Name: *Date:*

Written Exam — Logical Reasoning and Programming (sample)

Convention: Variables are denoted by capital letters while other symbols by lower case letters.

1. The following Prolog program is given:

```
pass(S):-clever(S).
pass(S1):-next_to(S1,S2),pass(S2).
clever(trevor).
next_to(trevor,bob).
next_to(alice,carol).
next_to(X,Y):-next_to(Y,X).
```

   (a) How many proofs are there for the query

   ```
   ?-pass(bob).
   ```

   Explain your answer and give one of the proof trees. *(3 points)*

   (b) Indicate all answers Prolog would find, and the order in which they are returned (duplicates included), for the query

   ```
   ?-pass(X).
   ```

   (3 points)

   (c) Discuss two possible adaptations to Prolog's search strategy, so that it would return all answers.

   (2 points)

   (d) Show that

   ```
   pass(alice)
   ```

   is not a logical consequence of the program. *(3 points)*

2. In the 3rd column, write the substitution under which the Prolog terms in the first two columns unify, or leave the cell empty if they do not unify.

$f([a,b])$	$f([X,Y,Z])$	$X =$, $Y =$, $Z =$
$[X\|\_]+1$	$[1,2,3]+X$	$X =$
$2+3$	X	$X =$
$[a\|X]$	$[a,b,[c,d]]$	$X =$

(4 points)

3. Show by resolution that from

$$\forall X(p(X,c,X) \vee p(c,X,X)),$$
$$\forall X \forall Y \forall Z(p(X,Y,Z) \rightarrow p(Y,X,Z)),$$
$$\forall X \forall Y \forall Z(p(X,Y,Z) \rightarrow p(X,f(Y),f(Z)))$$

   all the following formulae are provable

   (a) $\forall X(p(X,c,X))$,

   (b) $\exists X \exists Y \forall Z(p(X,Z,Y) \rightarrow p(Z,Y,X))$,

   (c) $\forall X(p(f(c),f(X),f(f(X))))$.

   Please use a separate sheet of paper for your proofs if needed. *(10 points)*

4. You want to find a counter-example for a small problem using a MACE-style model finder like Paradox. What is roughly (in powers of ten) the maximal size (in the number of elements) of models you can obtain

   (a) 10, (b) 100, (c) 1,000, (d) 10,000, (e) 100,000, (f) 1,000,000.

   (2 points)

5. Write down all the pairs of clauses (implicitly universally quantified) from the following list (6 clauses) such that the first clause subsumes the second one

   (a) $\{p(X, f(c)), \neg q(b, a), \neg q(c, a)\}$,
   (b) $\{p(f(X), X), \neg q(X, Z)\}$,
   (c) $\{p(d, f(X)), \neg q(X, a)\}$,
   (d) $\{p(X, f(Y)), \neg q(Y, a)\}$,
   (e) $\{p(Y, X), p(Z, X)\}$,
   (f) $\{\neg q(X, Y), r(Z)\}$.

   (3 points)

6. Write down the set of all the possible answer sets for the program

$$\begin{aligned} P = \{p(a) &\leftarrow \text{not}\, q(b). \\ q(b) &\leftarrow \text{not}\, p(a), \text{not}\, r(c). \\ r(c) &\leftarrow q(b). \\ r(c) &\leftarrow \text{not}\, p(a).\}. \end{aligned}$$

   (3 points)

7. In conflict-driven clause learning (CDCL), it is usually possible to learn more than one clause from a conflict. Briefly explain why and what is a popular criterion (heuristic) for selecting the most useful learned clause?

   (4 points)

8. Describe at least one widely used technique to improve the trustworthiness of proof assistants?

   (3 points)

9. Assume that we restrict the resolution rule in such a way that at least one input clause is always a negative clause (=contains no positive literal). Let Γ be a set of clauses (in FOL without equality) that does not contain the empty clause. If there is no pair of clauses in Γ such that the restricted resolution is applicable on them, does it mean that it is impossible to derive the empty clause from Γ by standard (unrestricted) resolution calculus? Briefly explain your answer and note that even incomplete/wrong attempts can get points.

   (5 points)

10. Let Γ and Δ be satisfiable sets of formulae in first-order logic. Decide which of the following claims are always true.

    (a) $\Gamma \subseteq \Delta$ or $\Delta \subseteq \Gamma$,
    (b) $\{\varphi \colon \Gamma \models \varphi\}$ is satisfiable,
    (c) $\Gamma \cup \Delta$ is satisfiable,
    (d) $\Gamma \cap \Delta$ is satisfiable,
    (e) there exists a formula φ such that $\Gamma \not\models \varphi$ and $\Delta \not\models \varphi$.

    (5 points)