DIFFRACTION OF LIGHT BY SINGLE SLIT AND GRATING
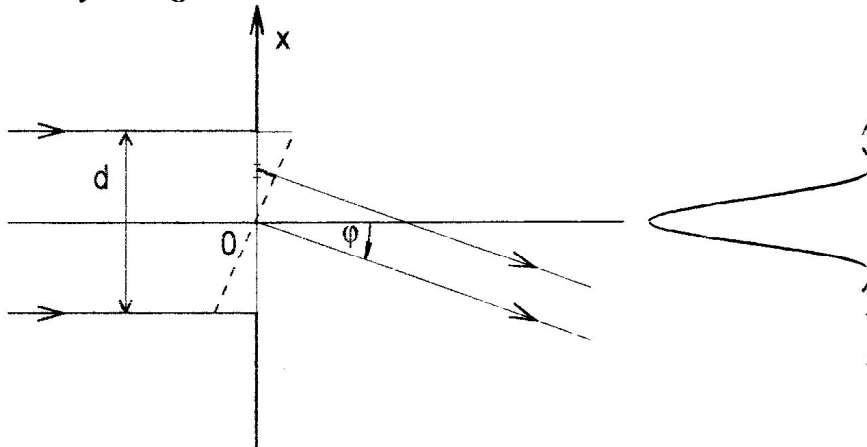
OBJECT

1. To measure the width of the slit.
2. To measure the constant of diffraction grating.

THEORY
Diffraction by a single slit

Diffraction of light by a slit

Assume that parallel rays (or a plane wave) of light fall on a narrow slit of width d (as shown in Fig.). The waves passing through the slit are spread out in all directions. The screen is located in the focal plane of the converging lens

It is known from the theory that the rays which pass through straightly ($\omega = 0$) must be in phase and so there is the central bright spot on the screen at which the maximum intensity will occurs. On the other side it is known that at the angles φ that correspond to the relation

$$\sin \varphi = m\frac{\lambda}{d} \ , \quad m = 1,2,3, \qquad (1)$$

when λ is the wavelength of the used light there will be the minima of intensity.

Diffraction by a grating

The diffraction grating consists of a large number of slits. It is made by ruling very fine lines (of width b) into glass with the diamond tip. The spaces between the lines serve as the slits (of width a). The sum $d = a + b$ represents the distance between the slits called the *grating constant*. The grating containing 10^4 lines/cm is suitable for highly precise measurement of wavelengths.

The rays that pass through each slit without deviation $\left(\Phi_i = 0^0\right)$ interfere constructively

and produce the bright spot at the center on the distant screen. For the angle Φ such that rays from adjacent slits travel the extra distance $\Delta l = m\lambda$, where m is an integer, the rays interfere again constructively.

Thus the following relation $\quad \sin \Phi = \dfrac{m}{d}\lambda \ , \qquad m = 0,1,2,\ldots \qquad (2)$

is the criterion for producing the bright maximum where λ is the wavelength of the used light. These bright maxima are much sharper and narrower for the grating compared to those for the single slit.


Diffraction of light by a grating.
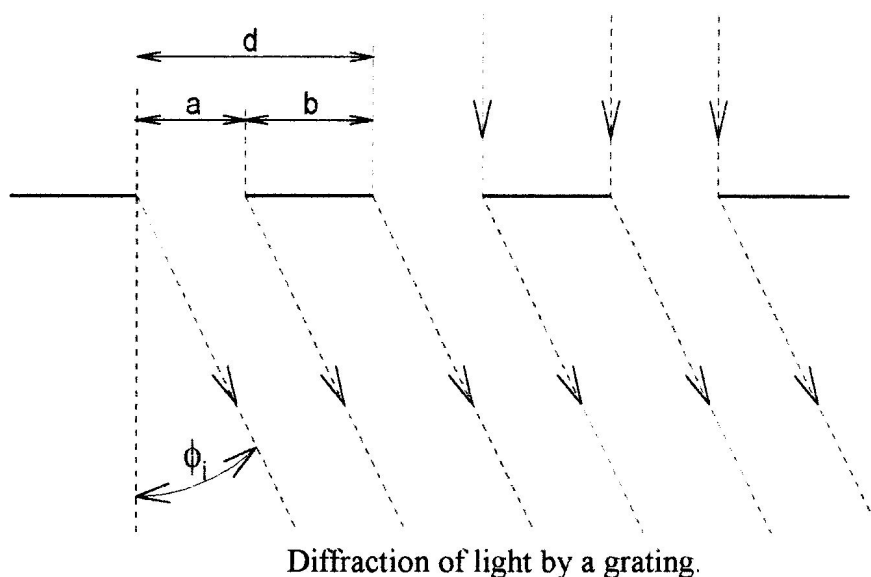
PROCEDURE

The He-Ne laser whose wavelength is $\lambda = 632.8$ nm is used as a source of light.

Experimental arrangement.
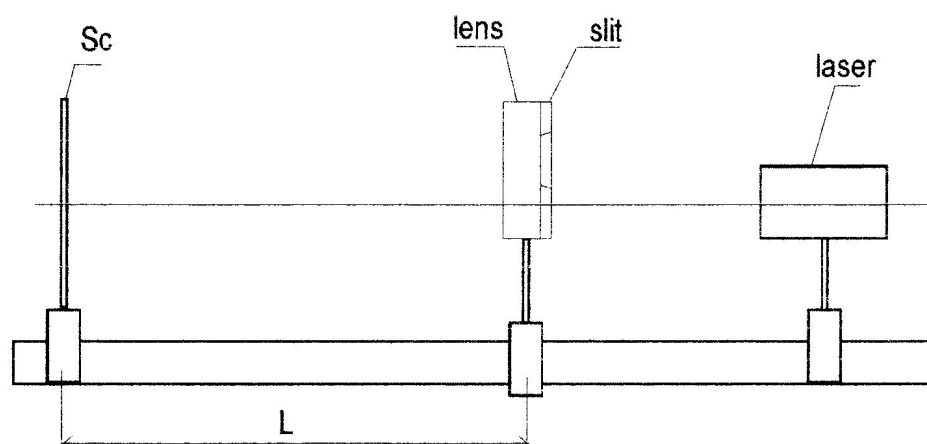
1. Adjust the width of the slit. Let the light fall on the slit. After passing through the slit the light passes through the converging lens. In the focal plane of the lens there is placed the screen on which you can observe the diffraction picture consisting of diffraction maxima separated from each other by dark spaces, as shown in the following figure.
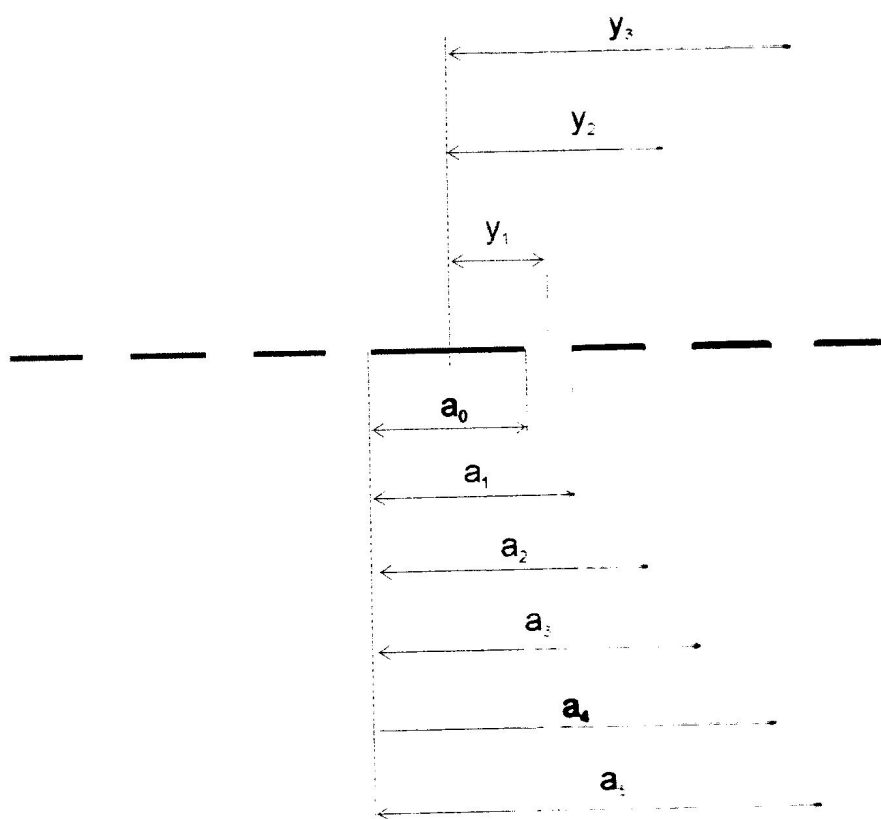
Diffraction on the screen

2. For each minimum $y_1, y_2, y_3, \ldots$, measure the distance of its center from the center of the first maximum.

For the distance y_1 of the 1st minimum we have

$$y_1 = \frac{a_0}{2} + \frac{a_1 - a_0}{2} .$$

for the distance y_2 of the 2nd minimum we have

$$y_2 = a_2 - \frac{a_0}{2} + \frac{a_3 - a_2}{2} = \frac{a_2 - a_1 - a_0}{2}$$

The angle of the i-th maximum can be calculated from the expression

$$tg\varphi \doteq \sin\varphi \doteq \frac{y_i}{L} ,$$

where L is the distance of the slit from the screen. Do not forget to measure this distance.

For $\Phi \langle 5^0$ we can use the approximation $\sin\Phi \approx \frac{y_i}{L}$ and the width of the slit can be found from Eq.1.

3. Repeat the procedure for another width of the slit.

4. Repeat the procedure but for the diffraction grating to obtain the grating constant.

CAUTION! Laser light is very harmful to the human eye. Never look directly into laser rays. All phenomena must be observed on the screen only.

FRESNEL DIFFRACTION

OBJECT

1. From the Fresnel diffraction of the He-Ne laser light on the circular aperture determine the radius of this aperture.
2. Repeat the same procedure for two other different radii of the aperture.
3. Evaluate the error of measurements.

THEORY

There are two different types of diffraction: Fraunhofer and Fresnel diffraction. Fraunhofer diffraction occurs when the diffracting aperture has dimensions much less than the distance to the screen, so that the secondary wavelets are assumed to be plane waves and rays from different points in the aperture are assumed to be parallel. When the distance to the screen is small enough in comparison with the dimensions of the aperture, we must consider the secondary wavelets as spherical, so that the rays to the screen from the different points in the aperture are not parallel. This is the case of Fresnel diffraction, which we shall treat here using a technique devised by Fresnel, in which the incident wavefront is divided into *half-period zones* or *Fresnel zones*. Each zone is considered as a source of secondary spherical wavelets out of which the propagated wavefront is constructed.

In order to analyze the distribution of the intensity of light of the diffraction pattern, we assume a spherical wave propagating from a source Z, see the following figure. We denote the spherical wavefront originating in the source and passing through point M_0 as S'.

Each point of this wavefront can be considered as a source of secondary spherical wavelets from which the wavefront passing through the observation point O is constructed. In this way we have substituted the effect of the light emitted from the source Z by the effect of the secondary wavelets originating on the spherical surface S'.
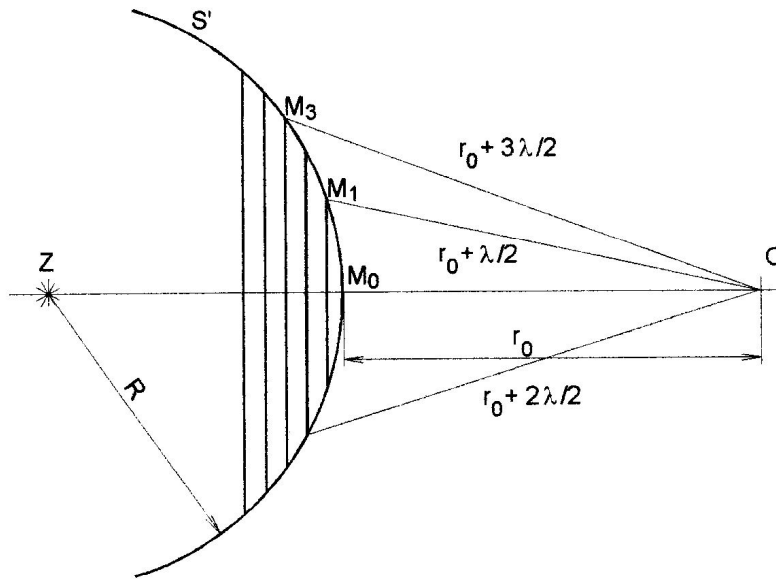
Fresnel zones.
Z - source of light, S' - head of the spherical wavefront, O - observation point.

To apply Fresnel's technique we divide the surface of the wavefront S' into annular zones in such a way that the distances r_n to point O from successive zones vary by half a wavelength, or

$$M_1O - M_0O = M_2O - M_1O = M_3O - M_2O = \frac{\lambda}{2}.$$

If the distance between point O and point M_0 is denoted as r_0 we can construct the k-th Fresnel zone as a sphere with its center at point O and a radius

$$r_k = r_0 + \frac{k\lambda}{2} \qquad \text{for} \quad k = 1, 2, 3 \dots$$

The intersections of these spheres with the wavefront S' determine the Fresnel zones on the wavefront S'. The radius ρ_k of the k-th Fresnel zone is given by the following expression:

$$\rho_k^2 = \frac{r_0 R}{r_0 + R} k\lambda,$$

and the area S_k of the *k-th* Fresnel zone is given by

$$S_k = \frac{\pi r_0 R}{r_0 + R} \, k \lambda.$$

If E_K is the amplitude of the electric field of the wave reaching the point O from the *k-th* Fresnel zone, then the total amplitude of this wave at this point is

$$E = E_1 - E_2 + E_3 - E_4 + \dots$$

After a little rearrangement we obtain

$$E = \frac{E_1}{2} + \left(\frac{E_1}{2} - E_2 + \frac{E_3}{2} \right) + \left(\frac{E_3}{2} - E_4 + \frac{E_5}{2} \right) + \dots \pm \frac{E_k}{2},$$

where the positive and negative signs before the last term are used for odd and even numbers of zones respectively. As long as the amplitudes of the waves from the neighboring zones are very close, we can write

$$E_k = \frac{E_{k-1} - E_{k+1}}{2}.$$

Taking into account this expression we can write for the total amplitude of the wave at point O

$$E = \frac{E_1}{2} \pm \frac{E_k}{2}.$$

For an infinitely large aperture, the contribution from the last zone is negligibly small, so, it may be disregarded.
Thus we can write

$$E = \frac{E_1}{2} = E_0.$$

Thus we see that the actual electric field amplitude of the unobstructed (i.e. undiffracted) wave at point O is only one-half the amplitude that would be due to the first Fresnel zone alone.

Diffraction on a circular aperture

To determine the intensity of the diffracted wave at point O we produce the normal from O to the plane of the aperture and then construct the Fresnel zones relative to O, see following figure. Then we consider which zones or parts of zones are „*visible*" as a result of the aperture. Note that, if we move O, its Fresnel zones move with it. We can regard the aperture as just a „*window*" moving across the Fresnel zones and revealing or concealing different areas as it does so. This also implies that, if we have two distinct points O and O', then each point has its own set of Fresnel zones determined by its position relative to the advancing wavefront and the aperture.
In general, when the circular opening contains an odd number of zones, there will be a bright spot at O, whereas an even number of zones in the aperture gives rise to destructive interference between the wavelets at P.
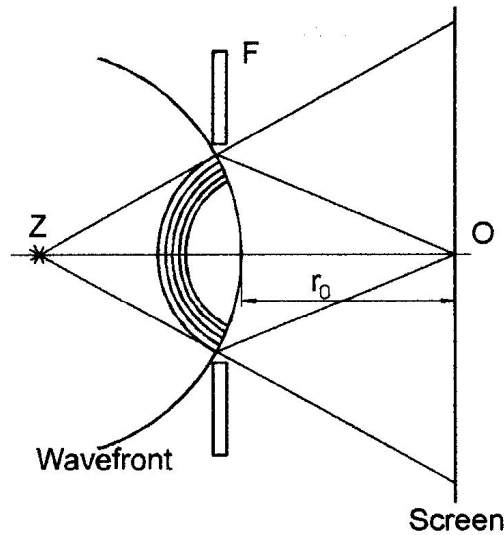
Construction of Fresnel zones for diffraction on a circular aperture.
Z – source of light, F - circular aperture.

For an aperture of fixed radius r, the number of zones contributing to the diffraction pattern at point O can be increased by moving the screen toward the aperture. The diffraction pattern for a small aperture consists of a set of bright and dark annular zones. The number of these zones (bright and dark) corresponds to the number of Fresnel zones uncovered by the aperture. If we calculate the number of these zones, we can determine, for example, the radius of the aperture following this expression:

$$\rho = \left(\frac{r_0 R}{r_0 + R} \, n \lambda \right)^{\frac{1}{2}},$$

where n is the number of zones, λ is the wavelength of the used light, r_0 is the distance between the screen and the aperture, and R is the distance between the aperture and the source of light.

In order to determine the interference pattern as seen at some point O that is not on the axis of the aperture, we must recall that the Fresnel zones are drawn about the normal from O to the plane of the aperture, so that now the aperture is out of alignment with the zones. Thus for point O located not on the axis of the aperture only radiation from uncovered portions of the zones can pass through the aperture to reach this point, with the amount depending upon the area of the zone that is uncovered.

PROCEDURE

1. Set up an experimental arrangement according the following figure. Be sure that all the optical elements are on the same optical axis. As a converging element use a lens +6.
2. Turn on the He-Ne laser. The wavelength of the light is 632.8 nm.
3. Set up the distance between the laser, the aperture and the screen in such a way that the annular bright and dark circles appear on the screen.

4. Move the aperture along the optical axis toward the screen and for each maximum and minimum read the values of r_0, R and n. Last maximum (closest to the screen) corresponds to only one (central) Fresnel zone.
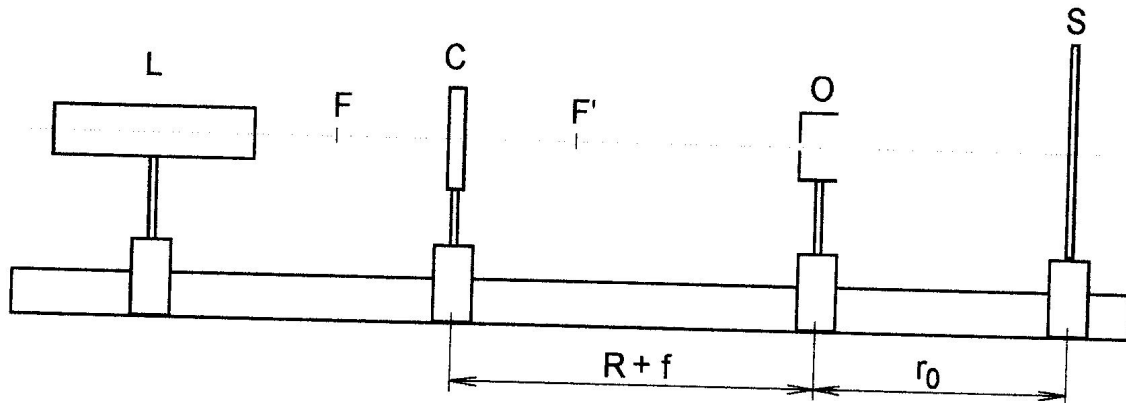
Experimental arrangement.
L - laser, C - lens, O - aperture, S - screen.

5. Repeat the steps 2 till 4 for two other different radii of the aperture.
6. Calculate the radii of the apertures as the average of the readings for at least three measurements of r_0, R and n.
7. Evaluate the error of measurements.

Warning: Laser light is dangerous to an eye. Never look into the beam! All described effects can be observed on the screen only!

SEMESTER PROJECT INSTRUCTIONS

In case of the Fraunhofer diffraction the program will calculate and depict the dependence of the light intensity on the screen as a function of the distance from the optical axis. The distance from the slit, slit width and laser wavelength will be parameters. In case of the Fresnel diffraction the program will calculate and depict the image of concentric dark and bright circles. Variable parameters will be distances, focal length of the lens and laser wavelength.