Logical reasoning and programming, lab session XI

(December 9, 2019)

XI.1 Skolemize the following formula

$$\forall X(p(a) \vee \exists Y(q(Y) \wedge \forall Z(p(Y,Z) \vee \exists U q(X,U)))) \vee \exists W q(a,W).$$

Why is it possible in this particular case to do that without producing an equivalent formula in prenex form?

XI.2 Unify the following pairs of formulae:

(a) $\{p(X,Y) \doteq p(Y,f(Z))\}$,
(b) $\{p(a,Y,f(Y)) \doteq p(Z,Z,U)\}$,
(c) $\{p(X,g(X)) \doteq p(Y,Y)\}$,
(d) $\{p(X,g(X),Y) \doteq p(Z,U,g(U))\}$,
(e) $\{p(g(X),Y) \doteq p(Y,Y), p(Y,Y) \doteq p(U,f(W))\}$.

XI.3 What is the size of the maximal term that is produced when you try to unify

$$\{f(g(X_1,X_1), g(X_2,X_2), \dots, g(X_{n-1},X_{n-1})) \doteq f(X_2,X_3,\dots,X_n)\}.$$

XI.4 Show that the resolution rule is correct.

XI.5 Derive the empty clause $\Box$ using the resolution calculus from:

(a) $\{\{\neg p(X), \neg p(f(X))\}, \{p(f(X)), p(X)\}, \{\neg p(X), p(f(X))\}\}$
(b) $\{\{\neg p(X,a), \neg p(X,Y), \neg p(Y,X)\}, \{p(X,f(X)), p(X,a)\}, \{p(f(X),X), p(X,a)\}\}$

XI.6 Prove using the resolution calculus that from

$$\forall X \forall Y (p(X,Y) \rightarrow p(Y,X))$$
$$\forall X \forall Y \forall Z((p(X,Y) \wedge p(Y,Z)) \rightarrow p(X,Z))$$
$$\forall X \exists Y p(X,Y)$$

follows $\forall X p(X,X)$.

XI.7 List all possible applications of the factoring rule on the clause

$$\{p(X,f(Y),Z), \neg s(Z,T), p(T,T,g(a)), p(f(b),S,g(W)), \neg s(c,d)\}.$$

If it is possible to use the factoring rule several times, then produce even these results.