Basics of Description Logic $\mathcal{ALC}$

Petr Křemen, Miroslav Blaško

November 21, 2019

1 Understanding $\mathcal{ALC}$

Consider the following $\mathcal{ALC}$ theory $\mathcal{K} = (\mathcal{T}, \{\})$, where $\mathcal{T}$ contains the following axioms:

$$\begin{aligned} Man &\sqsubseteq Person \\ Woman &\sqsubseteq Person \sqcap \neg Man \\ Father &\equiv Man \sqcap \exists hasChild \cdot Person \\ GrandFather &\equiv \exists hasChild \cdot \exists hasChild \cdot \top \\ Sister &\equiv Person \sqcap \neg Man \sqcap \exists hasSibling \cdot Person \end{aligned}$$

Ex. 1 — What is the meaning of these particular axioms ? Do they reflect your understanding of reality ? Formulate them in natural language.

Answer (Ex. 1) — For example, the third axiom defines a concept $Father$ as any Man that has some $Person$ as a child. The fourth axiom is not well defined – it allows grandfathers to be women. More precise version of the fourth axiom might be e.g. $GrandFather \equiv Man \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$.

Ex. 2 — Rewrite the last axiom into the semantically equivalent FOPL formula.

Answer (Ex. 2) — Each TBox axiom corresponds to a universally closed FOPL formula. Notice that two different variables are enough for encoding of any $\mathcal{ALC}$ axiom. The encoding of the last axiom is:
$(\forall x)(Sister(x) \equiv Person(x) \wedge \neg Man(x) \wedge (\exists y)(hasSibling(x, y) \wedge Person(y)))$

Ex. 3 — Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$:

$$\begin{aligned} \Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = \{B, A\} \\ Man^{\mathcal{I}} &= \{B\} \\ Woman^{\mathcal{I}} &= \{A\} \\ Father^{\mathcal{I}} &= GrandFather^{\mathcal{I}} = \{B\} \end{aligned}$$

$$\begin{aligned} hasChild^{\mathcal{I}} &= \{(B,B)\} \\ hasSibling^{\mathcal{I}} &= \{\} \\ Sister^{\mathcal{I}} &= \{B\} \end{aligned} \tag{1}$$

1. Is $\mathcal{I}$ a model $\mathcal{K}$? If yes, decide, whether $\mathcal{I}$ reflects reality.
2. We know that $\mathcal{ALC}$ has the *tree model property* and *finite model property.* In case $\mathcal{I}$ is a model, is $\mathcal{I}$ tree-shaped ? If not, find a model that is tree-shaped.

Answer (Ex. 3) — $\mathcal{I}$ is not a model of $\mathcal{K}$, as it does not satisfy the last axiom: $Sister^{\mathcal{I}} \neq Person^{\mathcal{I}} \cap (\Delta^{\mathcal{I}} \setminus Man^{\mathcal{I}}) \cap \{x \in \Delta^{\mathcal{I}} | (\exists y \in \Delta^{\mathcal{I}})((x,y) \in hasSibling^{\mathcal{I}} \wedge y \in Person^{\mathcal{I}})\}$

Ex. 4 — How does the situation change when we consider the same $\mathcal{I}$, except that $Sister^{\mathcal{I}} = \{\}$?

Answer (Ex. 4) — Now, $\mathcal{I}$ is a model of $\mathcal{K}$ as it satisfies all axioms. However, it does not reflect the reality well, as it states that a person B is his/her own child. This interpretation is finite, yet not tree-shaped. A tree-shaped model ensured by the *tree-model property* of $\mathcal{ALC}$ is e.g. the following infinite model $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$, where

$$\begin{aligned} \Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = Man^{\mathcal{I}} = Father^{\mathcal{I}} = GrandFather^{\mathcal{I}} = \{A_1, A_2, \ldots\}_{i=1\ldots\infty} \\ Woman^{\mathcal{I}} &= Sister^{\mathcal{I}} = \{\} \\ hasChild^{\mathcal{I}} &= \{(A_i, A_{i+1})\}_{i=1\ldots\infty} \\ hasSibling^{\mathcal{I}} &= \{\} \end{aligned} \tag{2}$$

Ex. 5 — Using the vocabulary from $\mathcal{K}$, define the concept "A father having just sons."

Answer (Ex. 5) — $FatherOfBoys \equiv Father \sqcap \forall hasChild \cdot Man$

Ex. 6 — Using the vocabulary from $\mathcal{K}$, define the concept "A man who has no brother, but at least one sister with at least one child."

Answer (Ex. 6) — $HappyUncle \equiv Man \sqcap \exists hasSibling \cdot (Woman \sqcap \exists hasChild \cdot \top) \sqcap \forall hasSibling \cdot \neg Man$

Ex. 7 — During knowledge modeling, it is often necessary to specify:

global domain and range of given role, e.g. "By *hasChild* (role) we always connect a *Person* (domain) with another *Person* (range)".

local range of given role, e.g. "Every father having only sons (domain) can be connected by *hasChild* (role) just with a *Man* (range)".

Show, in which way it is possible to model global domain and range of these roles in $\mathcal{ALC}$.

Answer (Ex. 7) — Global domain and range can be modeled as:

$$\begin{aligned} \exists hasChild \cdot \top &\sqsubseteq Person \\ \top &\sqsubseteq \forall hasChild \cdot Person \end{aligned} \quad (3)$$

Local range is similar and only replaces the top concepts in the global range axiom:

$$\begin{aligned} \exists hasChild \cdot FatherOfSons &\sqsubseteq Person \\ FatherOfSons &\sqsubseteq \forall hasChild \cdot Man \end{aligned} \quad (4)$$

2 Using Protégé

1. Go through the Protégé Crash Course on the tutorial web pages.
2. Create a new ontology in Protégé 4 and insert there all the definitions from Section 1. Verify correctness of your solution of the previous task (e.g. in the DL query tab).

3 Suggested excercise for the semestral work

1. For each of your RDF datasets that are final output of CP1 create a separate ontology describing schema of that data (you will need to use TBox axioms mostly).
2. Modify each of your RDF datasets to include statement importing related schema created in previous task. Hint: use *owl:imports*.
3. Create an ontology that imports all your datasets.
4. Open the ontology of all datasets in Protege to browse all your data.