Description Logics – Basics

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Outline

Towards Description Logics

Logics

Formal Ontologies

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- Most of them are based on some **logical calculus**.

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- ... what is the meaning of these formulas ?

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- Syntax – to *represent* concepts (*defining symbols*)

Logics trade-off

A logical calculus is always a trade-off between *expressiveness* and *tractability of reasoning*.

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- Proof Theory – to enforce the semantics

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How to check satisfiability of the formula $A \vee (\neg(B \wedge A) \vee B \wedge C)$?

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complexity – NP-Complete (Cook theorem)

First Order Predicate Logic

Example

What is the meaning of this sentence ?

$$(\forall x_1)((Student(x_1) \wedge (\exists x_2)(GraduateCourse(x_2) \wedge isEnrolledTo(x_1, x_2))) \Rightarrow (\forall x_3)(isEnrolledTo(x_1, x_3) \Rightarrow GraduateCourse(x_3)))$$

$$Student \sqcap \exists isEnrolledTo.GraduateCourse \sqsubseteq \forall isEnrolledTo.GraduateCourse$$

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complexity – undecidable (Goedel)

Open World Assumption

OWA

FOPL accepts Open World Assumption, i.e. whatever is not known is not necessarily false.

As a result, FOPL is *monotonic*, i.e.

monotonicity

No conclusion can be invalidated by adding extra knowledge.

This is in contrary to relational databases, or Prolog that accept Closed World Assumption.

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  - ☹ Prolog is not an implementation of FOPL – OWA vs. CWA, negation as failure, problems in expressing disjunctive knowledge, etc.

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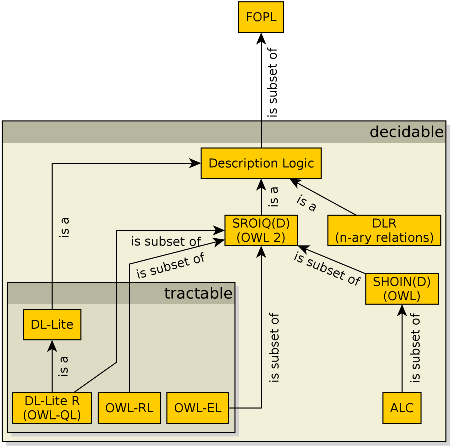
  - What is the semantics of a "slot" in a frame (relation in semantic networks) ? The slot **must/might** be filled **once/multiple times** ?

What are Description Logics ?



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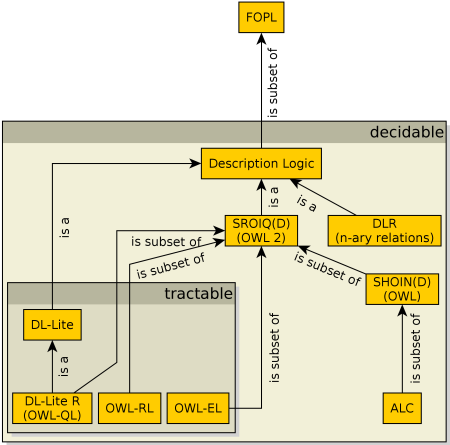

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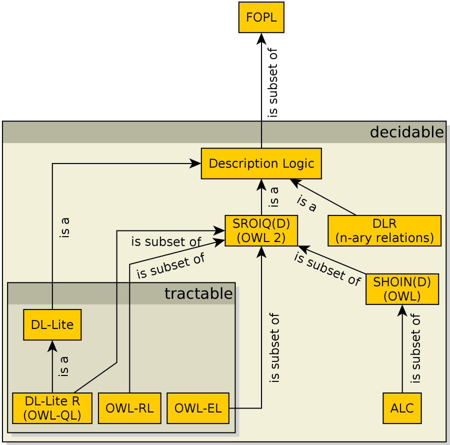

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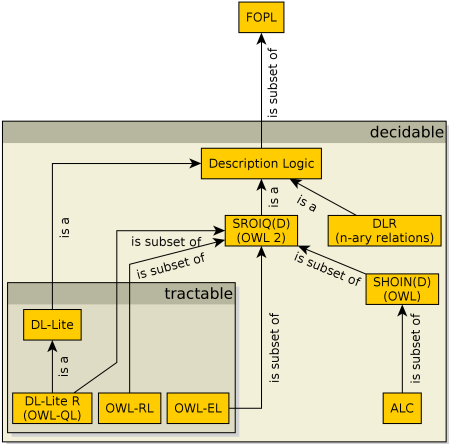

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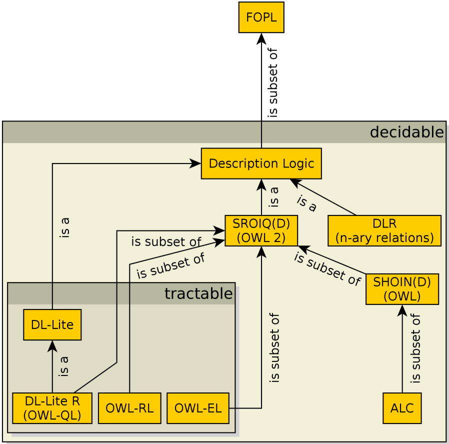

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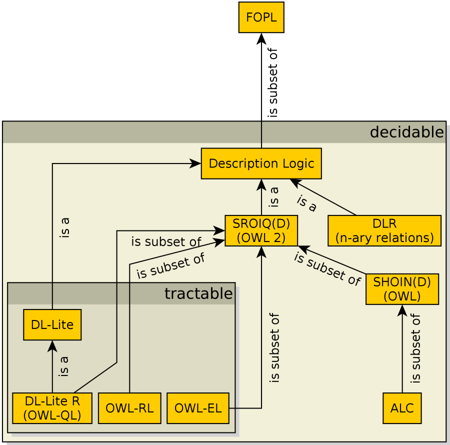

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$\mathcal{ALC}$ Language

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- DLs differ in their expressive power (concept/role constructors, axiom types).

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- Having *atomic* concept A, *atomic* role R and individual a, then

$$\begin{aligned} A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \\ R^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \\ a^{\mathcal{I}} &\in \Delta^{\mathcal{I}} \end{aligned}$$

$\mathcal{ALC}$ (= attributive language with complements)

Having concepts C, D, atomic concept A and atomic role R, then for interpretation $\mathcal{I}$:

concept	*concept*$^{\mathcal{I}}$	*description*
$\top$	$\Delta^{\mathcal{I}}$	(universal concept)
$\bot$	$\emptyset$	(unsatisfiable concept)
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	(negation)
$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	(intersection)
$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$	(union)
$\forall R \cdot C$	$\{a \mid \forall b((a,b) \in R^{\mathcal{I}} \implies b \in C^{\mathcal{I}})\}$	(universal restriction)
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ABOX (UNA = unique name assumption[1])

axiom	$\mathcal{I} \models$ axiom iff	*description*
$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	(concept assertion)
$R(a_1, a_2)$	$(a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in R^{\mathcal{I}}$	(role assertion)

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- S is consistent, if S has at least one model

$\mathcal{ALC}$ – Example

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- How to define concept *GrandParent* ? (specify an *axiom*)

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Consider an information system for genealogical data. Information integration from various sources is crucial – databases, information systems with *different data models*. As an integration layer, let's use a description logic theory. Let's have atomic concepts *Person*, *Man*, *GrandParent* and atomic role *hasChild*.

- Set of persons that have just men as their descendants, if any ? (specify a *concept*)
  - $Person \sqcap \forall hasChild \cdot Man$
- How to define concept *GrandParent* ? (specify an *axiom*)
  - $GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$

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- How to define concept $GrandParent$? (specify an *axiom*)
  - $GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$
- How does the previous axiom look like in FOPL ?

$$\forall x\,(GrandParent(x) \equiv (Person(x) \wedge \exists y\,(hasChild(x,y) \\ \wedge \exists z\,(hasChild(y,z)))))$$

$\mathcal{ALC}$ Example – $\mathcal{T}$

Example

$$
\begin{aligned}
\mathit{Woman} &\equiv \mathit{Person} \sqcap \mathit{Female} \\
\mathit{Man} &\equiv \mathit{Person} \sqcap \neg \mathit{Woman} \\
\mathit{Mother} &\equiv \mathit{Woman} \sqcap \exists \mathit{hasChild} \cdot \mathit{Person} \\
\mathit{Father} &\equiv \mathit{Man} \sqcap \exists \mathit{hasChild} \cdot \mathit{Person} \\
\mathit{Parent} &\equiv \mathit{Father} \sqcup \mathit{Mother} \\
\mathit{Grandmother} &\equiv \mathit{Mother} \sqcap \exists \mathit{hasChild} \cdot \mathit{Parent} \\
\mathit{MotherWithoutDaughter} &\equiv \mathit{Mother} \sqcap \forall \mathit{hasChild} \cdot \neg \mathit{Woman} \\
\mathit{Wife} &\equiv \mathit{Woman} \sqcap \exists \mathit{hasHusband} \cdot \mathit{Man}
\end{aligned}
$$

Interpretation – Example

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- Consider a theory $\mathcal{K}_1 = (\{GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top\}, \{GrandParent(JOHN)\})$. Find some model.

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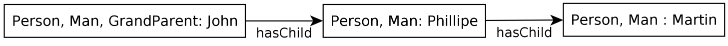
  - $JOHN^{\mathcal{I}_1} = \{John\}$
- this model is finite and has the form of a tree with the root in the node *John* :



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Both properties represent important characteristics of $\mathcal{ALC}$ that significantly speed-up reasoning.

In particular (generalized) TMP is a characteristics that is shared by most DLs and significantly reduces their computational complexity.

Example – CWA × OWA

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ABOX	$hasChild(JOCASTA, OEDIPUS)$	$hasChild(JOCASTA, POLYNEIKES)$
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$$JOCASTA \longrightarrow POLYNEIKES \longrightarrow THERSANDROS$$

$$JOCASTA \longrightarrow OEDIPUS \longrightarrow POLYNEIKES$$

Q1 $(\exists hasChild \cdot (Patricide \sqcap \exists hasChild \cdot \neg Patricide))(JOCASTA)$,

$$JOCASTA \longrightarrow \bullet \longrightarrow \bullet$$

Q2 Find individuals x such that $\mathcal{K} \models C(x)$, where C is

$$\neg Patricide \sqcap \exists hasChild^{-} \cdot (Patricide \sqcap \exists hasChild^{-}) \cdot \{JOCASTA\}$$

What is the difference, when considering CWA ?

$$JOCASTA \longrightarrow \bullet \longrightarrow x$$

From $\mathcal{ALC}$ to OWL(2)-DL

Extending . . . $\mathcal{ALC}$ ...

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- We have introduced $\mathcal{ALC}$, together with a decision procedure. Its expressiveness is higher than propositional calculus, still it is insufficient for many practical applications.
- Let's take a look, how to extend $\mathcal{ALC}$ while preserving decidability.

Extending ... $\mathcal{ALC}$ ... (2)

$\mathcal{N}$ (Number restructions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq\ n\ R)$	$\left\{ a \middle| \ \left|\{b \mid (a,b) \in R^{\mathcal{I}}\}\right| \geq n \right\}$
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- Concept $Woman \sqcap (\leq\ 3\ hasChild)$ denotes women who have at most 3 children.

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- ... and $Bicycle \equiv (=\ 2\ hasWheel)$?

Extending . . . $\mathcal{ALC}$ ... (3)

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Example

- Concept $Woman \sqcap (\geq\ 3\ hasChild\ Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq\ 4\ hasPart\ Wheel)$?
- Which qualified number restrictions can be expressed in $\mathcal{ALC}$?

Extending ... $\mathcal{ALC}$ ... (4)

$\mathcal{O}$ (Nominals) can be used for naming a concept elements explicitely.

syntax (concept)	semantics
$\{a_1, \ldots, a_n\}$	$\{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$

Example

- Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?

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- $Continent \equiv$ $\{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$?

Extending . . . $\mathcal{ALC}$ ... (5)

$\mathcal{I}$ (Inverse roles) are used for defining role inversion.

syntax (role)	semantics
R^-	$(R^\mathcal{I})^{-1}$

Example

- Role *hasChild*$^-$ denotes the relationship *hasParent*.

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$\cdot^{trans}$ (Role transitivity axiom) denotes that a role is transitive. Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

Example

- Role *isPartOf* can be defined as transitive, while role *hasParent* is not. What about roles *hasPart*, $hasPart^{-}$, $hasGrandFather^{-}$?

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- What is a transitive closure of a relationship ? What is the difference between a transitive closure of $hasDirectBoss^{\mathcal{I}}$ and $hasBoss^{\mathcal{I}}$.

Extending . . . $\mathcal{ALC}$ ...(7)

$\mathcal{H}$ (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

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Example

- Role *hasMother* can be defined as a special case of the role *hasParent*.
- What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

Extending ... $\mathcal{ALC}$ ... (8)

$\mathcal{R}$ (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
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Example

- How would you define the role *hasUncle* by means of *hasSibling* and *hasParent* ?
- how to express that R is transitive, using a role chain ?
- Whom does the following concept denote $Person \sqcap \exists likes \cdot Self$?

Global restrictions

- *Simple roles* have no (direct or indirect) subroles that are either *transitive* or are defined by means of property chains

$$\begin{aligned} hasFather \circ hasBrother &\sqsubseteq hasUncle \\ hasUncle &\sqsubseteq hasRelative \\ hasBiologicalFather &\sqsubseteq hasFather \end{aligned}$$

  hasRelative and *hasUncle* are not simple.

- Each concept construct and each axiom from this list contains only *simple roles*:
  - number restrictions – $(\geq\ n\,R)$, $(=\ n\,R)$, $(\leq\ n\,R)$ + their qualified versions
  - $\exists R \cdot Self$
  - specifying functionality/inverse functionality (leads to number restrictions)
  - specifying irreflexivity, asymmetry, and disjoint object properties.

Extending . . . $\mathcal{ALC}$ ... – OWL-DL a OWL2-DL

- From the previously introduced extensions, two prominent decidable supersets of $\mathcal{ALC}$ can be constructed:

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    data types – XSD datatypes are used

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DL-safe rules

DL-safe rules are decidable conjunctive rules where each variable **only binds individuals** (not domain elements themselves).

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Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

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- $$\Box(Man \implies Person \wedge \Box_{hasFather} Man) \tag{2}$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

Data Types ($\mathcal{D}$) allow integrating a data domain (numbers, strings), e.g. $Person \sqcap \exists hasAge \cdot 23$ represents the concept describing "23-years old persons".

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