

GRAPHICAL MARKOV MODELS (WS2019)
4. SEMINAR

Assignment 1. (breakpoint detection) Consider the following probabilistic model for real valued sequences $\mathbf{x} = (x_1, \dots, x_n)$, $x_i \in \mathbb{R}$ of fixed length n . Each sequence is a combination of a leading part $i \leq k$ and a trailing part $i > k$. The boundary $k = 1, \dots, n$ is random with some categorical distribution $\boldsymbol{\pi} \in \mathbb{R}_+^n$, $\sum_k \pi_k = 1$. The p.d.s for the leading and trailing parts of the sequence arise from two homogeneous HMM models:

$$p(x_{1:k}) = \sum_{s_{1:k}} p_1(x_{1:k}, s_{1:k}) \quad \text{and} \quad p(x_{k+1:n}) = \sum_{s_{k+1:n}} p_2(x_{k+1:n}, s_{k+1:n})$$

The HMMs p_1 and p_2 and the distribution $\boldsymbol{\pi}$ are known. Find an algorithm for inferring the boundary k for a given sequence \mathbf{x} , assuming that the loss function is $\ell(k, k') = (k - k')^2$.

Assignment 2. Let \mathbf{x} be a grey value image of size $n \times m$, where x_{ij} denotes the grey value of the pixel with coordinates (i, j) . The task is to segment such images into an upper and lower part by a boundary represented as a sequence of height values $s_j \in \{1, 2, \dots, n\}$ for all $j = 1, 2, \dots, m$.

The prior probability for boundaries is assumed to be a homogeneous Markov chain such that $p(s_j | s_{j-1}) = 0$ if $|s_j - s_{j-1}| > 1$. The appearance model for columns \mathbf{x}_j given the boundary value s_j , is assumed to be conditional independent

$$p(\mathbf{x}_j | s_j) = \prod_{i \leq s_j} p_1(x_{ij}) \cdot \prod_{i > s_j} p_2(x_{ij}),$$

where $p_1()$ and $p_2()$ are two distributions for grey values.

- a) Deduce an efficient algorithm for determining the most probable boundary.
- b) Suppose that the loss function $\ell(s, s')$ for incorrectly recognised boundaries is defined by

$$\ell(s, s') = \sum_{j=1}^m (s_j - s'_j)^2.$$

Formulate the segmentation task for this case. Deduce an efficient inference algorithm.

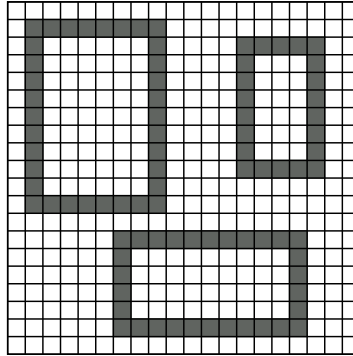
- c) When applying the inference rule derived in b), it may happen that the inferred boundary has zero probability in the model. Propose an augmented loss function which prevents this and derive the corresponding inference algorithm.

Assignment 3. Consider the language L of all b/w images $x: D \rightarrow \{b, w\}$ containing an arbitrary number of non-overlapping and non-touching one pixel wide rectangular frames (see figure).

- a) Prove that L is not expressible by a locally conjunctive predicate

$$x \in L \quad \text{if and only if} \quad f(x) = \bigwedge_{c \in \mathcal{C}} f_c(x_c) = 1$$

with predicates f_c , defined on image fragments x_c , where $c \subset D$ have bounded size $|c| < |D|$.



b) Show that L can be expressed by introducing a field $s: D \rightarrow K$ of non-terminal symbols, a locally conjunctive predicate on them and pixel-wise predicates g relating the non-terminal and terminal symbol in each pixel

$$x \in L \quad \text{if and only if} \quad \bigvee_{s \in K^D} \left[\bigwedge_{c \in \mathcal{C}} f_c(s_c) \wedge \bigwedge_{i \in D} g(x_i, s_i) \right] = 1$$

Find a suitable structure \mathcal{C} , an alphabet of non-terminal symbols K and predicates f_c, g .