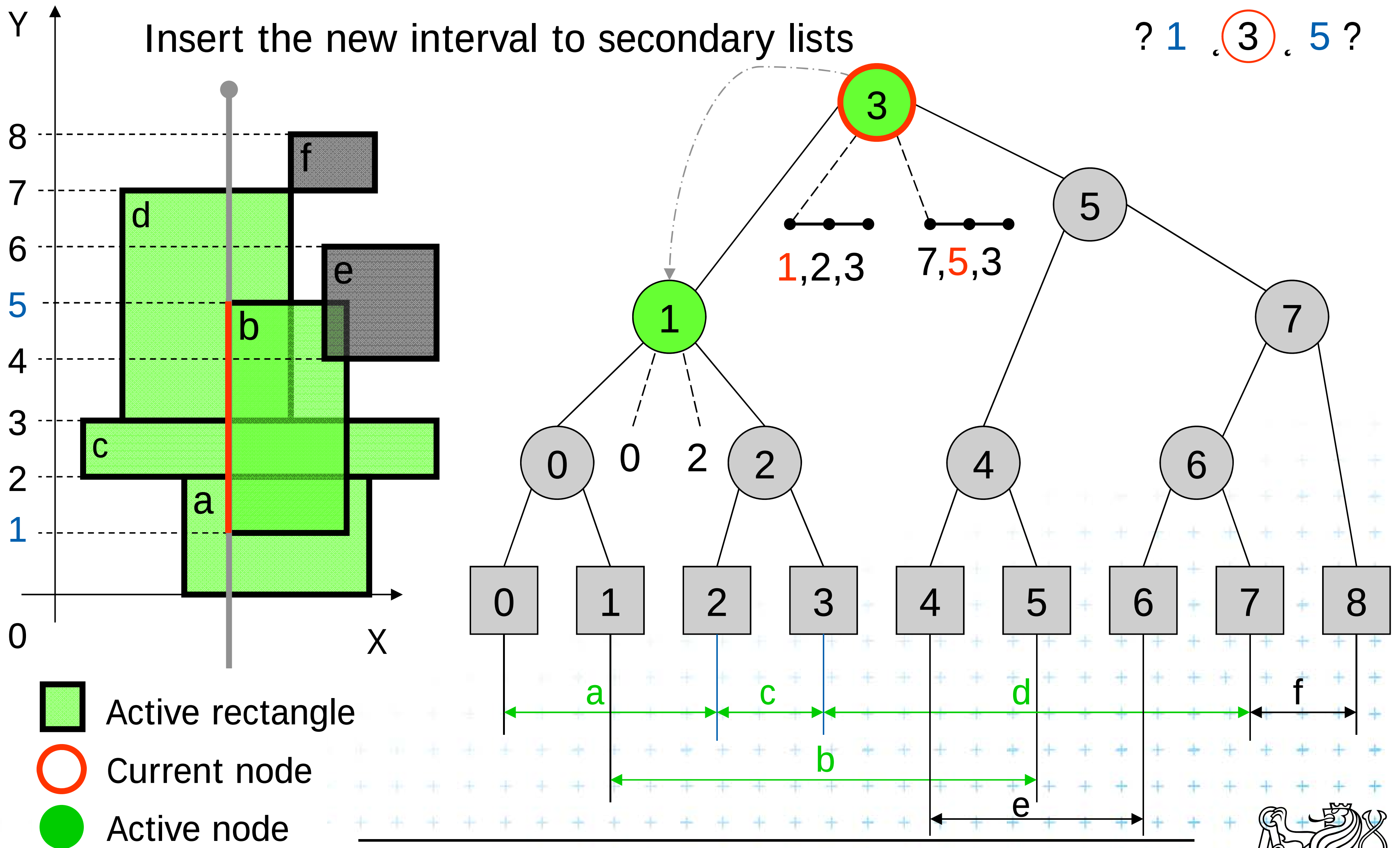
[Drtina]

Insert [1,5] b) Insert Interval

$b \le H(v) \le e$

Insert the new interval to secondary lists

? $1 \le \boxed{3} \le 5$?



🟩 Active rectangle

⭕ Current node

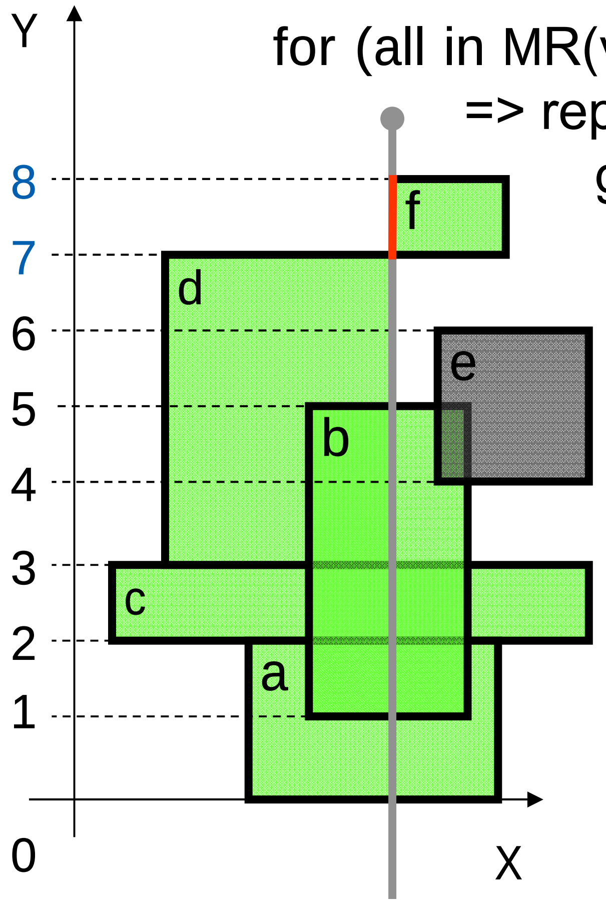
● Active node

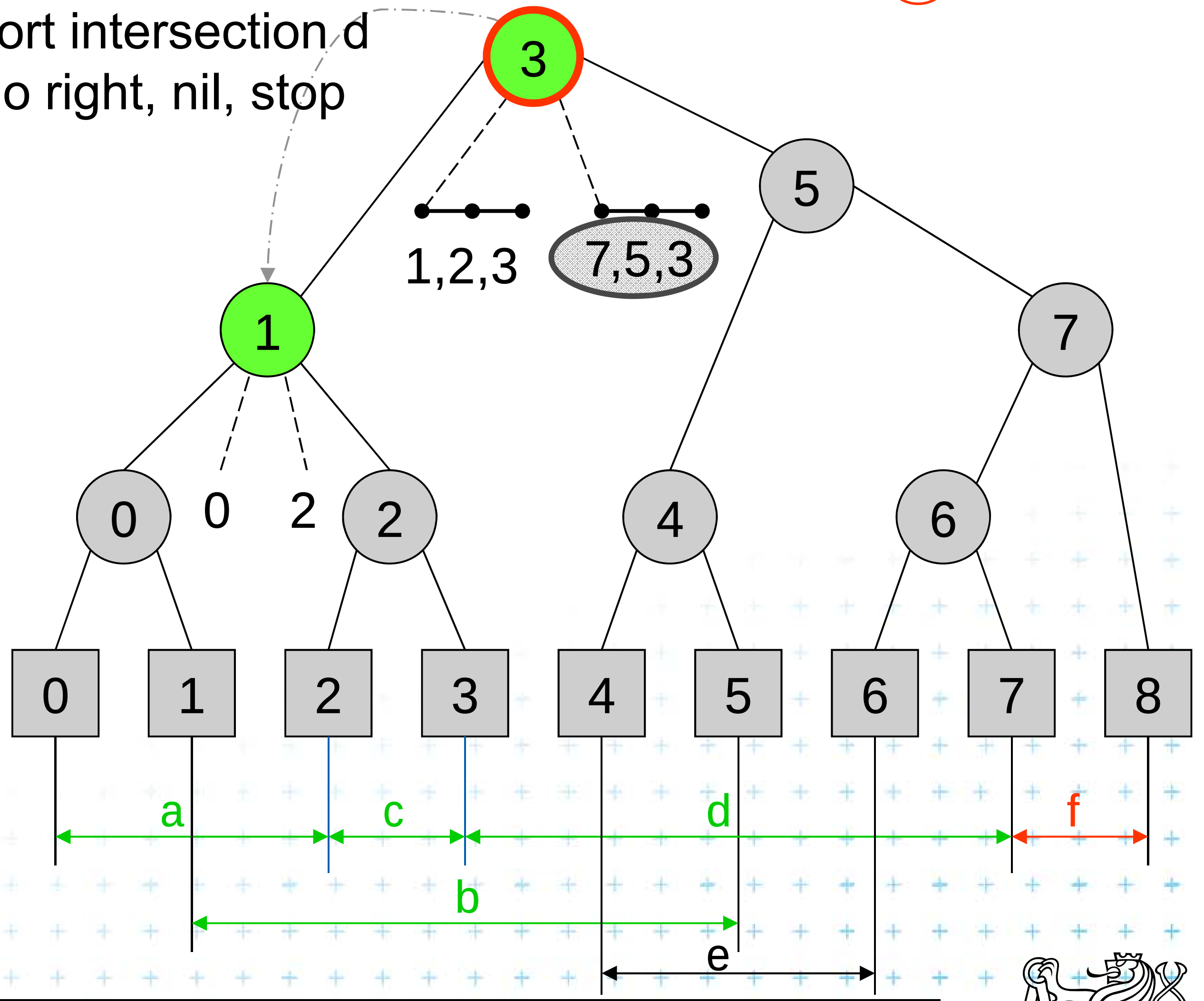
Insert [7,8] a) Query Interval

H(v) ¸ b ? e

? 3 ¸ 7 ? 8 ?

for (all in MR(v)) test MR(v).[i] – 7
=> report intersection d
go right, nil, stop





🟩 Active rectangle

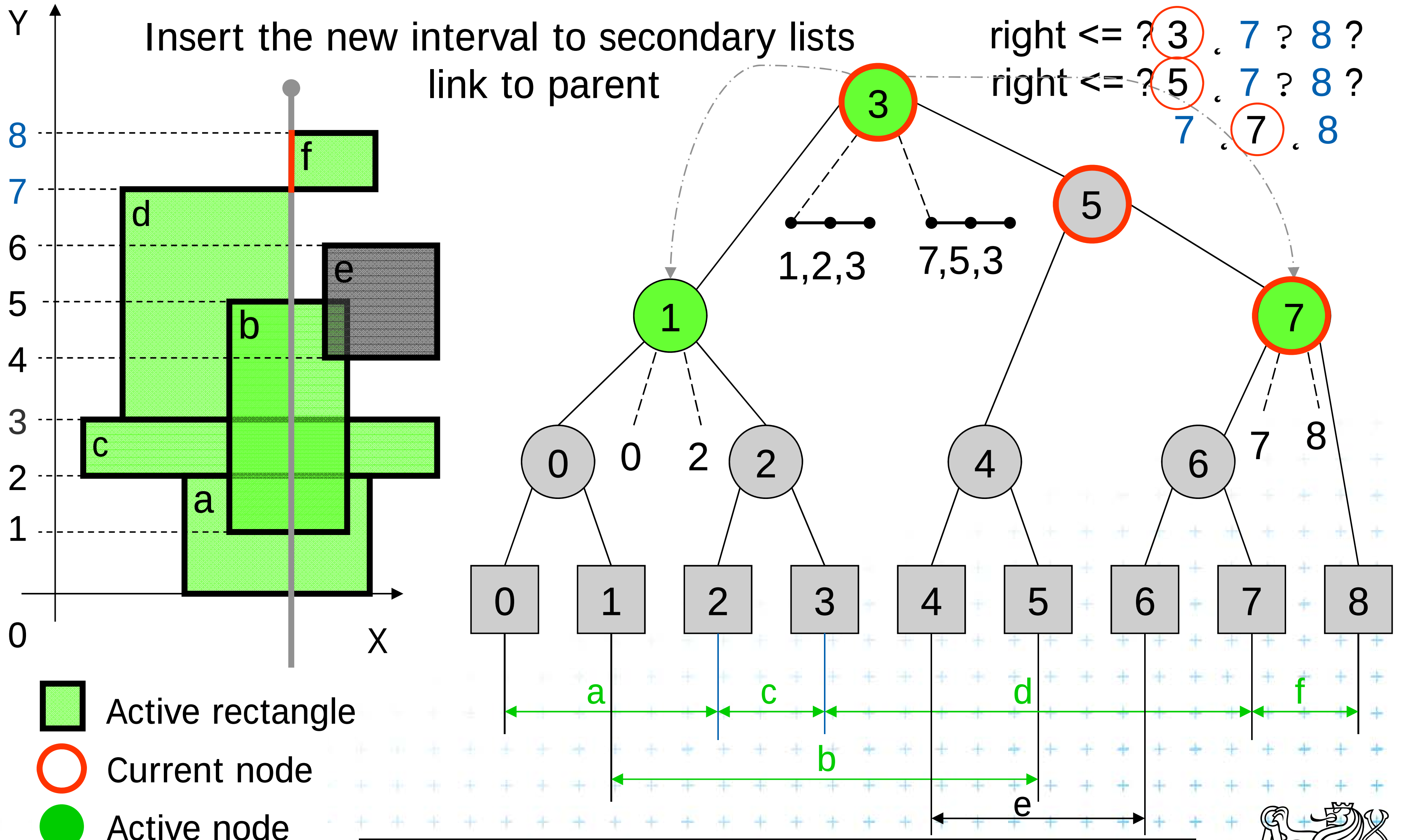
⭕ Current node

● Active node

Insert [7,8] b) Insert Interval

$b \le H(v) \le e$

Insert the new interval to secondary lists
link to parent



🟩 Active rectangle

⭕ Current node

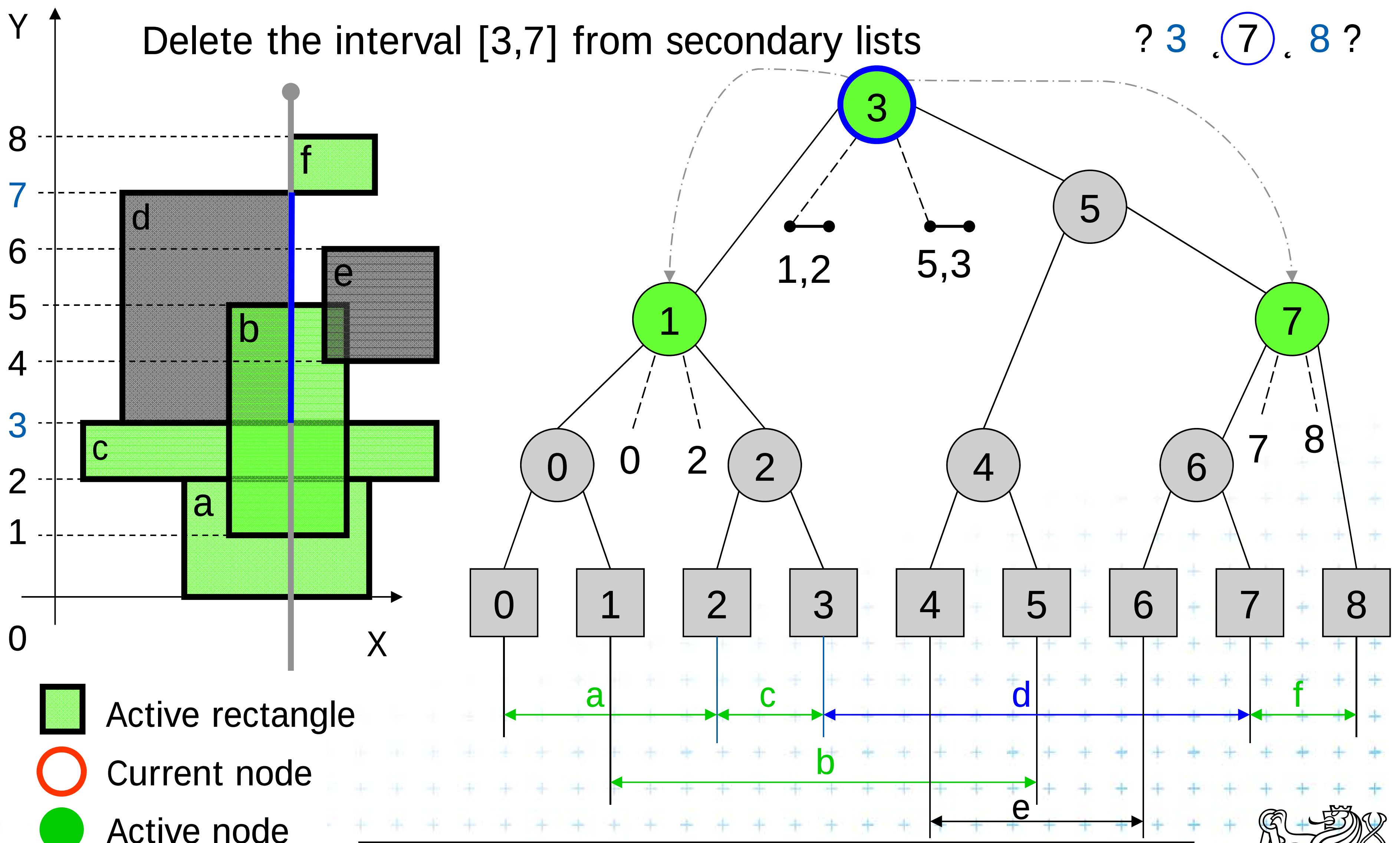
● Active node

Delete [3,7] Delete Interval

b ¸ H(v) ¸ e

Delete the interval [3,7] from secondary lists

? 3 ¸ 7 ¸ 8 ?



🟩 Active rectangle

○ Current node

● Active node

Insert [4,6] a) Query Interval

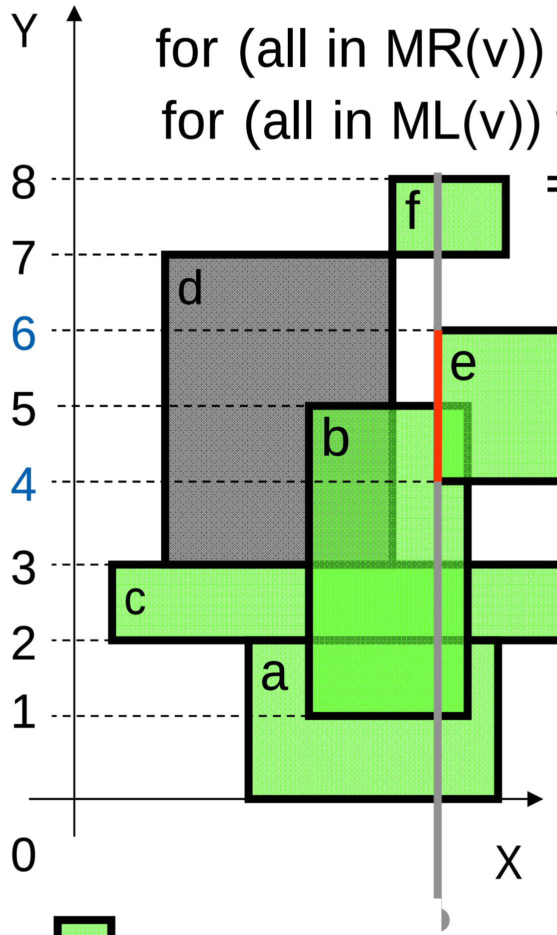
H(v) ¸ b ? e

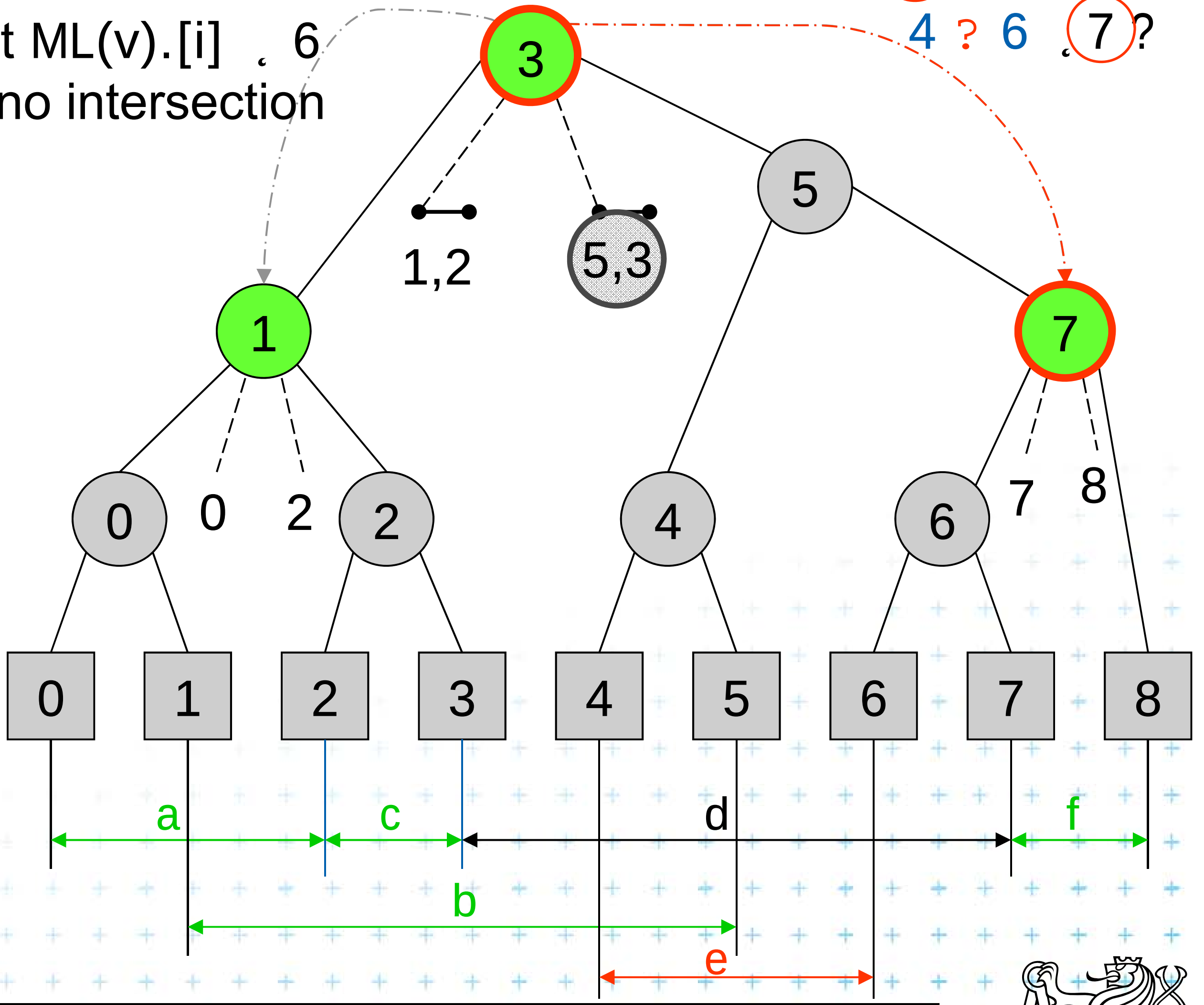
for (all in MR(v)) test MR(v).[i] – 4 => report intersection b 3 ¸ 4 ? 6 ?

4 ? 6 ¸ 7 ?

for (all in ML(v)) test ML(v).[i] ¸ 6

=> no intersection





🟩 Active rectangle

○ Current node

● Active node

[Drtina]

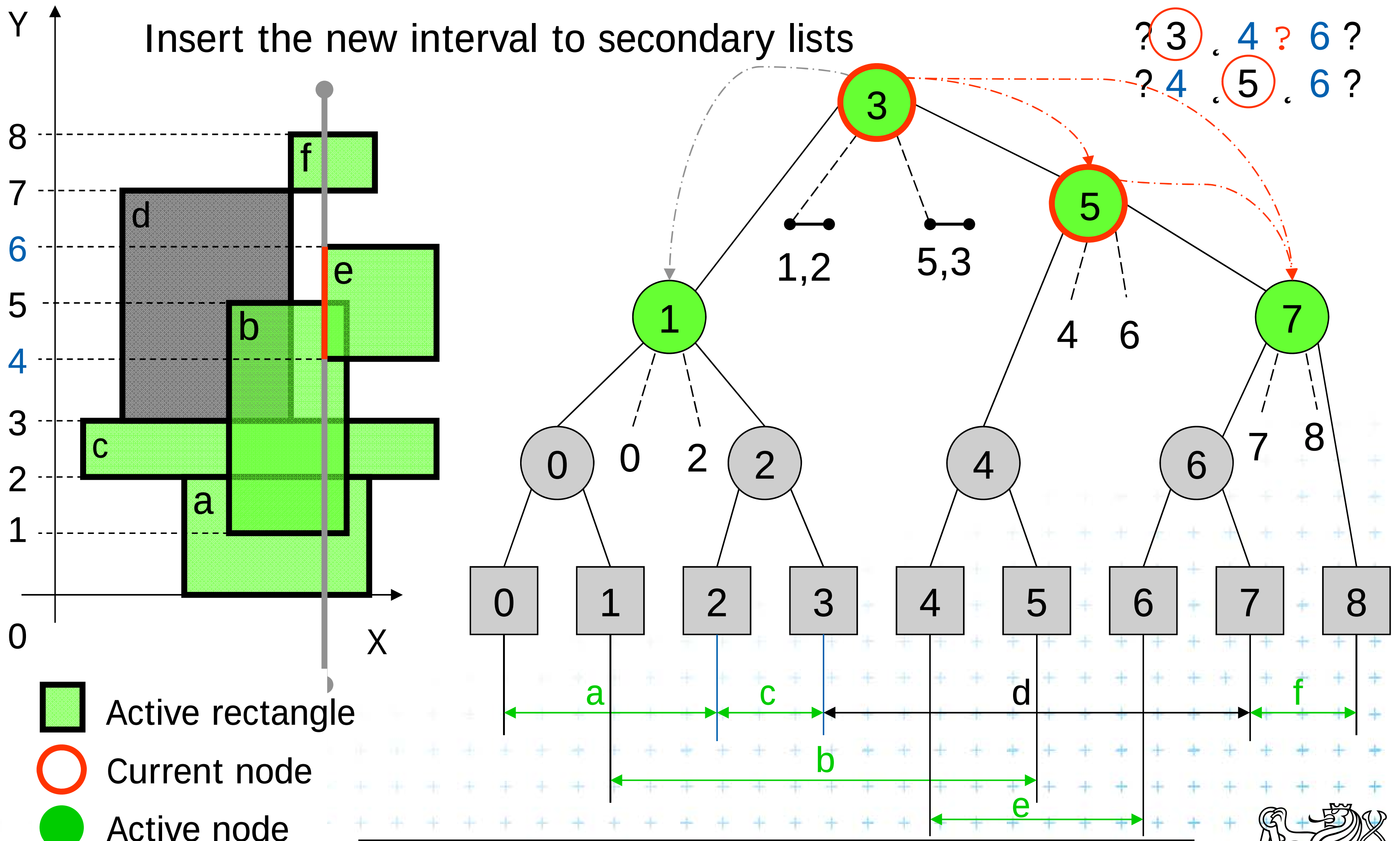
Insert [4,6] b) Insert Interval

H(v) ¸ b ? ¢

Insert the new interval to secondary lists

? 3 ¸ 4 ? 6 ?

? 4 ¸ 5 ¸ 6 ?



🟩 Active rectangle

⭕ Current node

● Active node

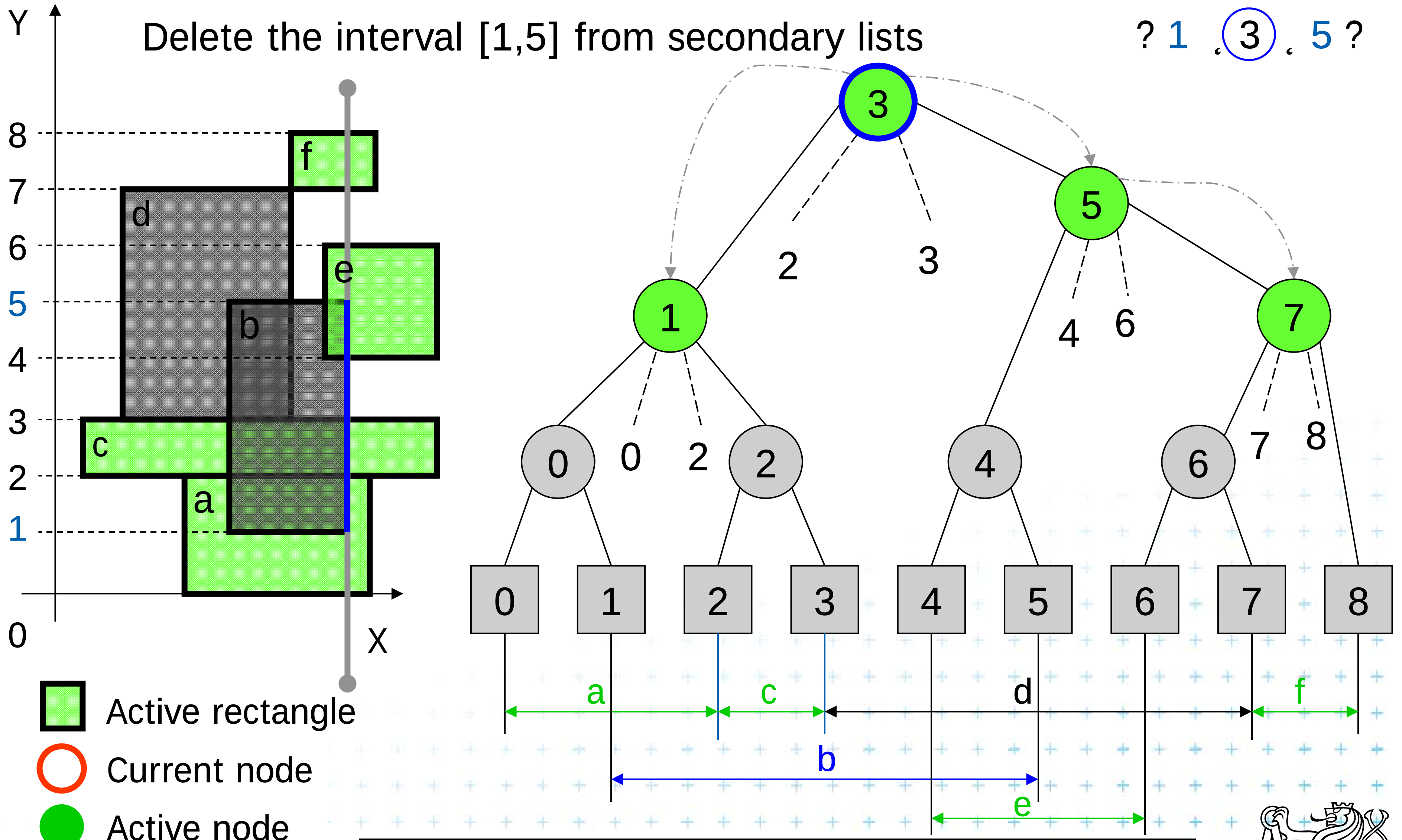
[Drtina]

Delete [1,5] Delete Interval

b ˛ H(v) ˛ e

Delete the interval [1,5] from secondary lists

? 1 ˛ 3 ˛ 5 ?



🟩 Active rectangle

○ Current node

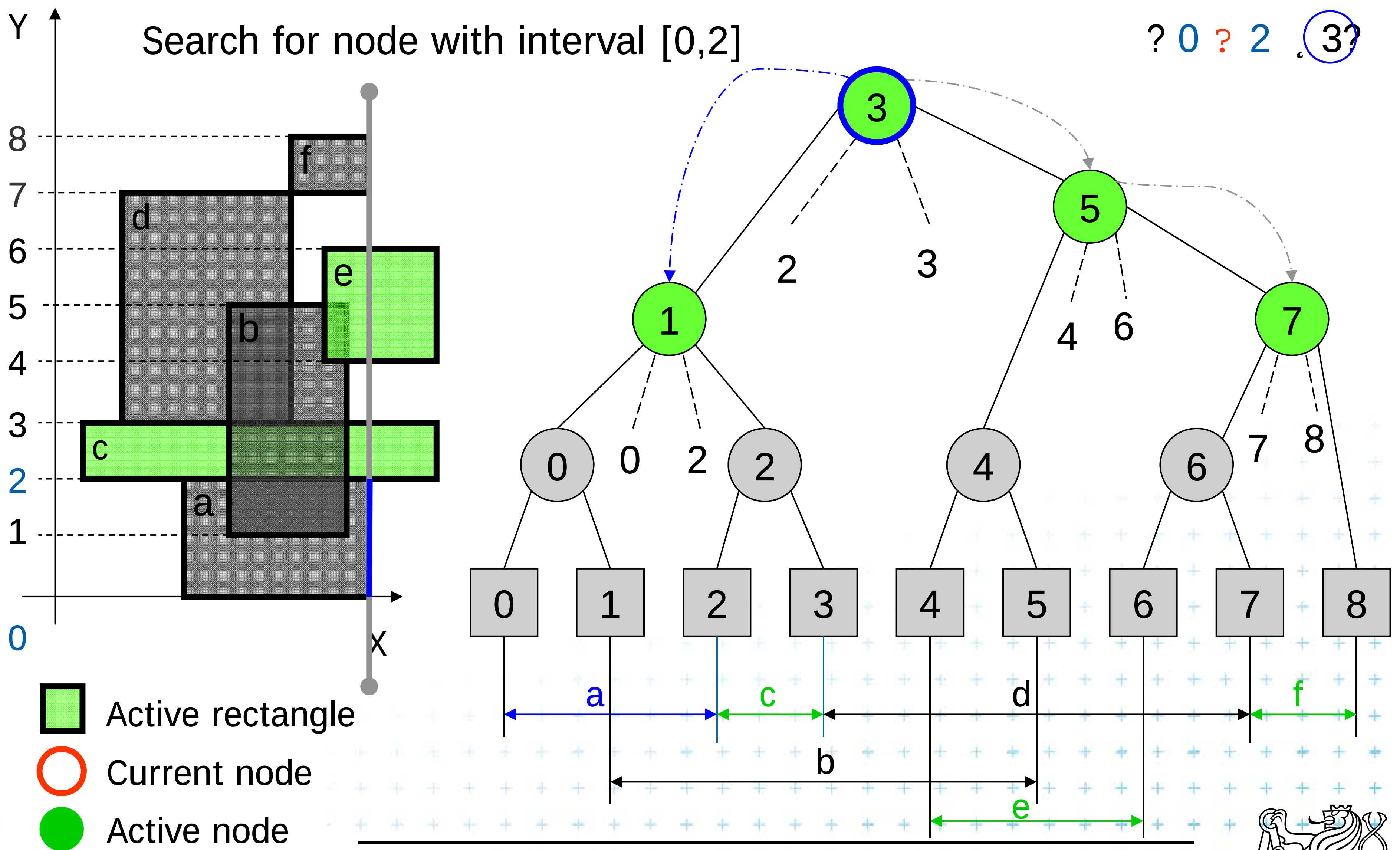
● Active node

Delete [0,2] Delete Interval 1/2

b ?∉ ¸ H(v)

Search for node with interval [0,2]

? 0 ? 2 ¸ 3?



🟩 Active rectangle

○ Current node

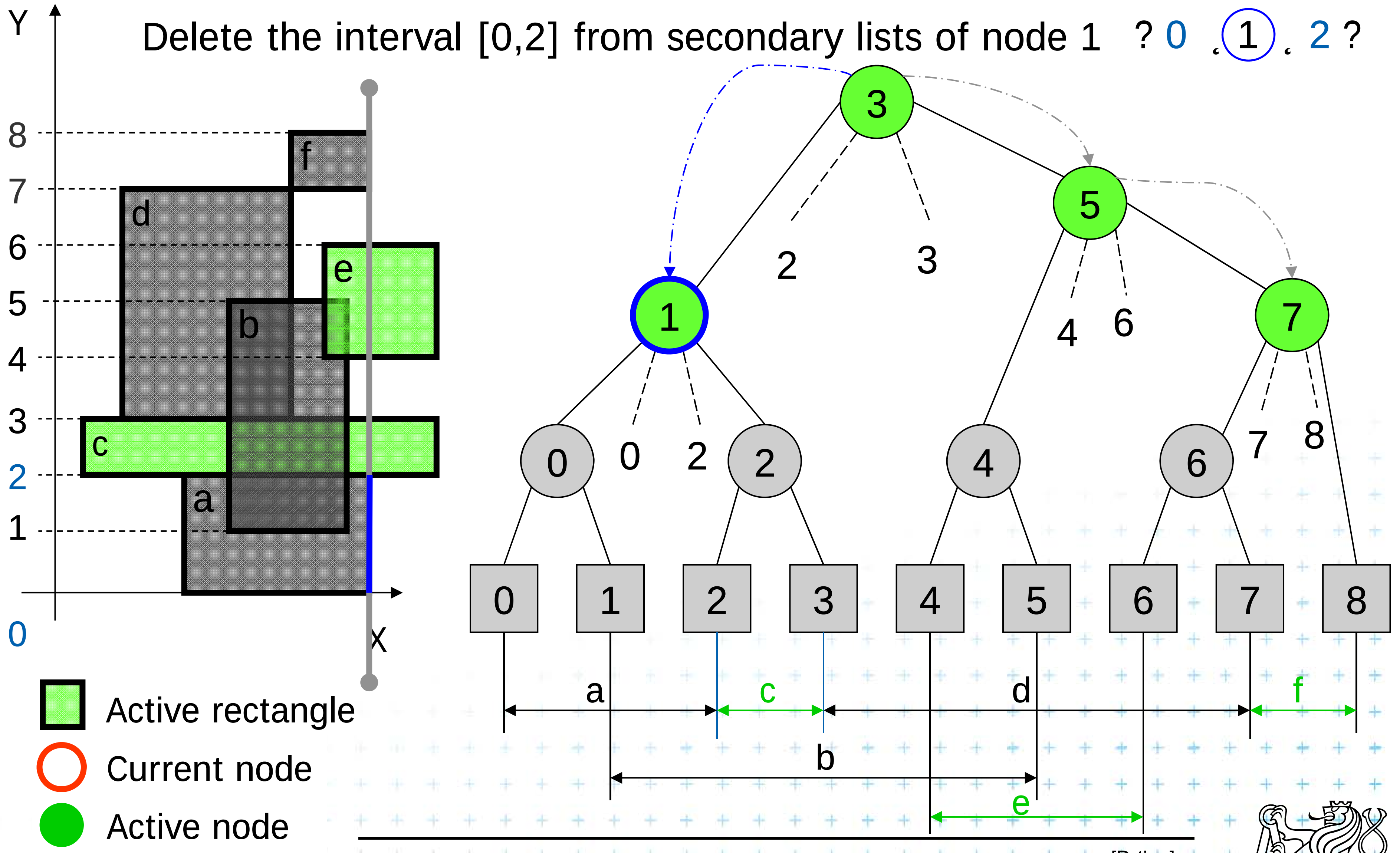
● Active node

[Drtina]

Delete [0,2] Delete Interval 2/2

b ¸ H(v) ¸ e

Delete the interval [0,2] from secondary lists of node 1 ? 0 ¸ 1 ¸ 2 ?



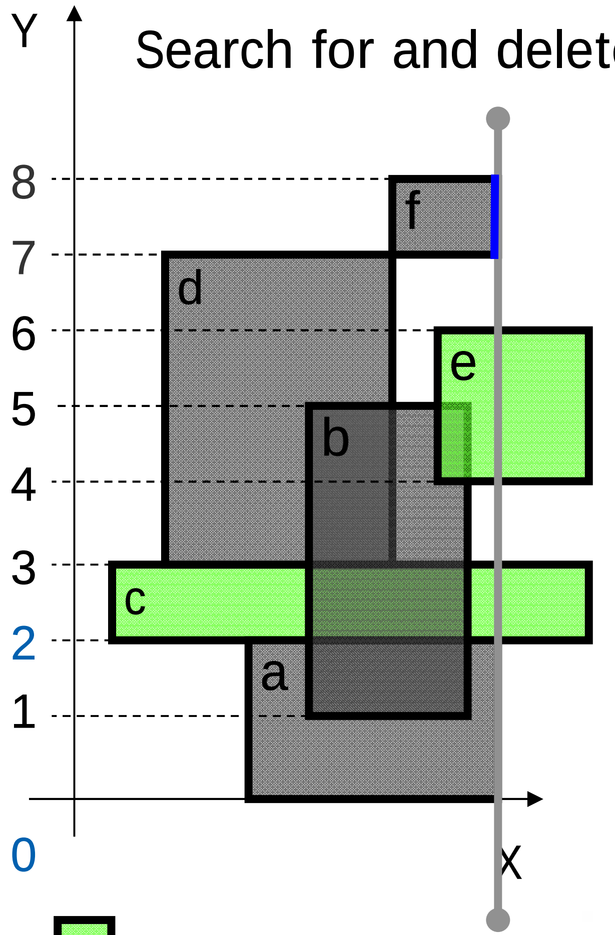
[Drtina]

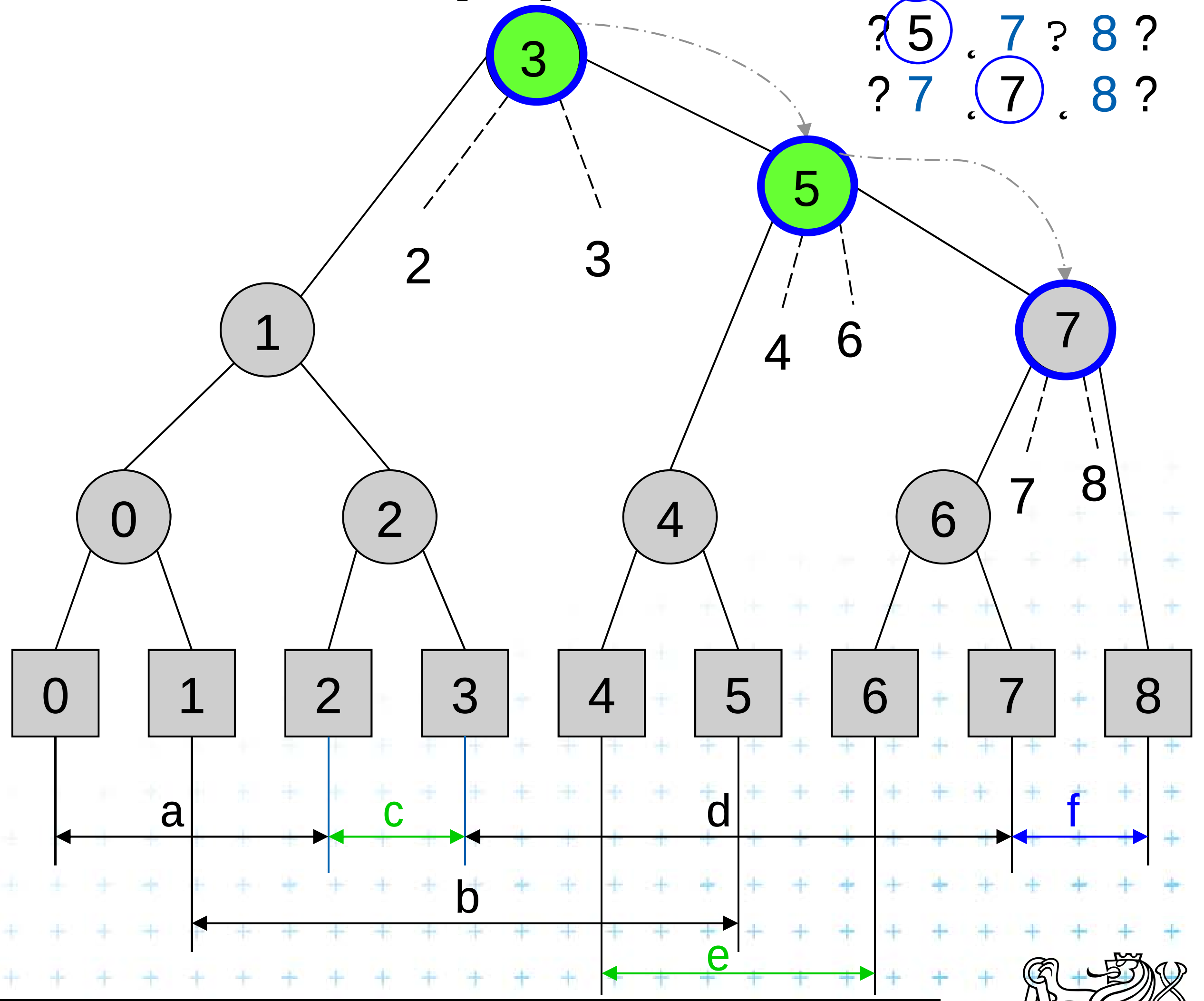
Delete [7,8] Delete Interval

b ¸ H(v) ¸ e

Search for and delete node with interval [7,8]

? 3 ¸ 7 ? 8 ?
? 5 ¸ 7 ? 8 ?
? 7 ¸ 7 ¸ 8 ?





🟩 Active rectangle
○ Current node
● Active node

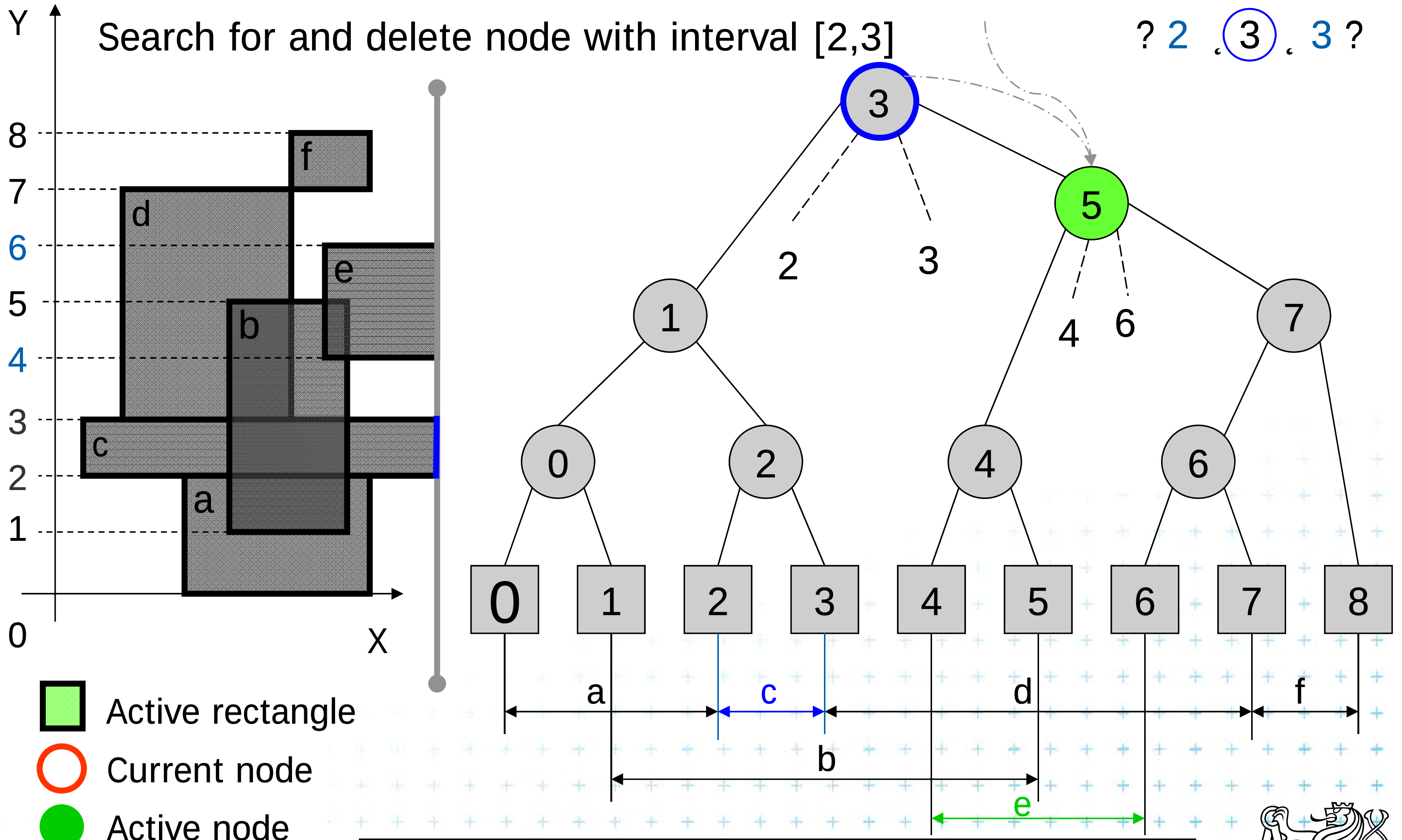
[Drtina]

Delete [2,3] Delete Interval

b ≤ H(v) ≤ e

Search for and delete node with interval [2,3]

? 2 ≤ 3 ≤ 3 ?



🟩 Active rectangle

⭕ Current node

● Active node

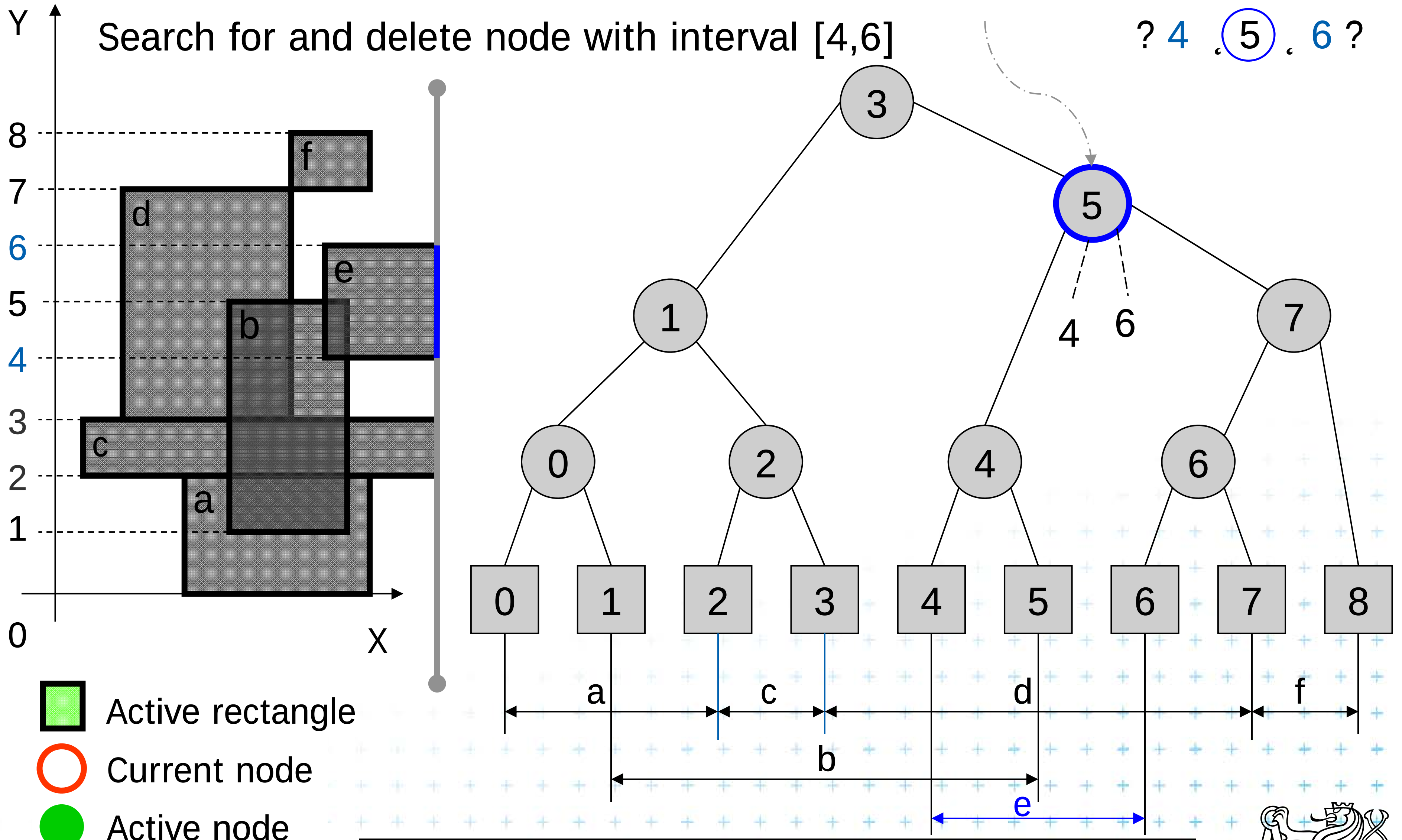
[Drtina]

Delete [4,6] Delete Interval

b ˛ H(v) ˛ e

Search for and delete node with interval [4,6]

? 4 ˛ 5 ˛ 6 ?



🟩 Active rectangle

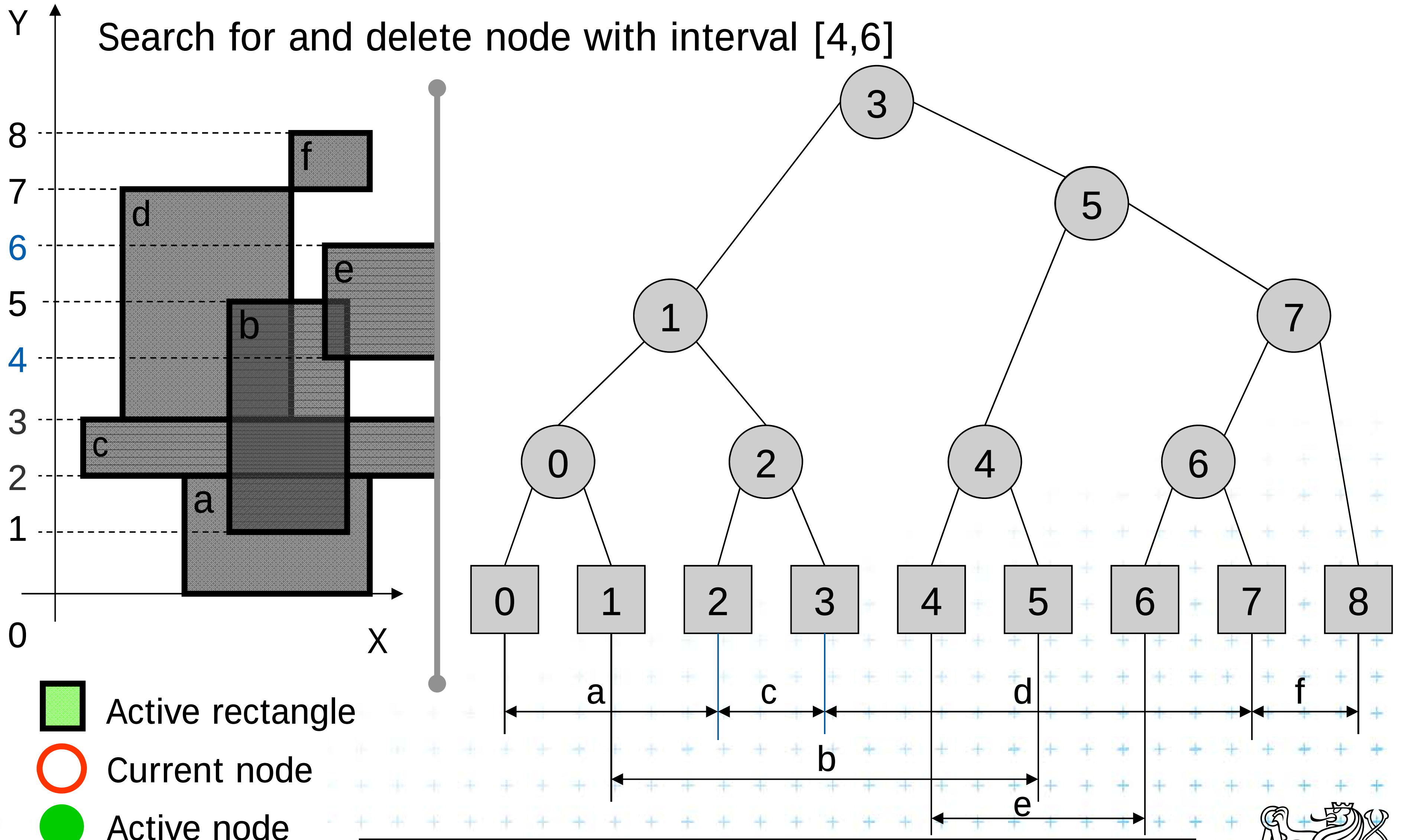
○ Current node

● Active node

[Drtina]

Delete [4,6] Delete Interval

Search for and delete node with interval [4,6]



🟩 Active rectangle

○ Current node

● Active node

[Drtina]

Complexities of rectangle intersections

- *n* rectangles, *s* intersected pairs found
- O(*n* log *n*) preprocessing time to separately sort
  - x-coordinates of the rectangles for the plane sweep
  - the y-coordinates for initializing the interval tree.
- The plane sweep itself takes O(*n* log *n* + *s*) time, so the overall time is O(*n* log *n* + *s*)
- O(*n*) space
- This time is optimal for a decision-tree algorithm (i.e., one that only makes comparisons between rectangle coordinates).

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[Vigneron] Segment trees and interval trees, presentation, INRA, France, http://w3.jouy.inra.fr/unites/miaj/public/vigneron/cs4235/slides.html