WINDOWING

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FEL CTU PRAGUE

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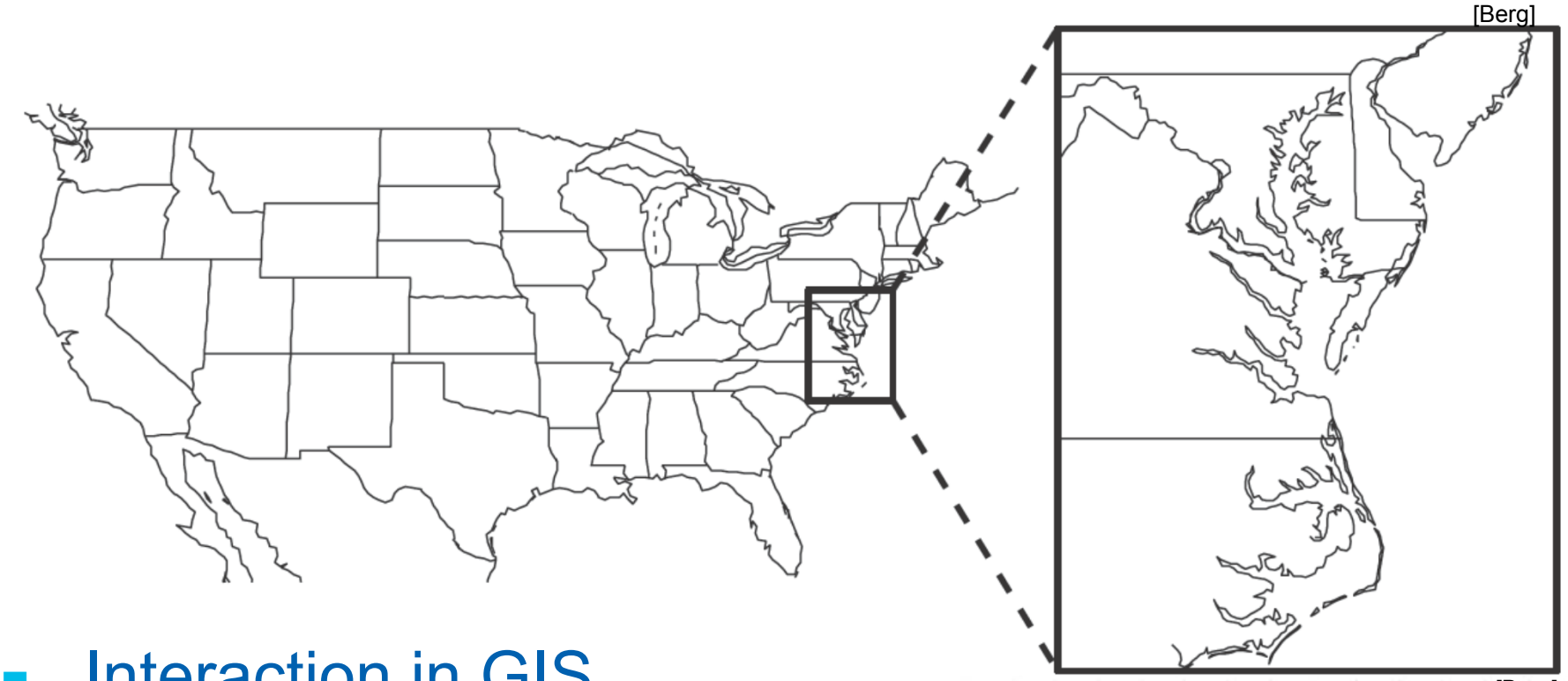
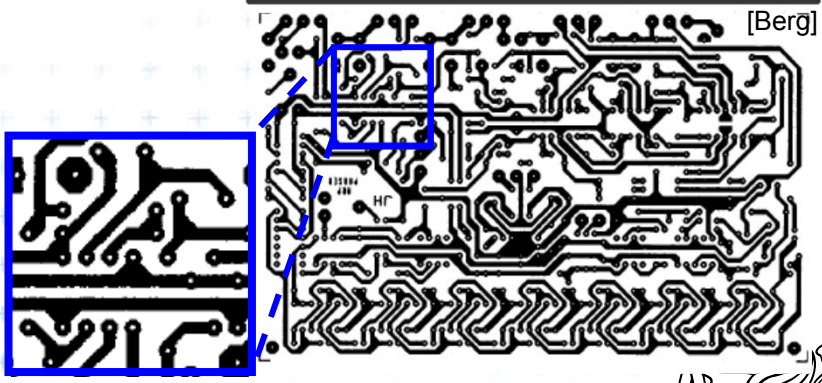
Based on [Berg], [Mount]

Version from 29.11.2019

Windowing queries - examples





- Interaction in GIS
  - Select subset by outlining
  - Zoom in and re-center
- Circuit board inspection,…

[Vakken]

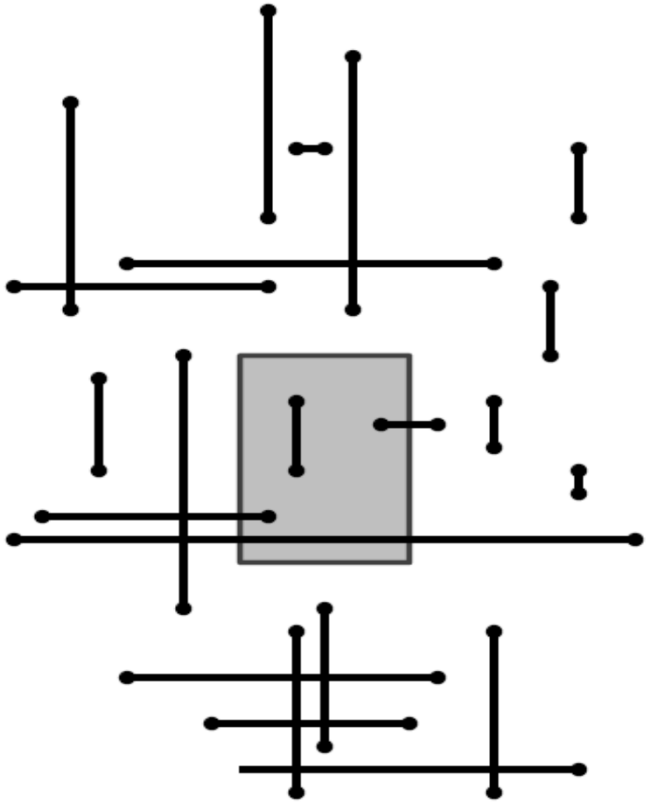
Windowing versus range queries

- Range queries (see range trees in Lecture 03)
  - Points
  - Often in higher dimensions
- Windowing queries
  - Line segments, curves, …
  - Usually in low dimension (2D, 3D)

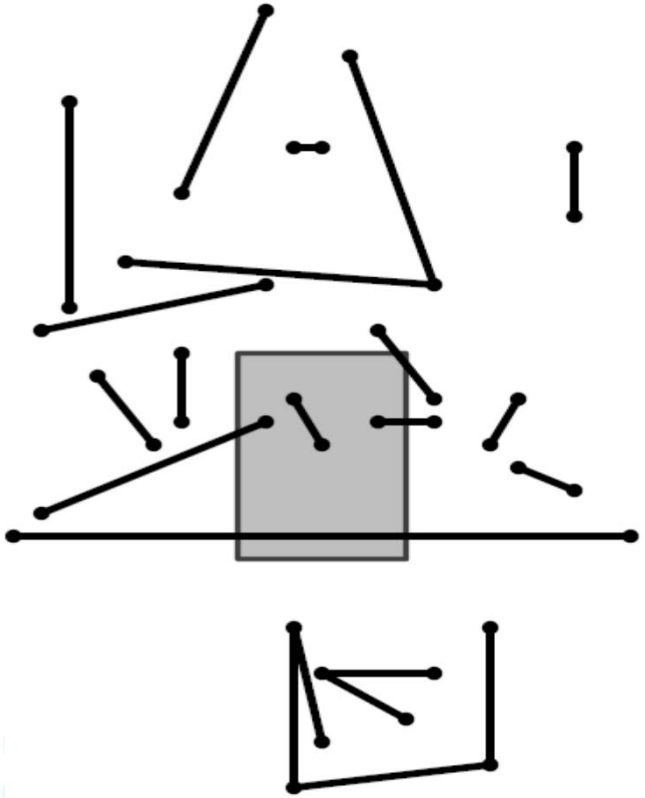
- The goal for both:
  Preprocess the data into a data structure
  - so that the objects intersected by the query rectangle can be reported efficiently

DCGI

Windowing queries on line segments



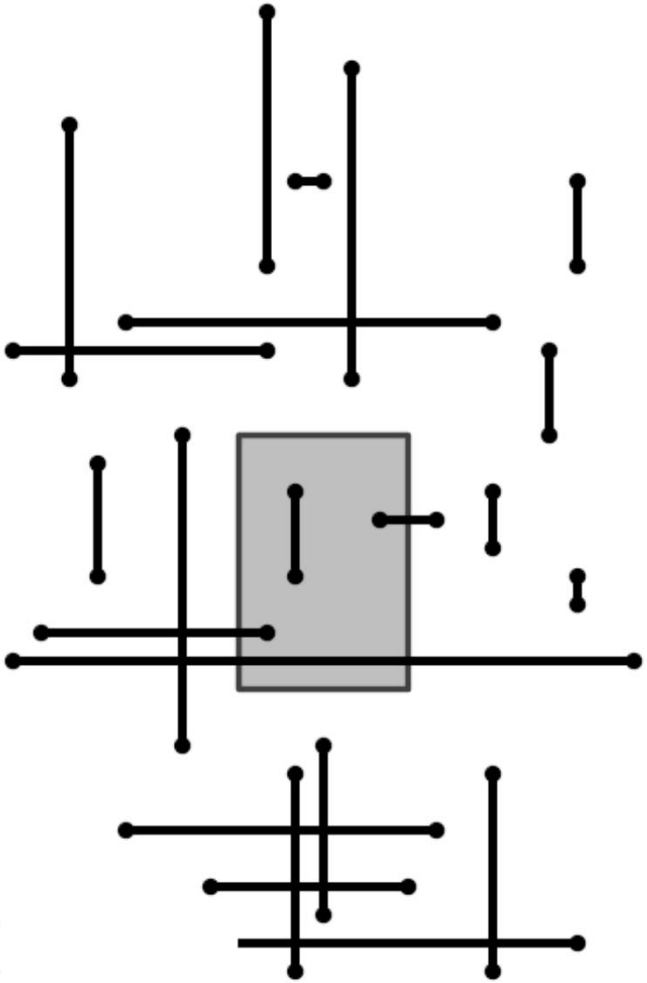
1. Axis parallel line segments



2. Arbitrary line segments (non-crossing)

[Vakken]

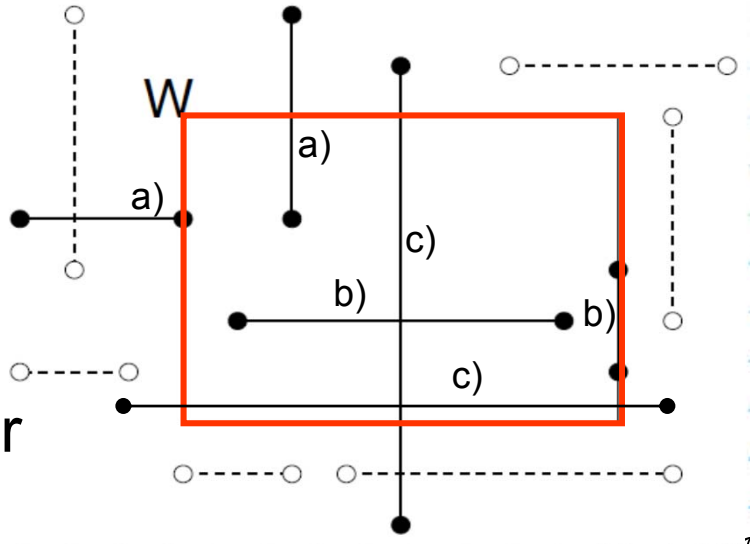
1. Windowing of axis parallel line segments



1. Windowing of axis parallel line segments

Window query

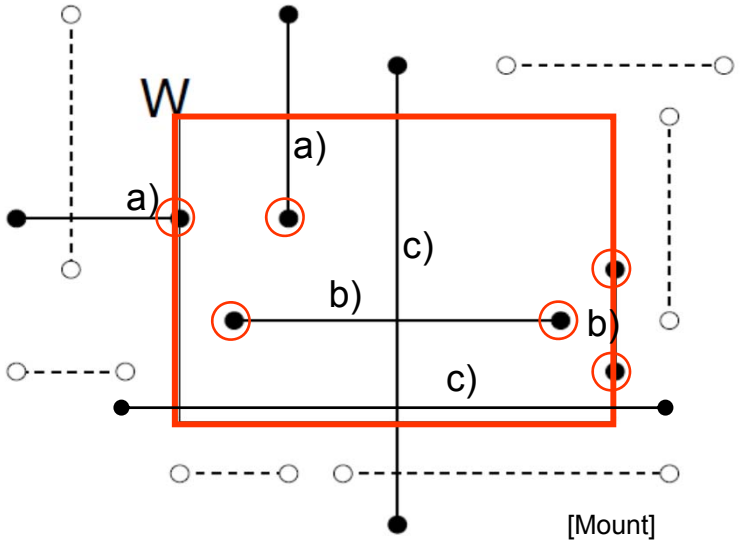
- Given
  - a set of orthogonal line segments S (preprocessed),
  - and orthogonal query rectangle $W = [\, x : x' \,] \times [\, y : y' \,]$
- Count or report all the line segments of S that intersect W
- Such segments have
  a) 1 endpoint in
  b) 2 end points in – Included
  c) no end point in – Cross over



Line segments with 1 or 2 points inside

a) 1 point inside

- Use a 2D range tree (lesson 3)
- O(n log n) storage
- O($\log^2 n + k$) query time or
- O($\log n + k$) with fractional cascading



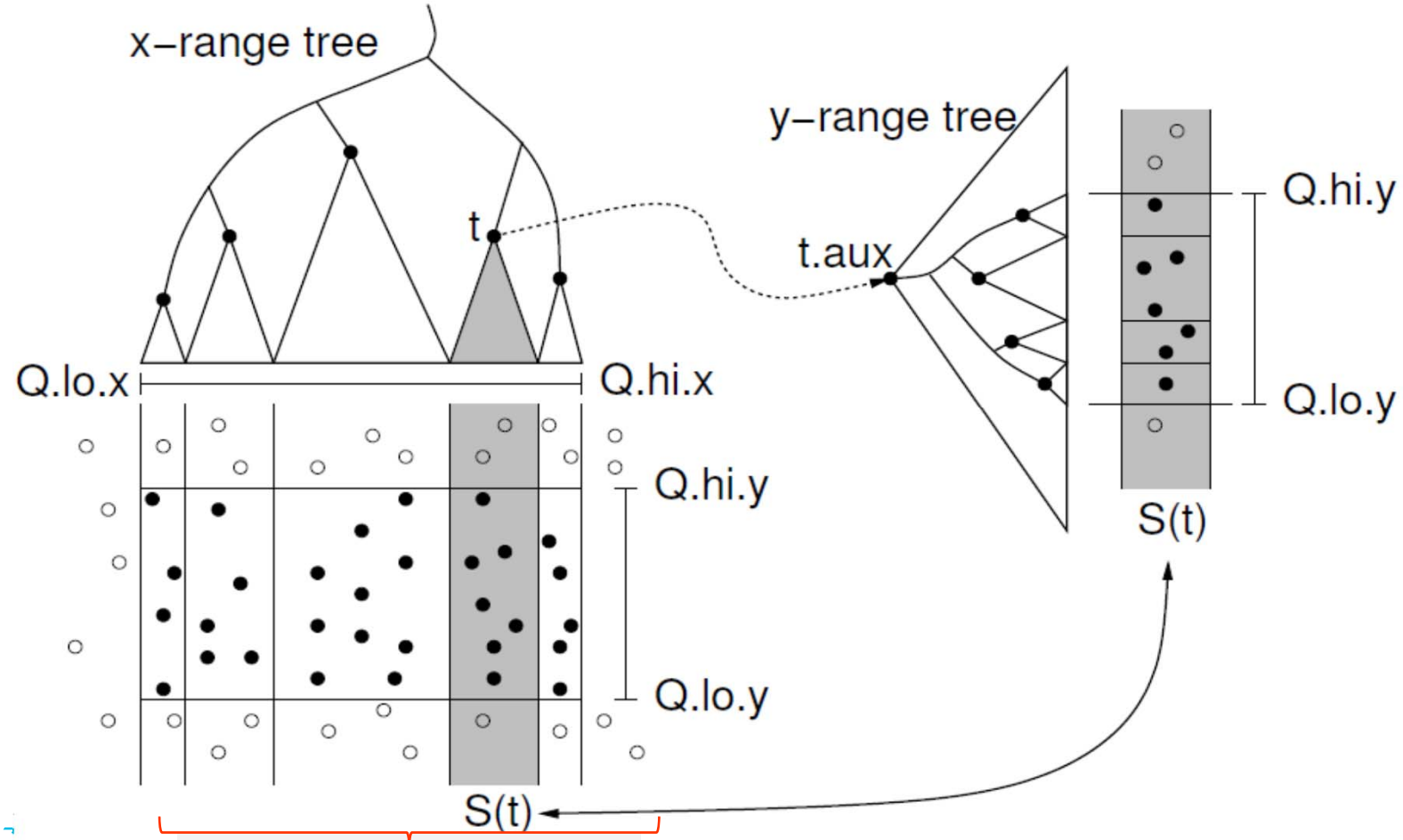
[Mount]

b) 2 points inside – as a) 1 point inside

- Avoid reporting twice
  1. Mark segment when reported (clear after the query)
  2. When end point found, check the other end-point. Report only the leftmost or bottom endpoint

2D range tree (without fractional cascading-more in Lecture 3)

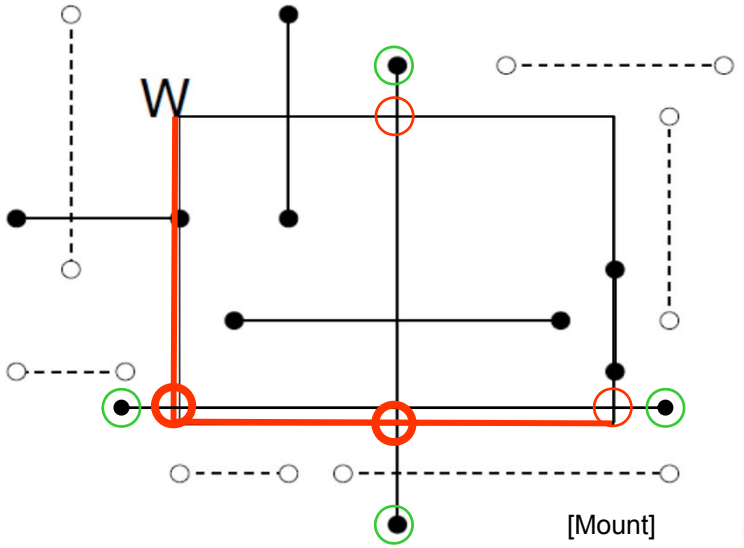
Search space: points
Query. Ortogonal intervals



Line segments that cross over the window

c) No points inside

- Such segments not detected using end-point range tree
- Cross the boundary twice
  or
  contain one boundary edge



[Mount]

- It is enough to detect segments intersected by the left and bottom boundary edges (not having end point inside)
- For left boundary: Report the segments intersecting vertical query *line segment* (1/ii.)
- Let's discuss vertical query *line* first (1/i.)
- Similarly for bottom boundary – rotated 90°

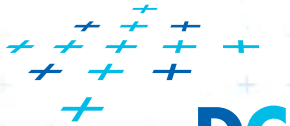
Windowing problem summary

Cases a) and b)

- Segment end-point in the query rectangle (window)
- Solved by 2D range trees

We will discuss case c)

- Segment crosses the window
- Three variants of data structure for storage of segments
  - 2x interval tree
  - 1x segment tree



Data structures for case c)

- Interval tree (1D IT)
  - stores 1D intervals (end-points in sorted lists)
  - computes intersections with query interval
    - known from intersection of axis angle rectangles – y-overlap there, x-overlap here
- We must extend IT to 2D
  - variants differ in storage of interval end-points
    - 2D range trees
    - priority search trees
- Segment tree
  - splits the plane to slabs in x in elementary intervals

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Talk overview

1. Windowing of axis parallel line segments in 2D

- 3 variants of *interval tree – IT in x-direction*
- Differ in storage of segment end points M_L and M_R

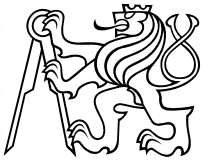
1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D ii. Line segment stabbing (*IT* with *range trees*)

2D iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D – *segment tree + BST*



Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree - IT*)
   i. Line stabbing (standard *IT* with *sorted lists*)
   ii. Line segment stabbing (*IT* with *range trees*)
   iii. Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*

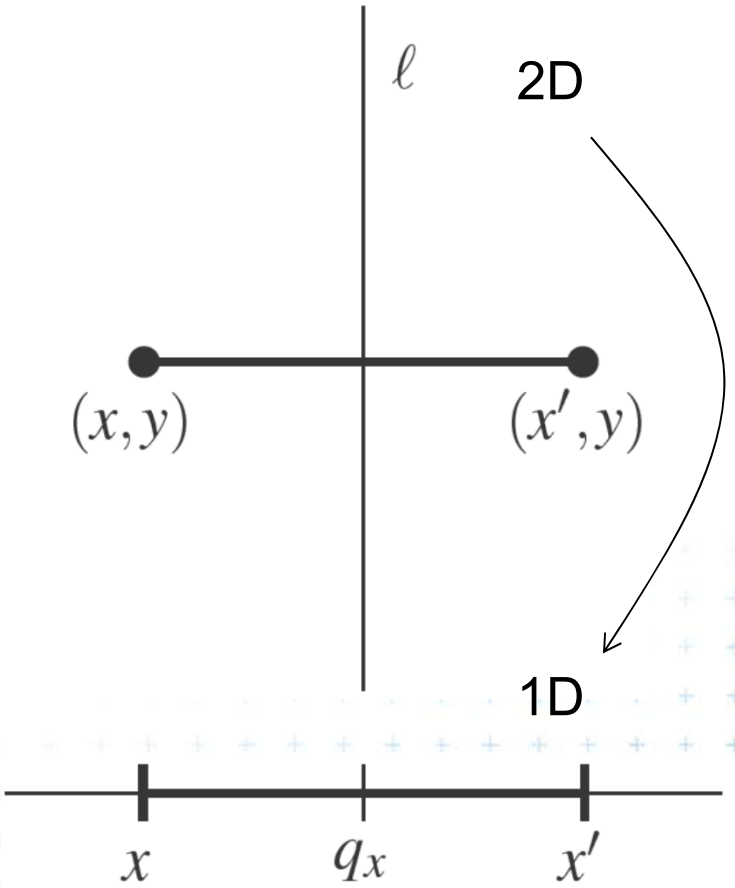
i. Segment intersected by vertical line

- Query line $\ell := (x = q_x)$

  Report the segments stabbed by a vertical line

  = 1 dimensional problem

  (ignore y coordinate)

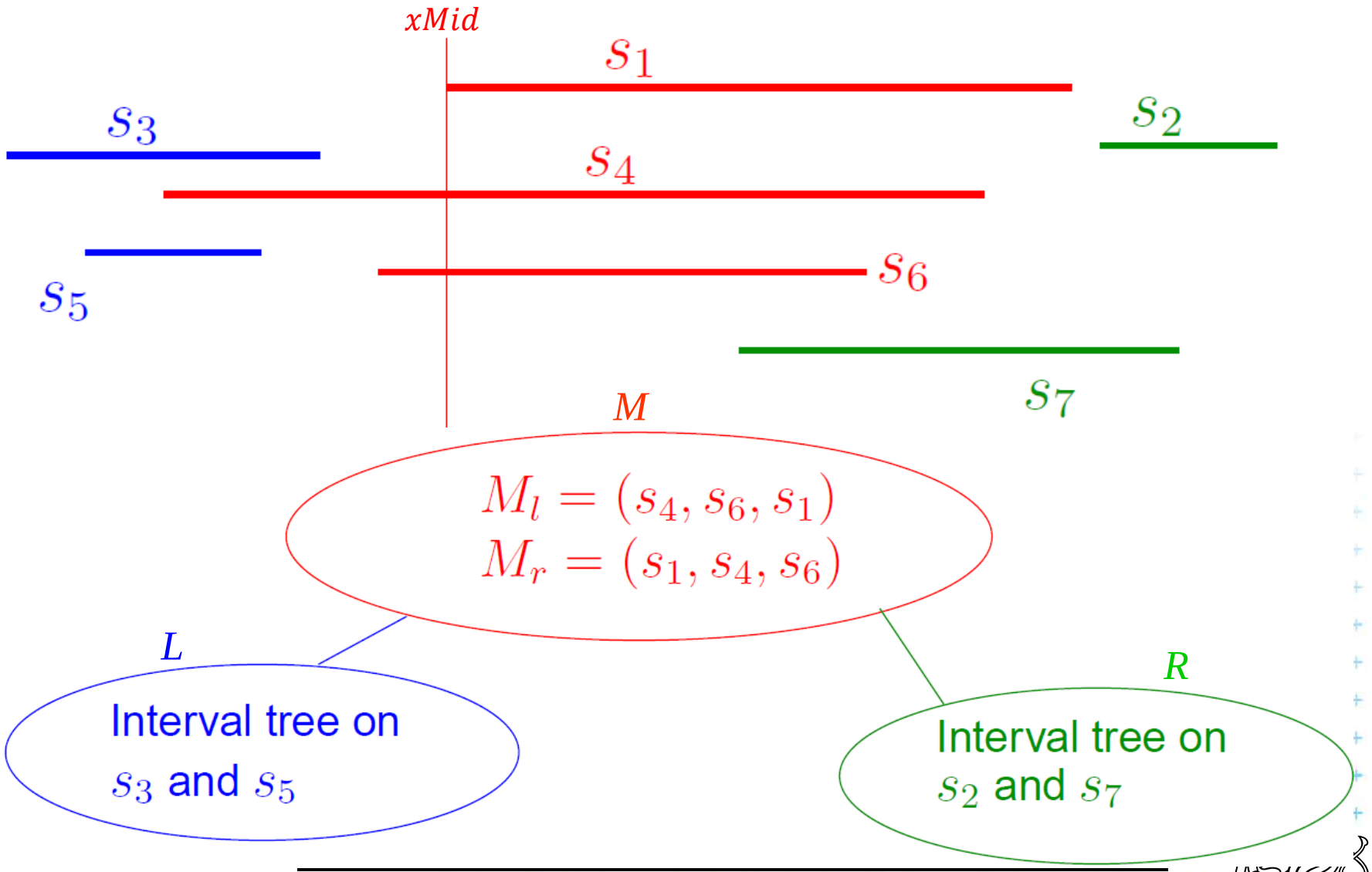


⇒ Report the interval containing query point q_x

DS: Interval tree with sorted lists

Interval tree principle

(see lecture 9 - intersections)



[Vigneron]

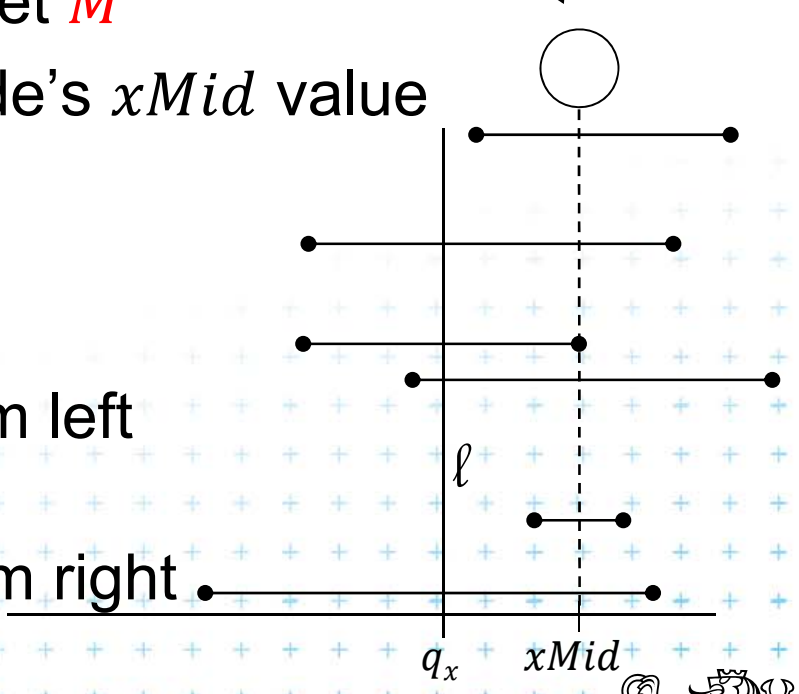
i. Segment intersected by vertical line

Principle

- Store input segments in interval tree
- In each interval tree node
  - Check the segments in the set M
  - These segments contain node's $xMid$ value
    - M_L are left end-points
    - M_R are right end-points
  - q_x is the query value
  - If $(q_x < xMid)$ Sweep M_L from left

    p ∈ M_L: if $p_x \leq q_x \Rightarrow$ intersection
  - If $(q_x > xMid)$ Sweep M_R from right

    p ∈ M_R: if $p_x \geq q_x \Rightarrow$ intersection



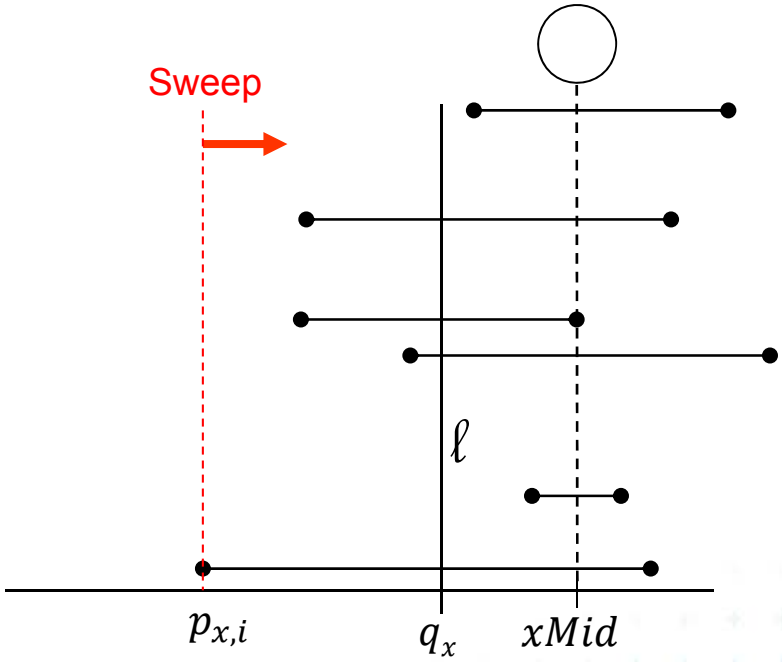
Inspired by [Berg]

Segment intersection (left from $xMid$)

All line segments from M pass through $xMid$

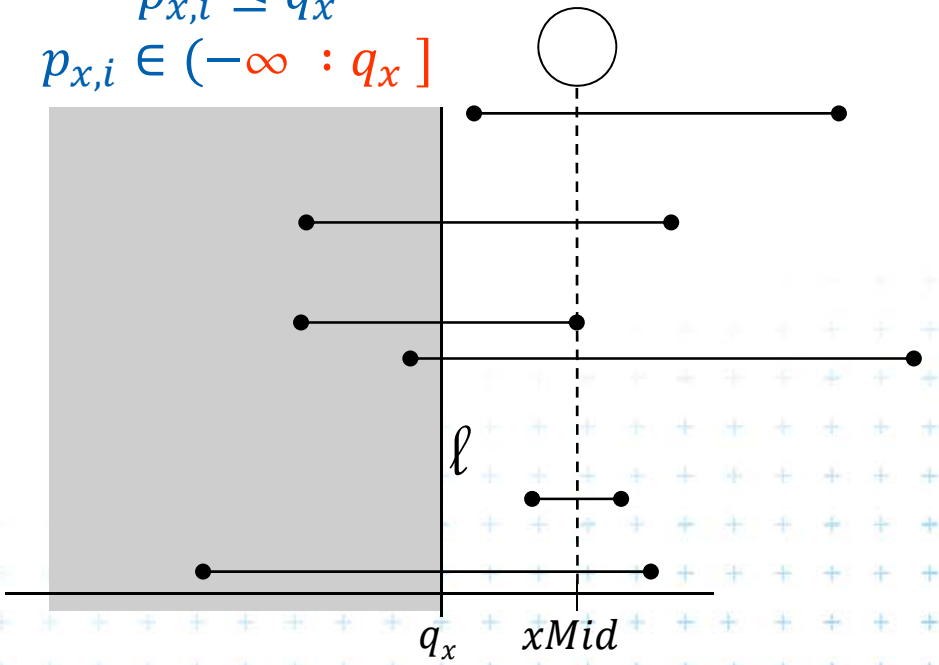
$\Rightarrow q_x$ must be between $p_{x,i}$ and $xMid$ to intersect the line segment i

$\Rightarrow p_{x,i} \leq q_x \Rightarrow$ intersection



$p_{x,i} \leq q_x$

$p_{x,i} \in (-\infty : q_x]$



Intersection with line ℓ means $\ell := q_x \times [-\infty : \infty]$

Intersection with half space q $q := (-\infty : q_x] \times [-\infty : \infty]$

Inspired by [Berg]

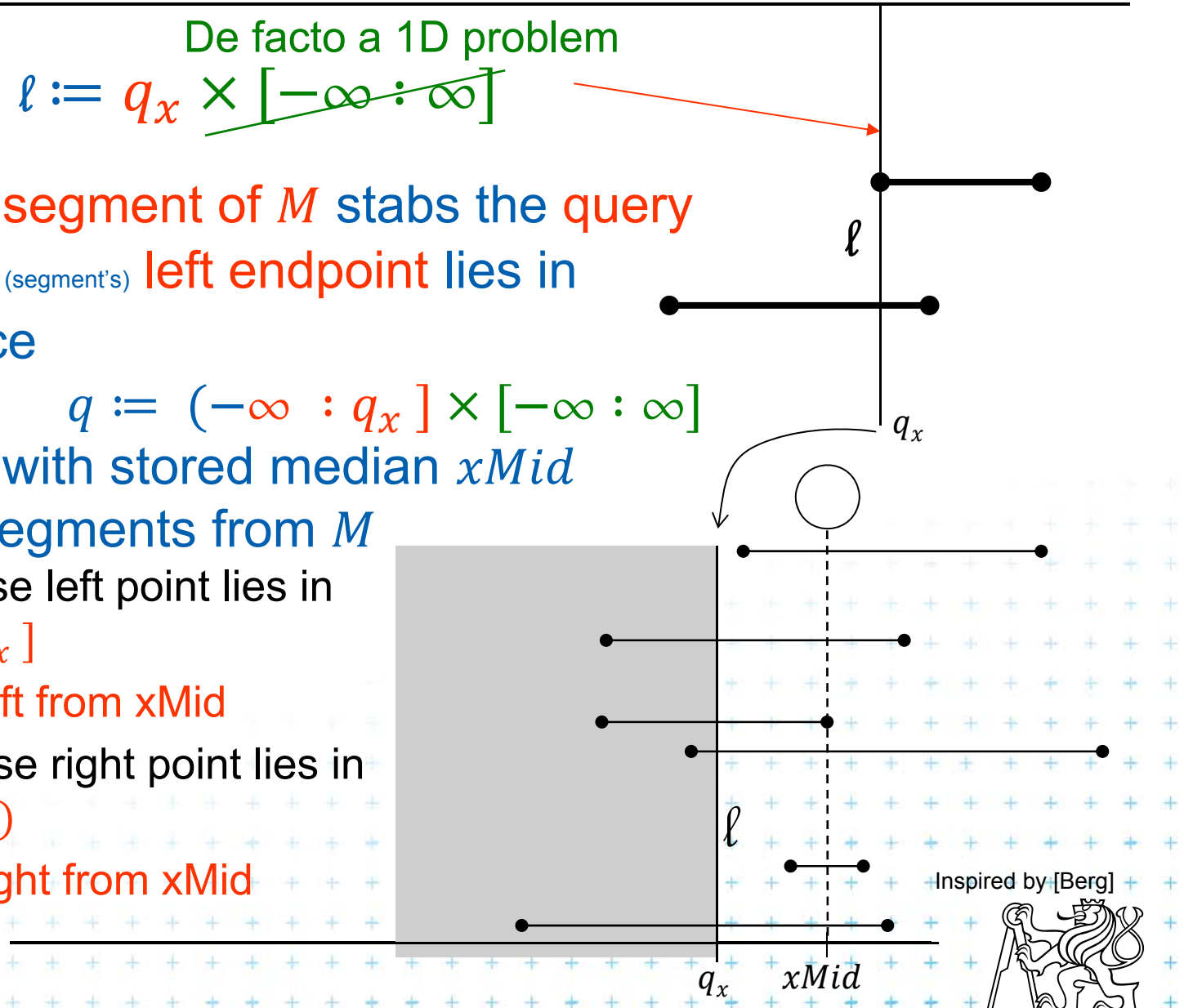
i. Segment intersected by vertical line

De facto a 1D problem

- Query line $\ell := q_x \times [-\infty : \infty]$
- Horizontal segment of M stabs the query line ℓ **iff** its (segment's) left endpoint lies in halph-space

$$q := (-\infty : q_x] \times [-\infty : \infty]$$

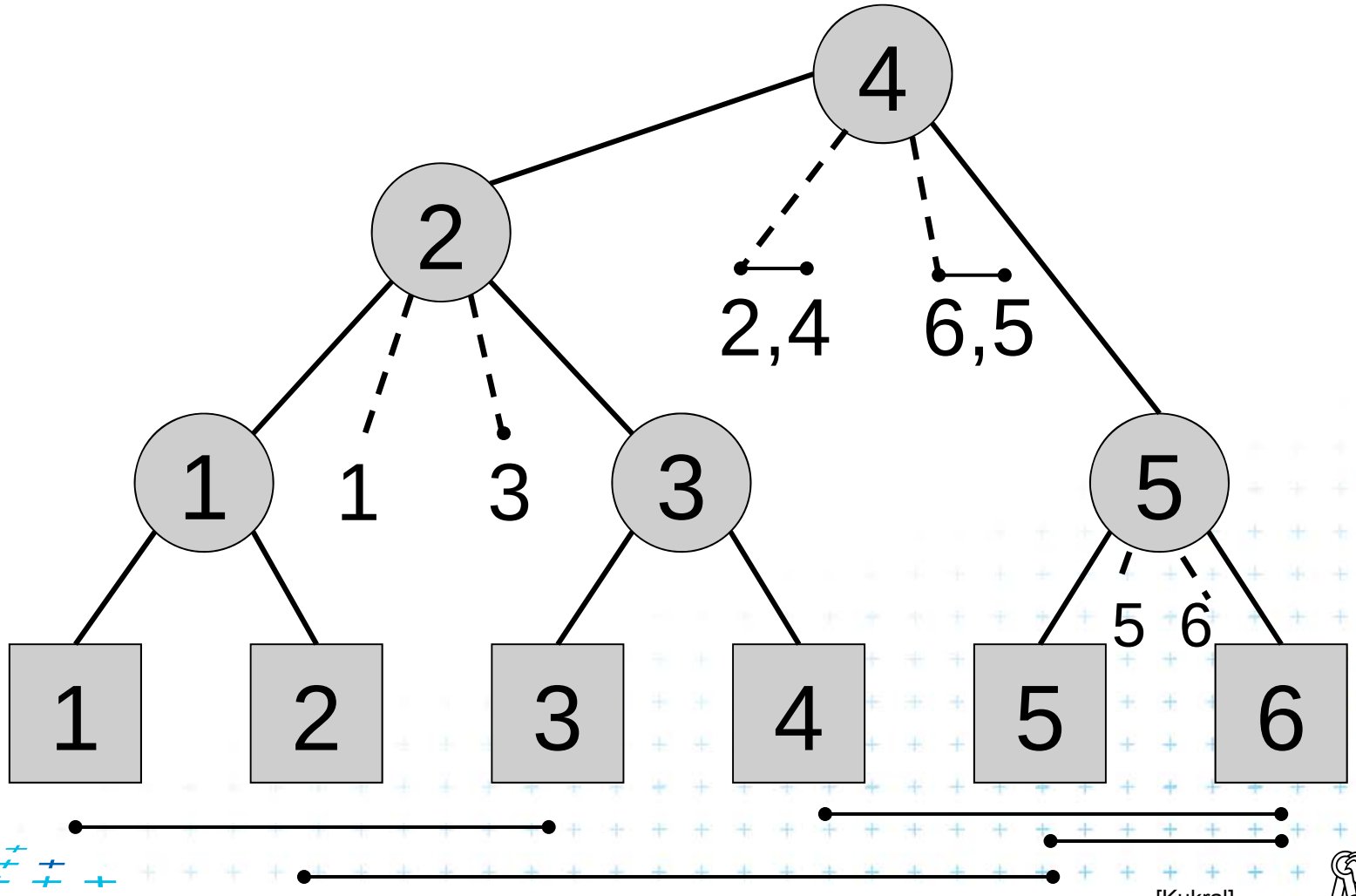
- In IT node with stored median $xMid$ report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x]$ if ℓ lies left from xMid
  - M_R: whose right point lies in $[q_x : +\infty)$ if ℓ lies right from xMid



Inspired by [Berg]

Static interval tree [Edelsbrunner80]

Tree over sorted segment end-points



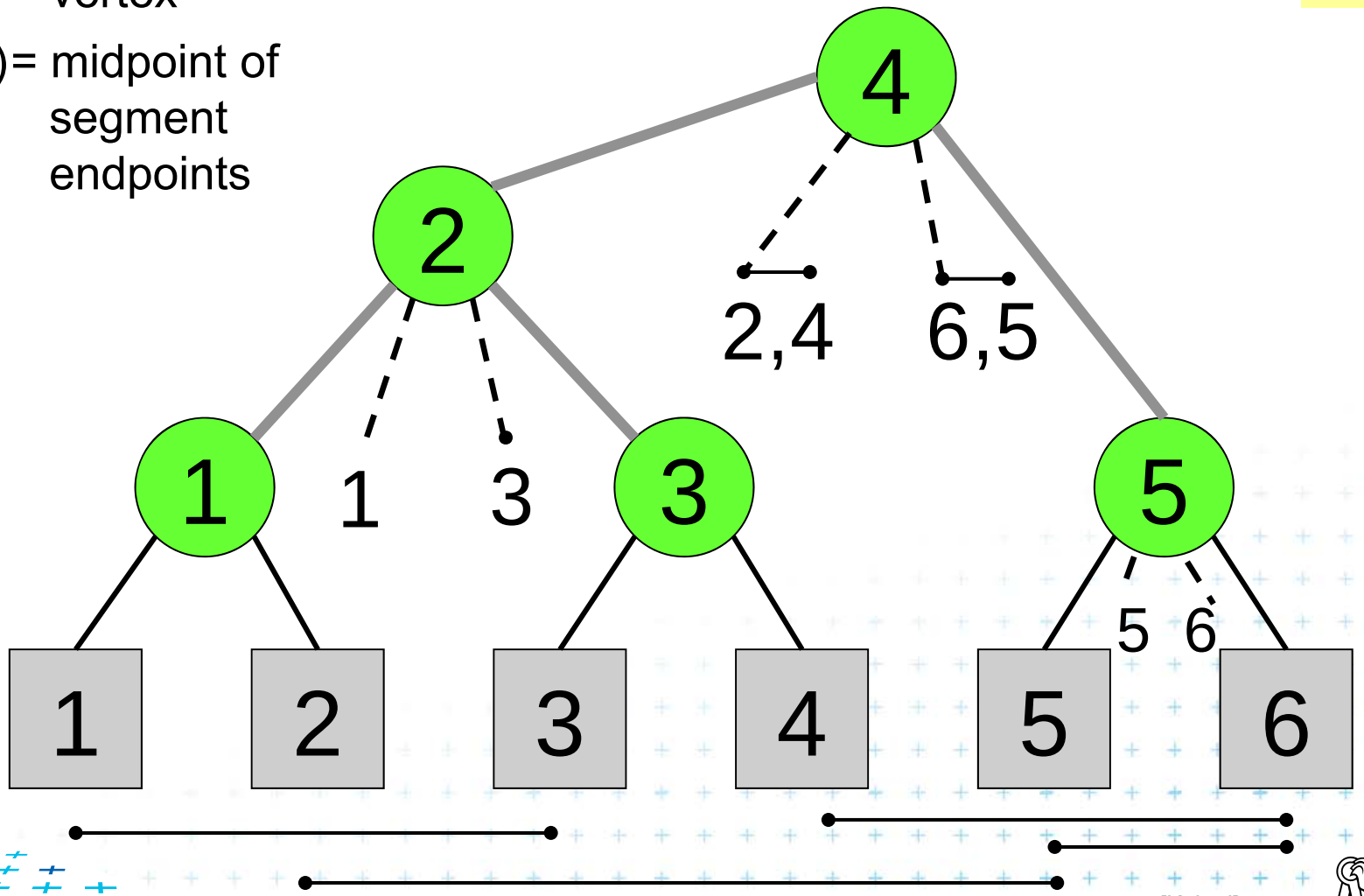
[Kukral]

Primary structure – static tree for endpoints

Static

v = vertex

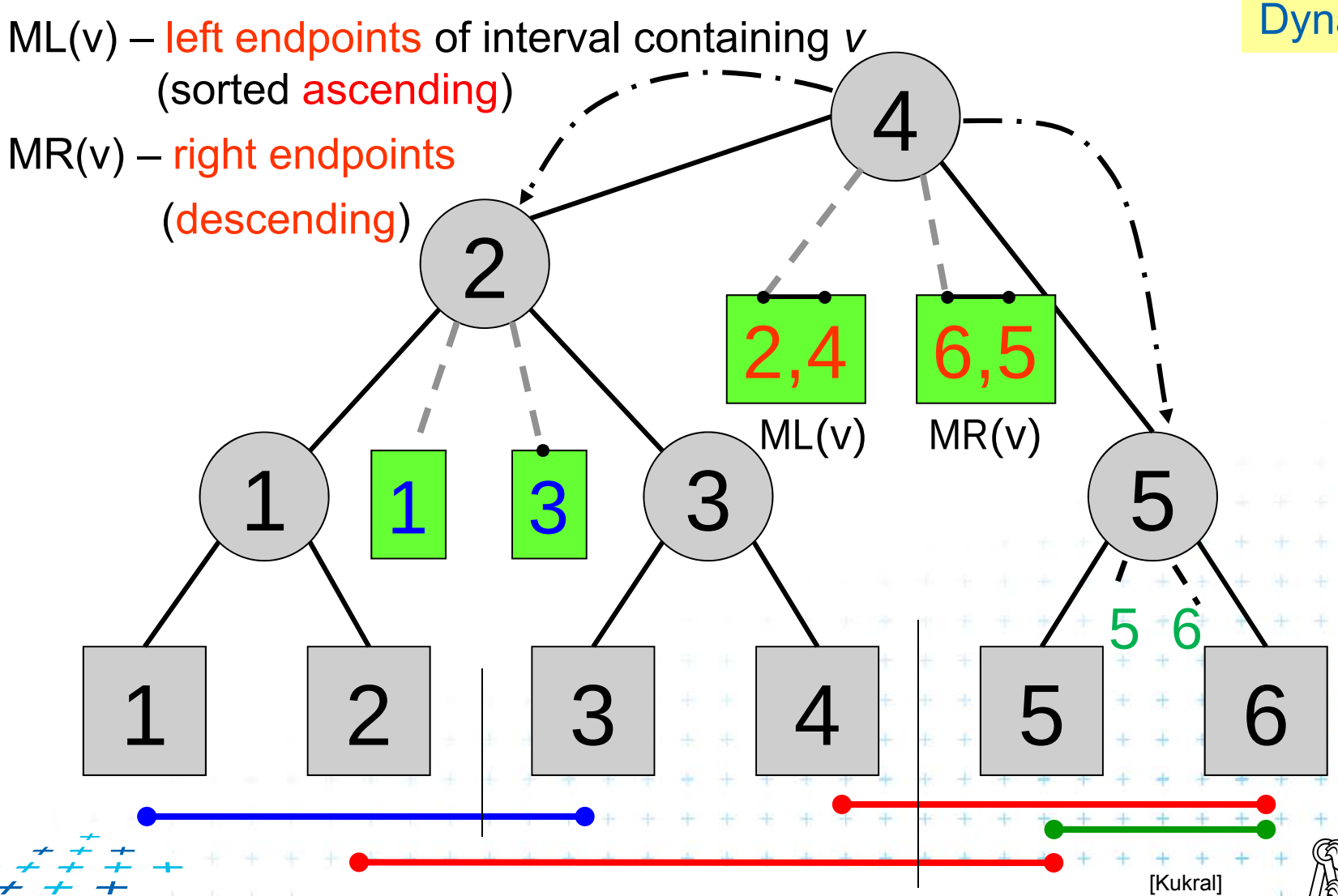
d(v)= midpoint of segment endpoints



[Kukral]

Secondary lists of incident interval end-pts.

Dynamic



Interval tree construction

Merged procedures from in lecture 09
- PrimaryTree(*S*) on slide 33
- InsertInterval (*b, e, T*) on slide 35

ConstructIntervalTree(*S*) // Intervals all active – no active lists

Input: Set S of intervals on the real line – on *x-axis*

Output: The root of an interval tree for *S*

```
1.  if (|S| == 0) return null                          // no more intervals
2.  else
3.      xMed = median endpoint of intervals in S       // median endpoint
4.      L = { [xlo, xhi] in S | xhi < xMed }           // left of median
5.      R = { [xlo, xhi] in S | xlo > xMed }           // right of median
6.      M = { [xlo, xhi] in S | xlo <= xMed <= xhi }   // contains median
7.      ML = sort M in increasing order of xlo         // sort M
8.      MR = sort M in decreasing order of xhi
9.      t = new IntTreeNode(xMed, ML, MR)              // this node
10.     t.left  = ConstructIntervalTree(L)             // left subtree
11.     t.right = ConstructIntervalTree(R)             // right subtree
12.     return t
```

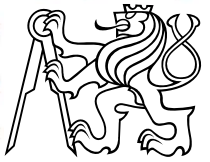
steps 4.,5.,6. done in one step if presorted [Mount]

Line stabbing query for an interval tree

Less effective variant of QueryInterval (b, e, T) on slide 34 in lecture 09 with merged parts: fork and search right

```
Stab( t, xq)
Input:    IntTreeNode t, Scalar xq
Output:   prints the intersected intervals
1.   if (t == null) return                          // no leaf: fell out of the tree
2.   if (xq < t.xMed)                               // left of median?
3.       for (i = 0; i < t.ML.length; i++)          // traverse M_L left end-points
4.           if (t.ML[i].lo ≤ xq) print(t.ML[i])    // ..report if in range
5.           else break                             // ..else done
6.       stab(t.left, xq)                           // recurse on left
7.   else  // (xq ≥ t.xMed)                         // right of or equal to median
8.       for (i = 0; i < t.MR.length; i++) {        // traverse M_R right end-points
9.           if (t.MR[i].hi ≥ xq) print(t.MR[i])    // ..report if in range
10.          else break                             // ..else done
11.      stab(t.right, xq)                          // recurse on right
```

Note: Small inefficiency for xq == t.xMed – recurse on right



Complexity of line stabbing via interval tree

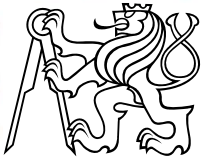
with *sorted lists*

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves or less (minus elements of M) => tree of height $h = O(\log n)$
  - If presorted endpoints in three lists L,R, and M then median in O(1) and copy to new L,R,M in O(n)
- Vertical line stabbing query - $O(k + \log n)$ time
  - One node processed in $O(1 + k')$, k' reported intervals
  - v visited nodes in $O(v + k)$, k total reported intervals
  - $v = h =$ tree height = $O(\log n)$ $k = \Sigma k'$
- Storage - $O(n)$
  - Tree has $O(n)$ nodes, each segment stored twice (two endpoints)

DCGI

Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree – IT*)
   i. Line stabbing (standard *IT* with *sorted lists*)
   ii. Line segment stabbing (*IT* with *range trees*)
   iii. Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*



Line segment stabbing (*IT* with *range trees*)

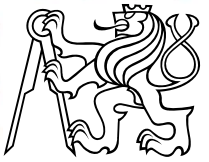
Enhance 1D interval trees to 2D

– 1D test $q_x \in \langle x, x' \rangle$ done by interval tree with sorted lists M_L and M_R as into $q_x \in (-\infty : q_x]$ for M_L

$q_x \in [q_x : +\infty)$ for MR

– and change lines to segments $q_x \times [-\infty : \infty]$ (no y-test)

$q_x \times [q_y : q'_y]$ (additional y-test)



i. Segment intersected by vertical line

De facto a 1D problem

- Query line $\ell := q_x \times [-\infty : \infty]$
- Horizontal segment of M_L stabs the query line ℓ **iff** its left endpoint lies in half-space

$$q := (-\infty : q_x] \times [-\infty : \infty]$$

- In IT node with stored median xMid report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x]$ if ℓ lies left from xMid
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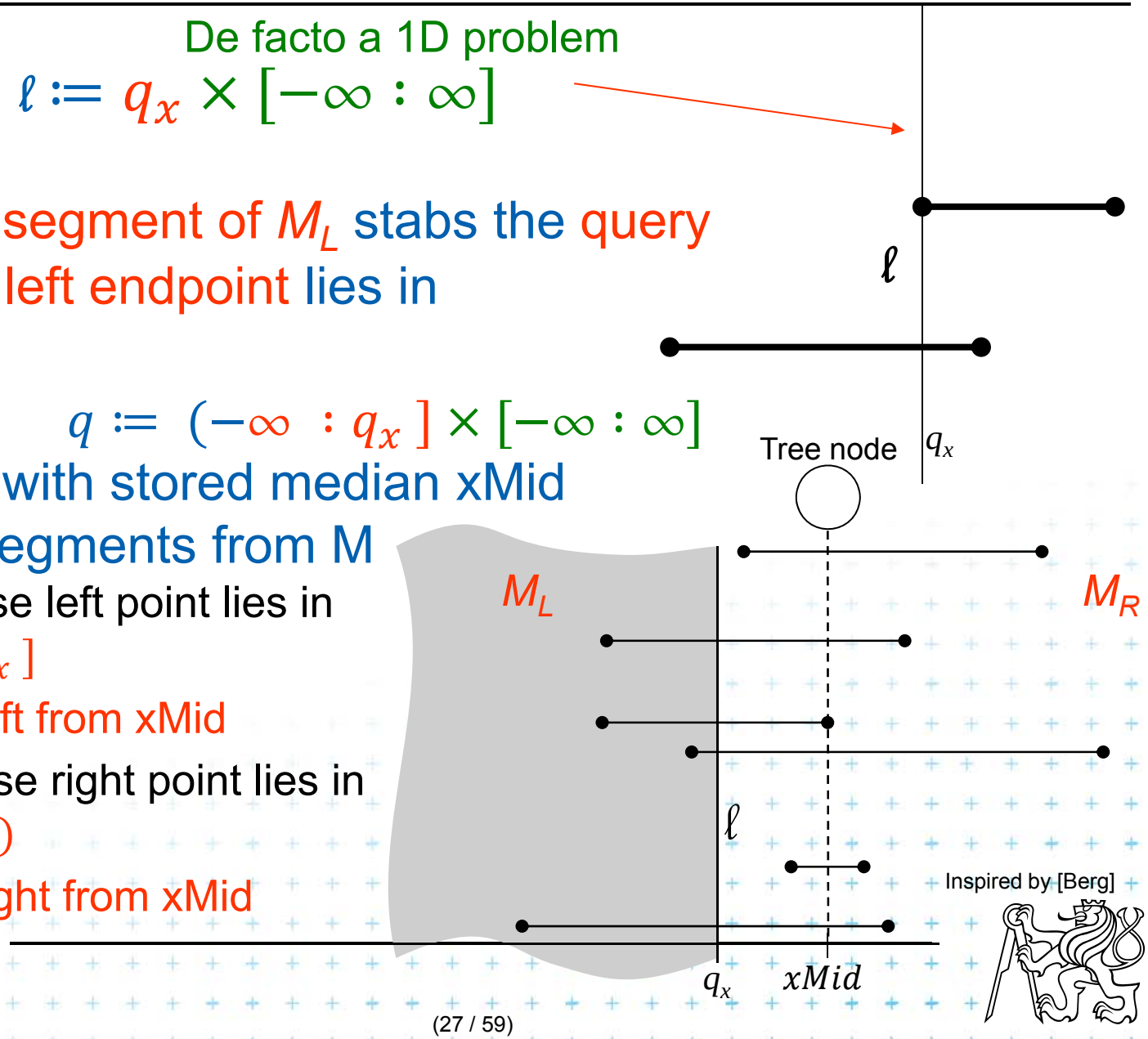
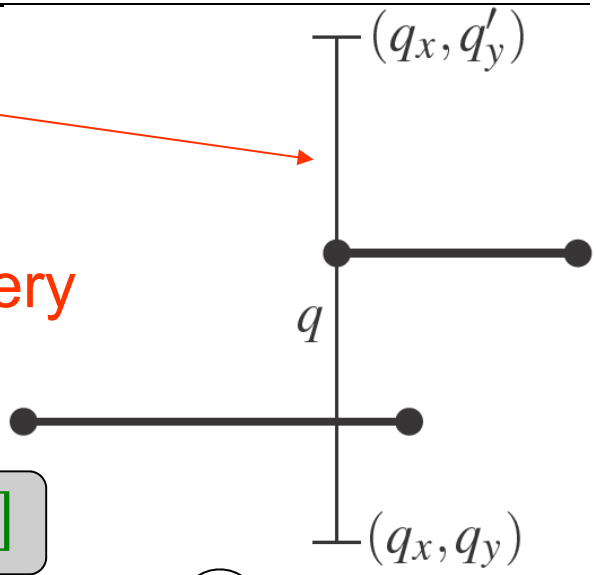


Inspired by [Berg]

ii. Segment intersected by vertical line segment

A 2D problem

- Query segment $q := q_x \times [q_y : q'_y]$

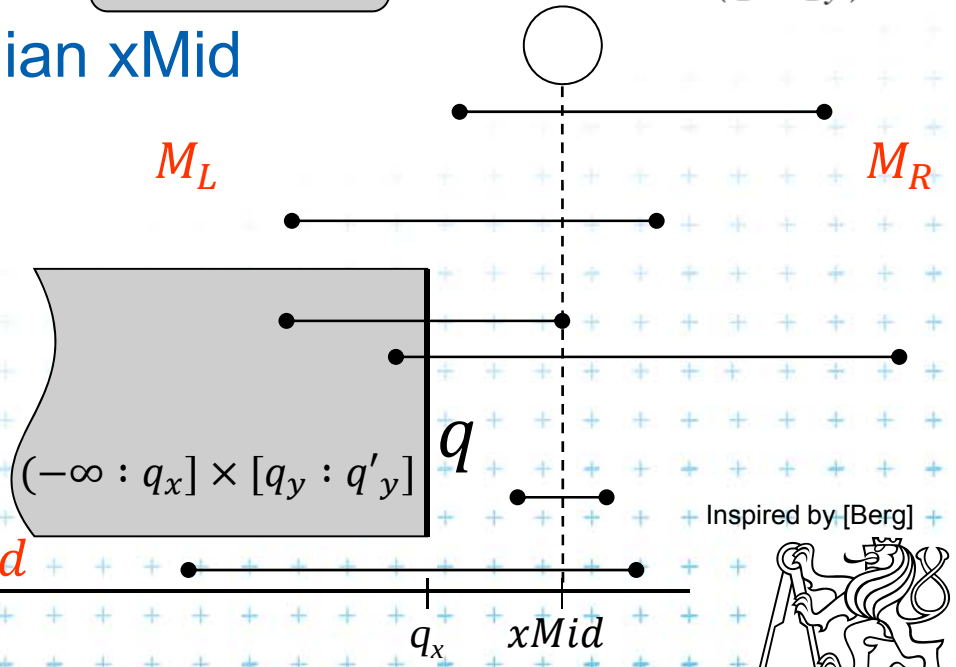


- Horizontal segment of M_L stabs the query segment q **iff** its left endpoint lies in semi-infinite rectangular region

New test

$$q := (-\infty : q_x] \times [q_y : q'_y]$$

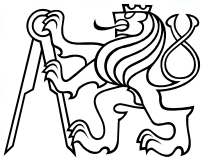
- In IT node with stored median xMid report all segments
  - M_L: whose left points lie in $(-\infty : q_x] \times [q_y : q'_y]$ where q_x lies left from $xMid$
  - M_R: whose right point lies in $[q_x : +\infty) \times [q_y : q'_y]$ where q_x lies right from $xMid$



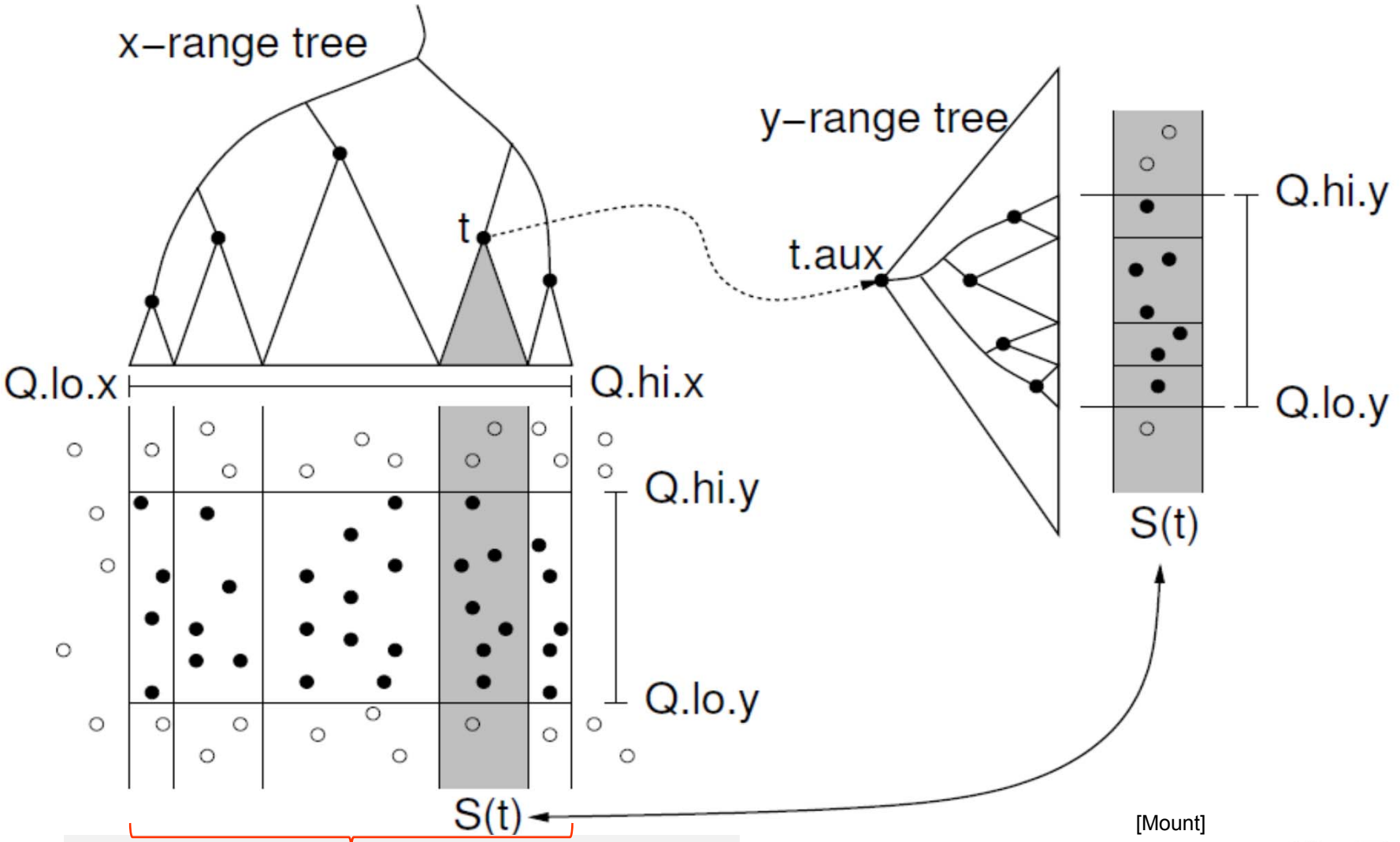
Inspired by [Berg]

Data structure for endpoints

- Storage of M_L and M_R
  - 1D Sorted lists is not enough for line segments
  - Use two 2D range trees
- Instead $O(n)$ sequential search in M_L and M_R perform $O(\log n)$ search in range tree with fractional cascading



2D range tree (without fractional cascading-more in Lecture 3)

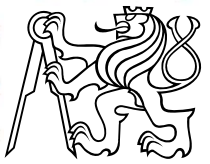


[Mount]

Segment left end-points for M_L
or segment right end-points for M_R

Complexity of line segment stabbing

- Construction - O(n log n) time
  - Each step divides at maximum into two halves L,R or less (minus elements of M) => tree height O(log n)
  - If the range trees are efficiently build in O(n) after points sorted
- Vertical line segment stab. q. - O(k + log^2 n) time
  - One node processed in $O(\log n + k')$, k' reported segm. (interval tree; 2D range tree search with Fractional Cascading)
  - v-visited nodes in $O(v \log n + k)$, k total reported segm. (interval tree)
  - v = interval tree height = $O(\log n)$ $\quad k = \sum k'$
  - $O(k + \log^2 n)$ time - range tree with fractional cascading
  - $O(k + \log^3 n)$ time - range tree without fractional casc.
- Storage - $O(n \log n)$ Can be done better?
  - Dominated by the range trees



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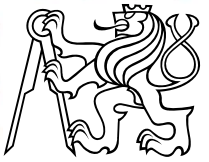
   iii. Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*

iii. Priority search trees

[McCreight85]

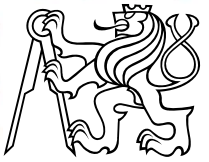
- Another variant for case c) on slide 9

  - Exploit the fact that query rectangle in range queries is unbounded (in x direction)
- Priority search trees
  - as secondary data structure for both left and right endpoints (ML and MR) of segments in nodes of interval tree – one for ML, one for MR
  - Improve the storage to O(n) for horizontal segment intersection with window edge (2D range tree has O(n log n))
- For cases a) and b) - O(n log n) storage remains
  - we need range trees for windowing segment endpoints



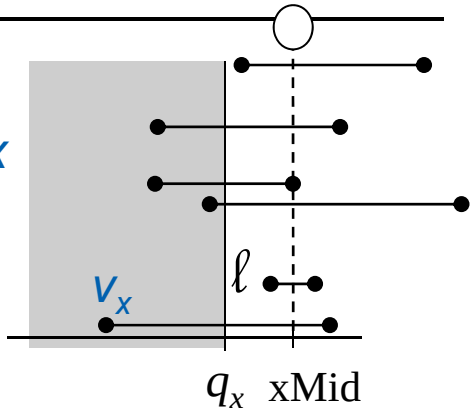
Rectangular range queries variants

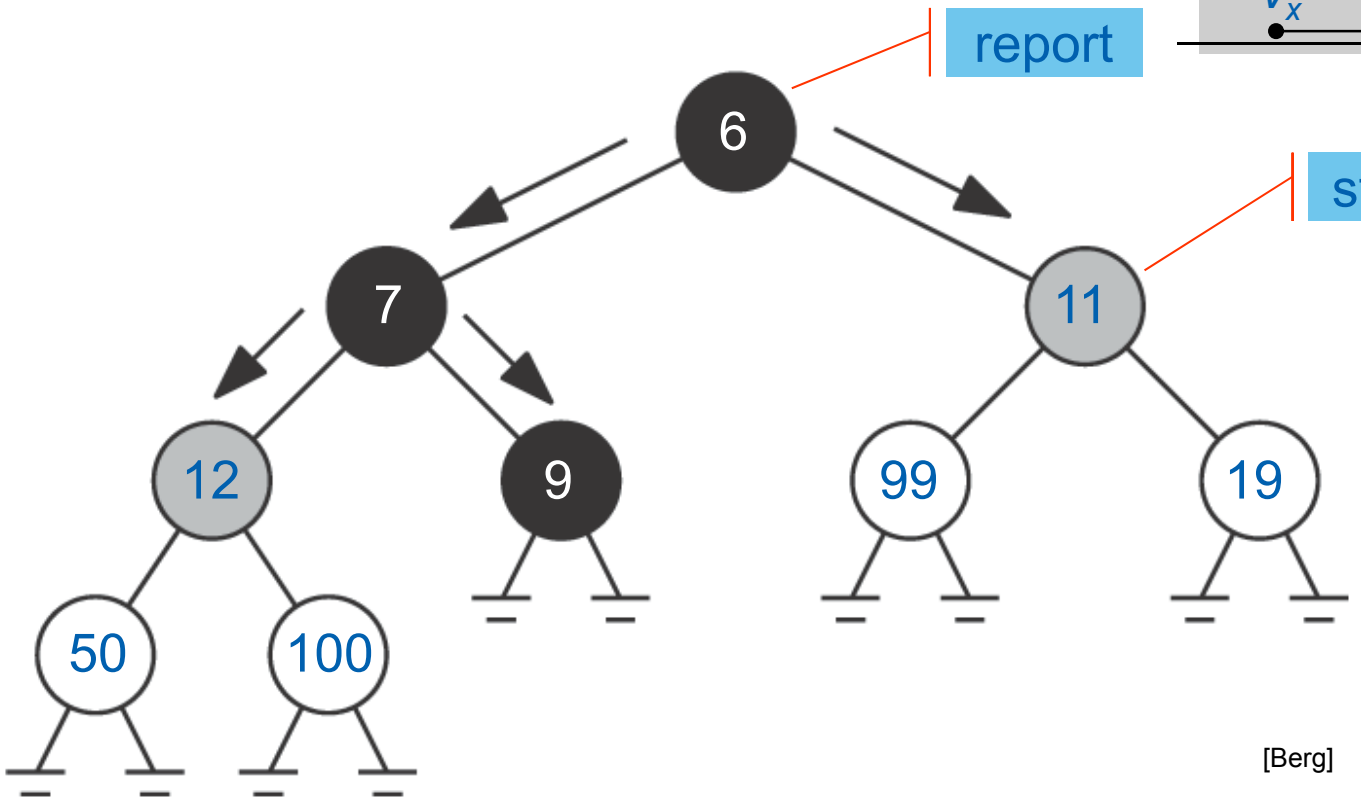
- Let $P = \{p_1, p_2, \ldots, p_n\}$ is set of points in plane
- Goal: rectangular range queries of the form
  $(-\infty : q_x] \times [q_y : q'_y] -$ unbounded (in x direction)
- In 1D: search for nodes v with $v_x \in (-\infty : qx]$
  - range tree $O(\log n + k)$ time (search the end, report left)
  - ordered list $O(1 + k)$ time 1 is for possibly fail test of the first
    (start in the leftmost, stop on v with $v_x > q_x$)
  - use heap $O(1 + k)$ time !
    (traverse all children, stop when $v_x > q_x$)
- In 2D – use heap for points with $x \in (-\infty : q_x]$
  + integrate information about y-coordinate
  = Priority search tree



Heap for 1D unbounded range queries

- Traverse all children, stop when $v_x > q_x$
- Example: Query $(-\infty:10]$, $q_x = 10$

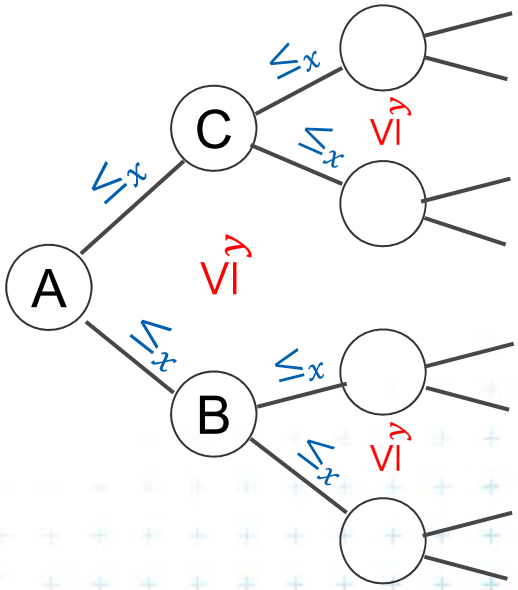




[Berg]

Principle of priority search tree

- Heap $\leq_x$
  - relation between parent and its child nodes only
  - no relation between the child nodes themselves
- Priority search tree
  - relate the child nodes according to y $\leq_y$



A $\leq_x$ B

A $\leq_x$ C

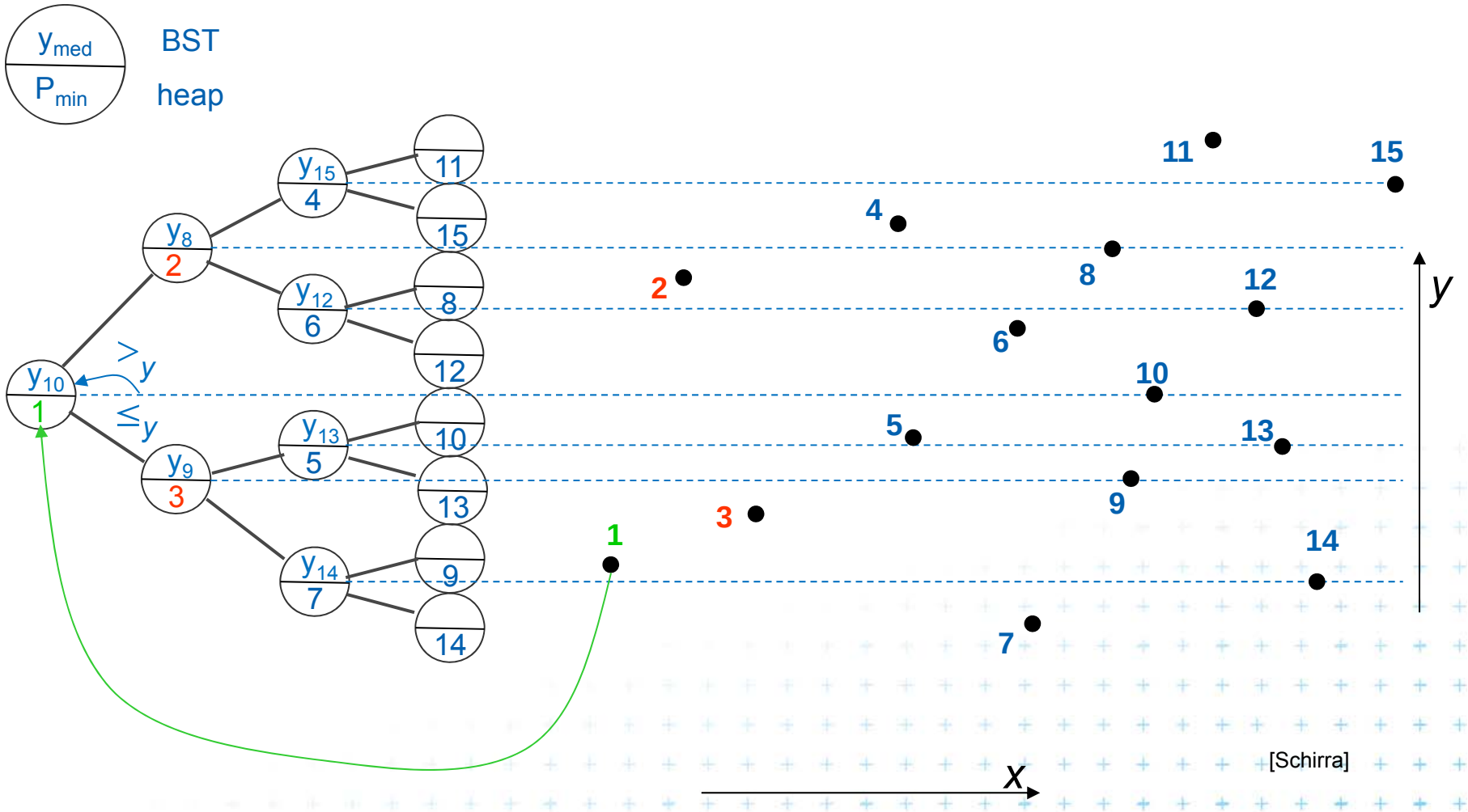
B $\leq_y$ C

Priority search tree (PST)

- Heap in 2D can incorporate info about both *x,y*
  - BST on *y*-coordinate (horizontal slabs) ~ range tree
  - Heap on *x*-coordinate (minimum *x* from slab along *x*)
- If P is empty, PST is empty leaf
- else
  - p_{min} = point with smallest x-coordinate in P --- a heap root
  - y_{med} = *y*-coord. median of points $P \setminus \{p_{min}\}$ --- BST root
  - $P_{below} := \{ p \in P \setminus \{p_{min}\} : p_y \leq y_{med}\}$
  - $P_{above} := \{ p \in P \setminus \{p_{min}\} : p_y > y_{med}\}$
- Point p_{min} and scalar y_{med} are stored in the PST root
- The left subtree is PST of P_{below}
- The right subtree is PST of P_{above}

DCGI

Priority search tree construction example



[Schirra]

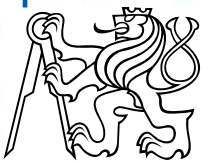
Priority search tree construction

PrioritySearchTree(P)
Input: set P of points in plane
Output: priority search tree T

1. if $P=\emptyset$ then PST is an empty leaf
2. else
3. p_{min} = point with smallest x-coordinate in P // heap on x root
4. y_{med} = y-coord. median of points $P \setminus \{p_{min}\}$ // BST on y root
5. Split points $P \setminus \{p_{min}\}$ into two subsets – according to y_{med}
6. $P_{below} := \{ p \in P \setminus \{p_{min}\} : p_y \leq y_{med}\}$
7. $P_{above} := \{ p \in P \setminus \{p_{min}\} : p_y > y_{med}\}$
8. T = newTreeNode()
9. $T.p = p_{min}$ // point [x, y]
10. $T.y = y_{mid}$ // skalar
11. $T.left$ = PrioritySearchTree(P_{below})
12. $T.rigft$ = PrioritySearchTree(P_{above})

… Notation on next slide:
… p(v)
… y(v)
… lc(v)
… rc(v)

13. O(n log n) , but O(n) if presorted on y-coordinate and bottom up



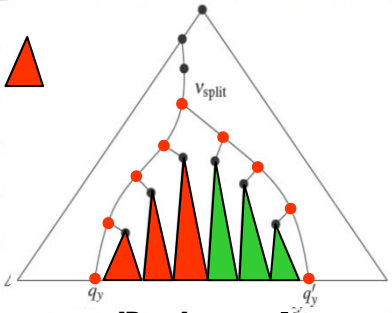
Query Priority Search Tree

QueryPrioritySearchTree(T, $(-\infty : q_x] \times [q_y ; q'_y]$)
Input: A priority search tree and a range, unbounded to the left
Output: All points lying in the range

1. Search with q_y and q'_y in T // BST on y-coordinate – select y range
   Let v_{split} be the node where the two search paths split (split node)

2. for each node v on the search path of q_y or q'_y // points • along the paths
3. if $p(v) \in (-\infty : q_x] \times [q_y ; q'_y]$ then Report $p(v)$ // starting in tree root

4. for each node v on the path of q_y in the left subtree of v_{split} // inner trees
5. if the search path goes left at v
6. ReportInSubtree($rc(v)$, q_x) // report right subtree
7. for each node v on the path of q'_y in right subtree of v_{split}
8. if the search path goes right at v
9. ReportInSubtree($lc(v)$, q_x) // rep. left subtree



[Berg]

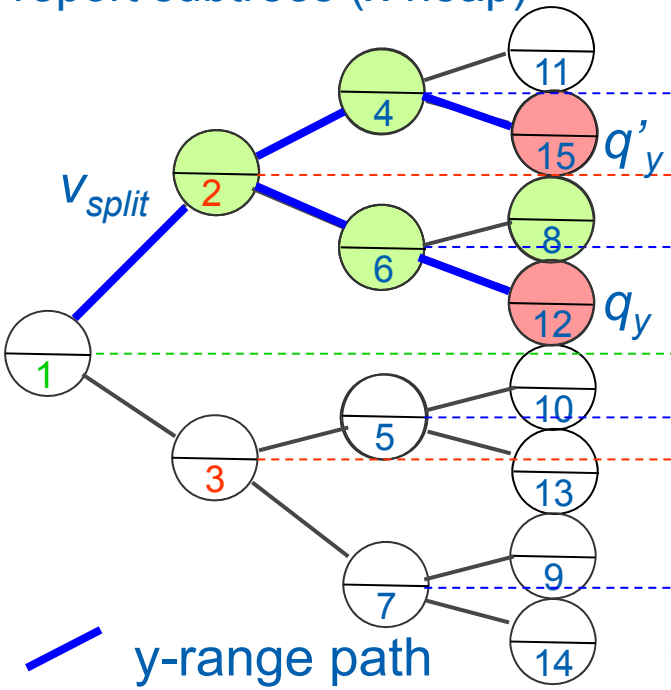
Reporting of subtrees between the paths

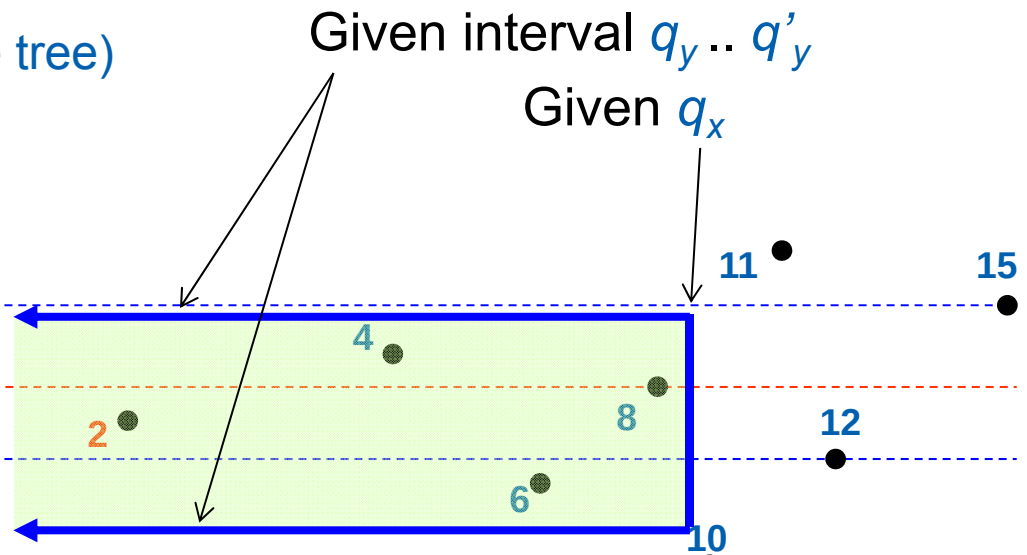
ReportInSubtree(v, q_x)
Input: The root v of a subtree of a priority search tree and a value q_x.
Output: All points in the subtree with x-coordinate at most q_x.

1. if v is not a leaf and $x(\,p(v)\,) \leq q_x$ // $x \in (-\infty : q_x]$ -- heap condition
2. Report $p(v)$.
3. ReportInSubtree($lc(v)$, q_x)
4. ReportInSubtree($rc(v)$, q_x)

Priority search tree query $(-\infty : q_x] \times [q_y ; q'_y]$

1. select *y* range (y-BVS~ 1D range tree)
2. report points on paths (x-heap)
3. report subtrees (x-heap)



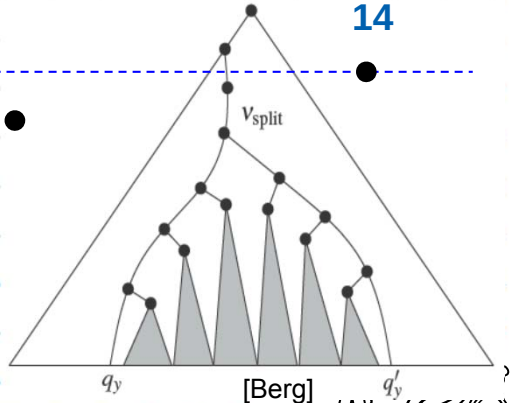


y-range path

x ok – report this point

x too high – stop

Segment left end-points



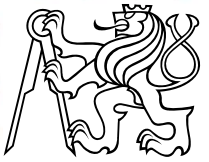
Based on [Schirra] [Berg]

Priority search tree complexity

For set of n points in the plane

- Build O(n log n)
- Storage O(n)
- Query O(k + log n)
  - points in query range $(-\infty : q_x] \times [q_y ; q'_y])$
  - k is number of reported points

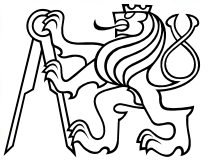
- Use Priority search tree as associated data structure for interval trees for storage of M (one for M_L, one for M_R)



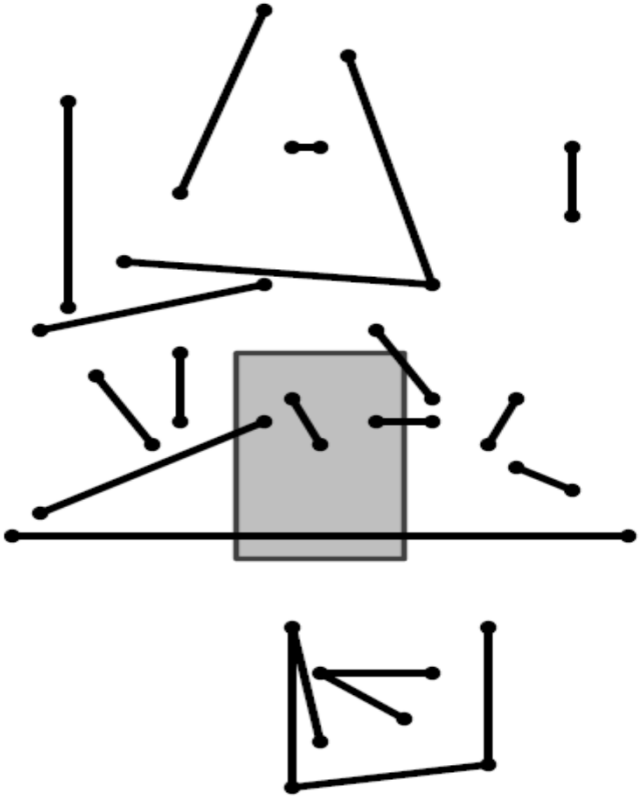
Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree - IT*)
   i. Line stabbing (standard *IT* with *sorted lists*)
   ii. Line segment stabbing (*IT* with *range trees*)
   iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position
   – *segment tree*



2. Windowing of line segments in general position



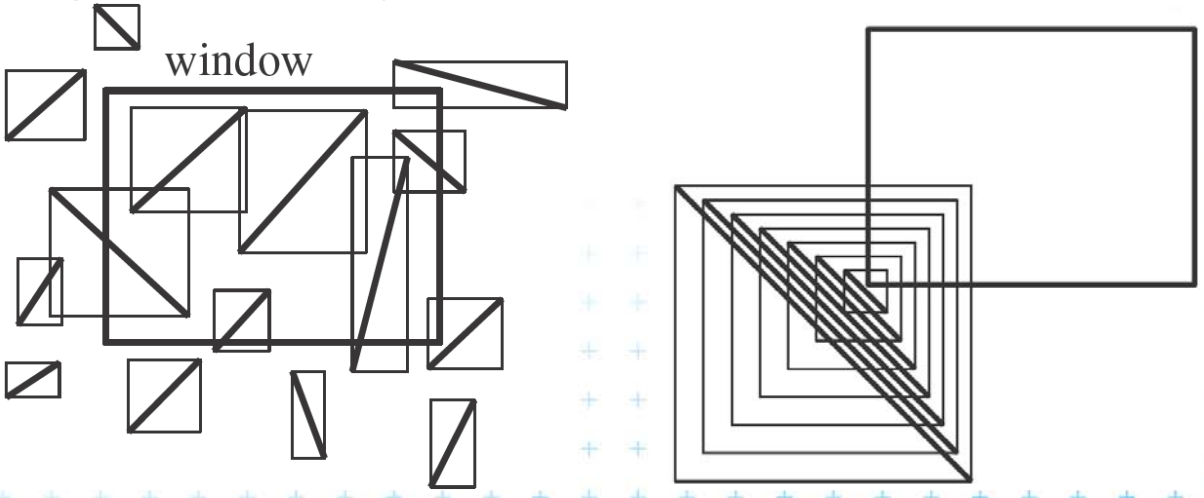
Windowing of arbitrary oriented line segments

- Two cases of intersection

  a,b) Endpoint inside the query window => range tree

  c) Segment intersects side of query window => ???

- Intersection with BBOX (segment bounding box)?
  - Intersection with 4n sides of the segment BBOX?
  - But segments may not intersect the window –> query y



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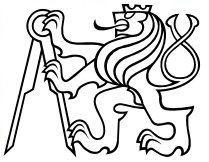
2. Windowing of line segments in general position
   – *segment tree*

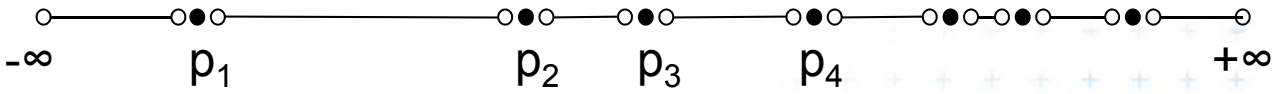
   Note: segment = interval
   it consist of elementary intervals



Segment tree

[Bentley, 1977]

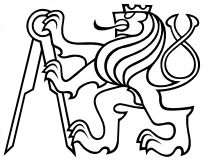
- Exploits locus approach
  - Partition parameter space into regions of same answer
  - Localization of such region = knowing the answer
- For given set *S* of *n* intervals (segments) on real line
  - Finds *m* elementary intervals (induced by interval end-points)
  - Partitions 1D parameter space into these elementary intervals



$$(-\infty : p_1), [p_1 : p_1], (p_1 : p_2), [p_2 : p_2], \ldots, (p_{m-1} : p_m), [p_m : p_m], (p_m : +\infty)$$

  - Stores segments s_i with the elementary intervals
  - Reports the segments s_i containing query point q_x.

Plain is partitioned into vertical slabs

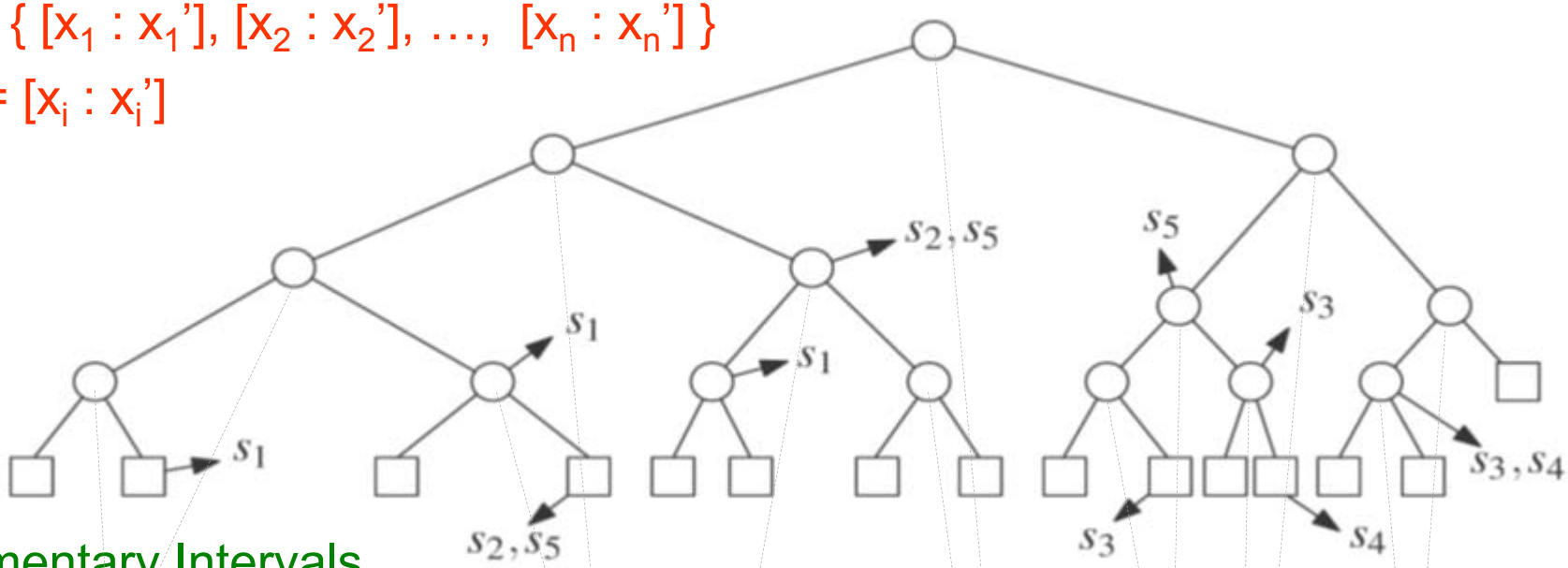


Segment tree example

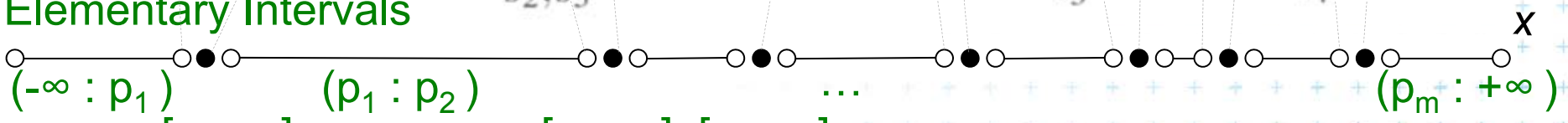
Intervals for segments

$S = \{ [x_1 : x_1'], [x_2 : x_2'], \ldots, [x_n : x_n'] \}$

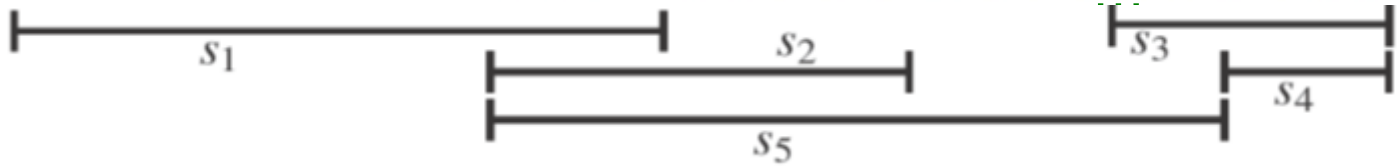
$s_i = [x_i : x_i']$



Elementary Intervals

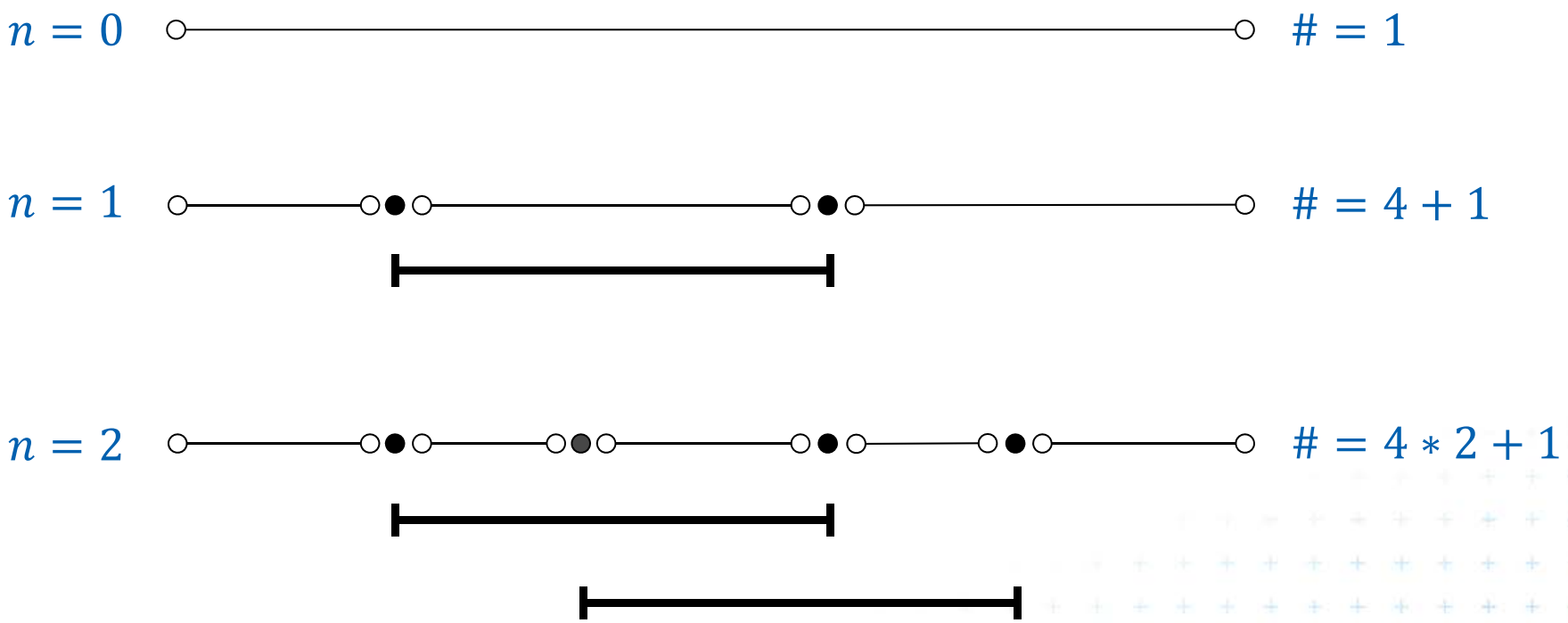


Intervals



[Berg]

Number of elementary intervals for n segments



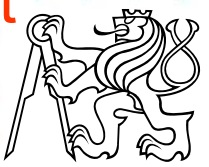
n Each end-point adds two elementary intervals $\# = 4n + 1$

Each segment four…

Segment tree definition

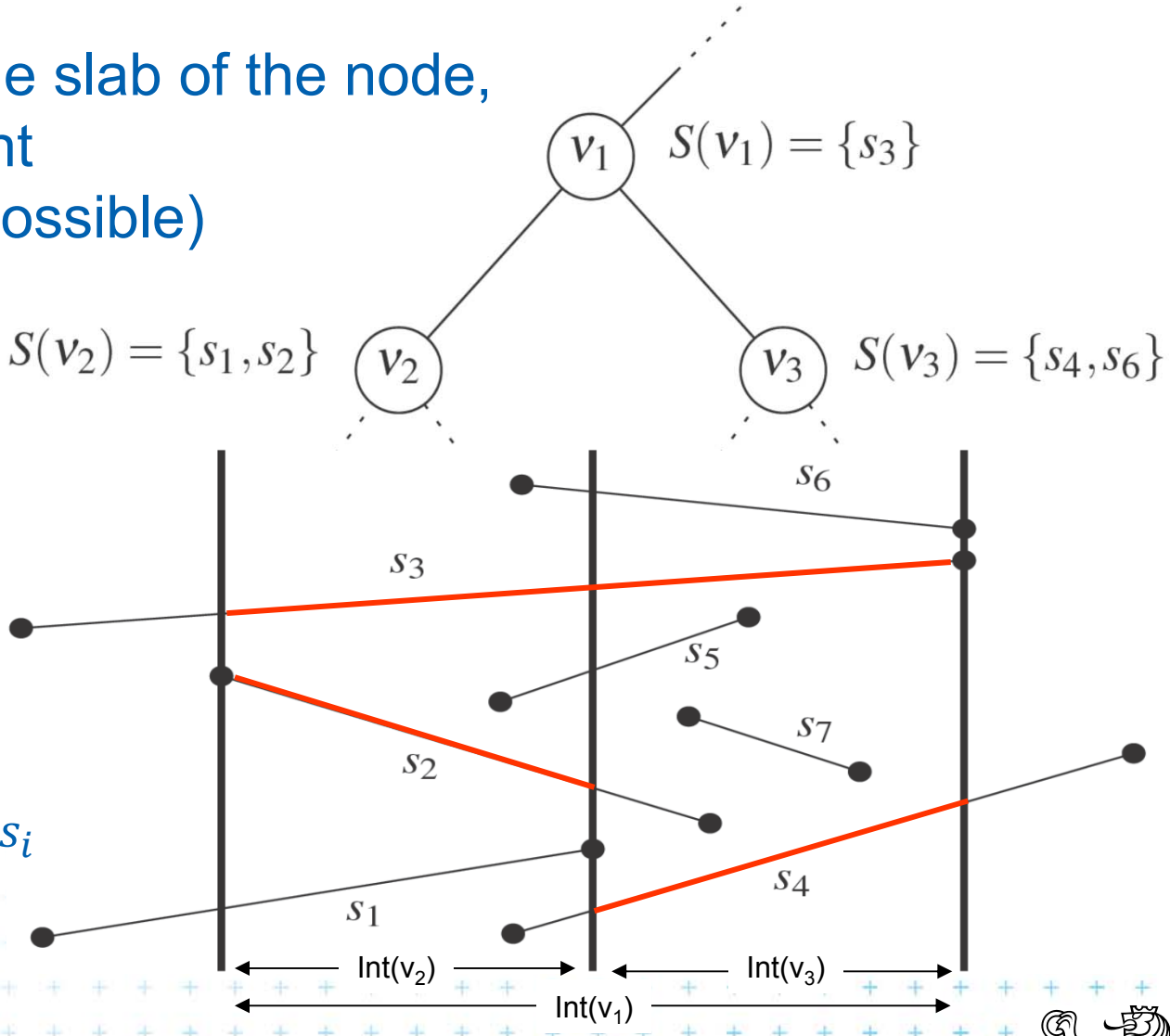
Segment tree

- Skeleton is a balanced binary tree T
- Leaves ~ elementary intervals $Int(v)$
- Internal nodes v
  ~ union of elementary intervals of its children
  - Store: 1. interval $Int(v)$ = union of elementary intervals of its children
    2. canonical set $S(v)$ of segments $[x_i : x_i']$ segments s_i $\in S$
  - Holds $Int(v) \subseteq [x_i : x_i']$ and $Int(parent(v)) \not\subseteq [x_i : x_i']$ (node interval is not larger than the segment)
  - Segments $[x_i : x_i']$ are stored as high as possible, such that $Int(v)$ is completely contained in the segment



Segments span the slab

Segments span the slab of the node,
but not of its parent
(stored as up as possible)



$Int(v_j) \subseteq s_i$

and

$Int(parent(v)) \nsubseteq s_i$

[Berg]

Query segment tree – stabbing query (1D)

QuerySegmentTree(v, q_x)
Input: The root of a (subtree of a) segment tree and a query point q_x
Output: All intervals (=segments) in the tree containing q_x.

1. Report all the intervals s_i in $S(v)$. // current node
2. **if** *v* is not a leaf
3. if $q_x \in$ Int(*lc*(v)) // go left
4. QuerySegmentTree(*lc*(v), q_x)
5. else // or go right
6. QuerySegmentTree(*rc*(v), q_x)

Query time $O(\log n + k)$, where *k* is the number of reported intervals
$O(1 + k_v)$ for one node
Height $O(\log n)$

Segment tree construction

ConstructSegmentTree(S)
Input: Set of intervals (segments) S
Output: segment tree

1. Sort endpoints of segments in S -> get elementary intervals…O(n log n)
2. Construct a binary search tree T on elementary intervals …O(n)
   (bottom up) and determine the interval Int(v) it represents
3. Compute the canonical subsets for the nodes (lists of their segments):
4. v = root(T)
5. for all segments $s_i = [x : x'] \in S$
6. InsertSegmentTree(v, $[x : x']$)

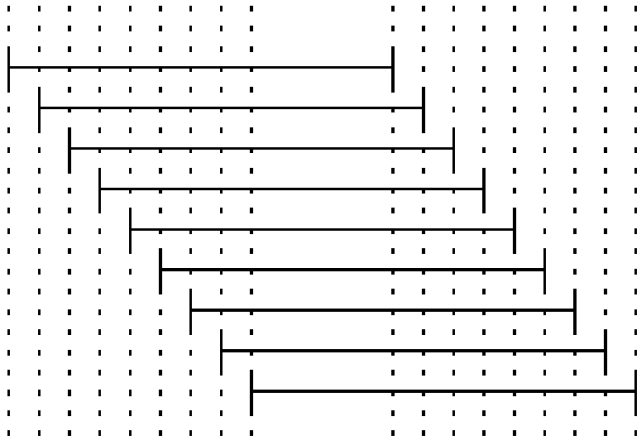
Segment tree construction – interval insertion

InsertSegmentTree(v, [$x : x'$])
Input: The root of (a subtree of) a segment tree and an interval.
Output: The interval will be stored in the subtree.

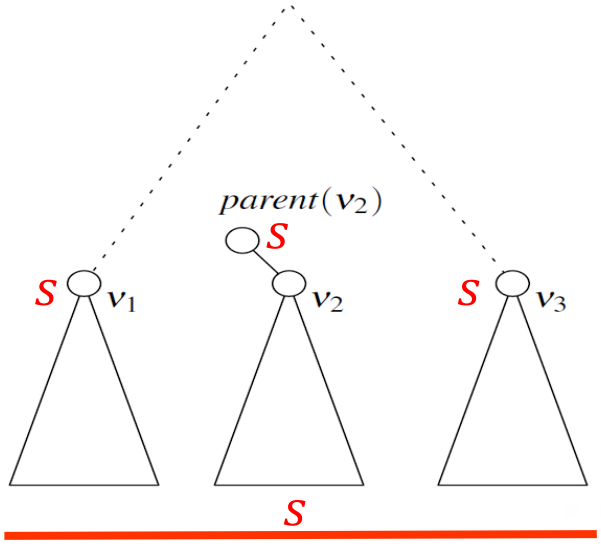
1. **if** Int(v) $\subseteq$ [$x : x'$] // *Int(v) contains* s_i = [$x : x'$]
2. store [$x : x'$] at v
3. **else if** Int($lc(v)$) $\cap$ [$x : x'$] $\neq \emptyset$
4. InsertSegmentTree($lc(v)$, [$x : x'$])
5. **if** Int($rc(v)$) $\cap$ [$x : x'$] $\neq \emptyset$
6. InsertSegmentTree($rc(v)$, [$x : x'$])

One interval is stored at most twice in one level =>
Single interval insert $O(\log n)$, insert n intervals $O(2n \log n)$
Construction total $O(n \log n)$
Storage $O(n \log n)$
Tree height $O(\log n)$, name stored max 2x in one level
Storage total $O(n \log n)$ – see next slide

Space complexity - notes



[Berg]



[Berg]

Worst case – $O(n^2)$ segments in leafs

But

Store segments as high, as possible

Segment max 2 times in one level ⇐

$\max 4n + 1$ elementary intervals (leaves)

⇒ $O(n)$ space for the tree

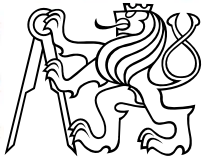
⇒ $O(n \log n)$ space for interval names

s covered by v_1 and v_3

⇒ v_2 covered, $Int(v_2) \in s$

As v_2 lies between v_1 and v_3

⇒ $Int(parent(v_2)) \in s$ ⇒ segment s will not be stored in v_2



Segment tree complexity

A segment tree for set S of n intervals in the plane,

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log n)$
  - Report all intervals that contain a query point
  - k is number of reported intervals

Segment tree versus Interval tree

- Segment tree
  - O(n log n) storage x O(n) of Interval tree
  - But returns exactly the intersected segments s_i, interval tree must search the lists ML and/or MR
- Good for
  1. extensions (allows different structuring of intervals)
  2. stabbing counting queries
     - store number of intersected intervals in nodes
     - O(n) storage and O(log n) query time = optimal
  3. higher dimensions – multilevel segment trees

     (Interval and priority search trees do not exist in ^dims)

DCGI

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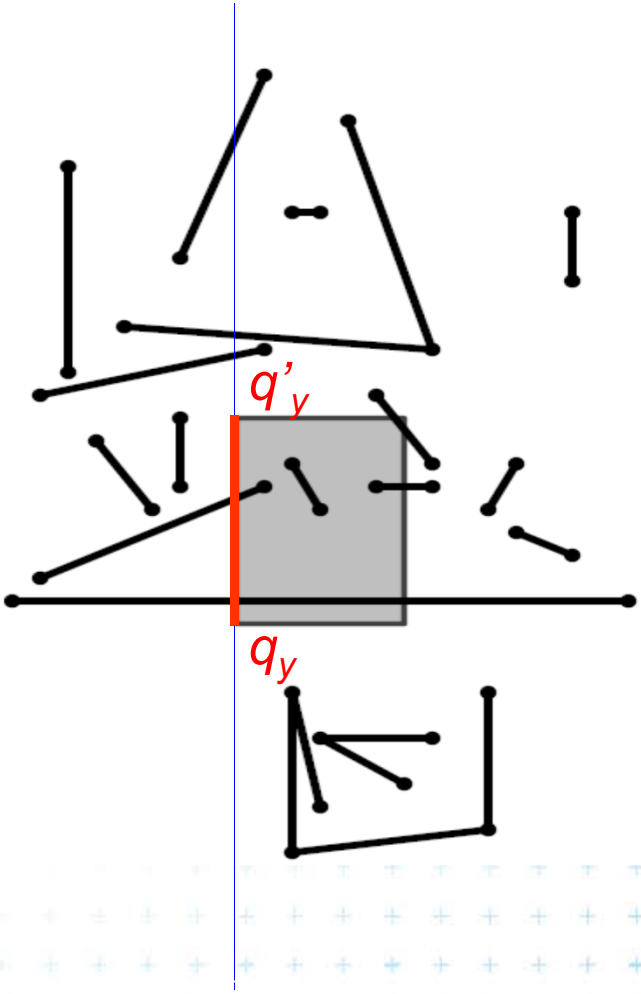
2. Windowing of line segments in general position
   – *segment tree*
   – the algorithm

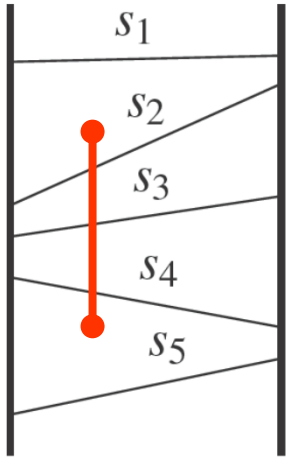
2. Windowing of line segments in general position

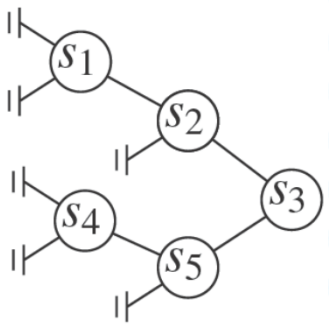


Windowing of arbitrary oriented line segments

- Let S be a set of arbitrarily oriented line segments in the plane.
- Report the segments intersecting a vertical query segment $q := q_x \times [q_y : q'_y]$
- Segment tree T on x intervals of segments in S
  - node v of T corresponds to vertical slab $Int(v) \times (-\infty : \infty)$
  - segments span the slab of the node, but not of its parent
  - segments do not intersect

  => segments in the slab (node) can be vertically ordered – BST





[Berg]

Segments between vertical segment endpoints

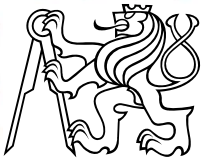
- Segments (in the slab) do not mutually intersect

  => segments can be vertically ordered and stored in BST

  – Each node v of the x segment tree (vertical slab) has an associated y BST

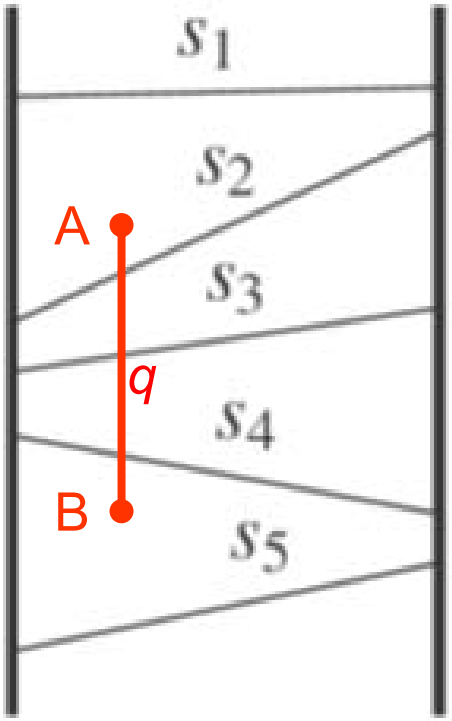
  – BST $T(v)$ of node v stores the canonical subset $S(v)$ according to the vertical order

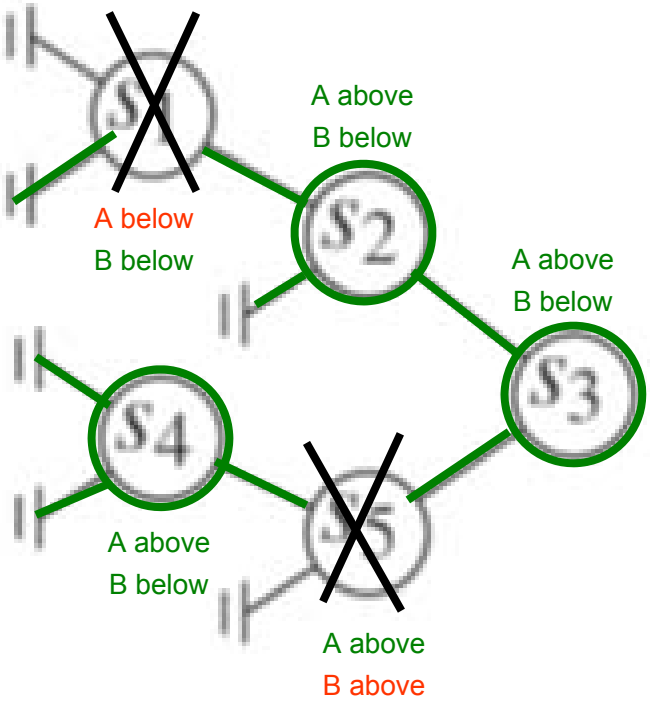
  – Intersected segments can be found by searching $T(v)$ in $O(k_v + \log n)$, k_v is the number of intersected segments



Segments between vertical segment endpoints

- Segment s is intersected by vert.query segment q iff
  - The lower endpoint (B) of q is below s and
  - The upper endpoint (A) of q is above s





Windowing of arbitrary oriented line segments complexity

Structure associated to node (BST) uses storage linear in the size of $S(v)$

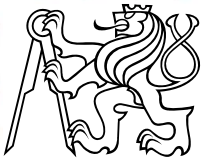
- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log^2 n)$
  - Report all segments that contain a query point
  - k is number of reported segments

DCGI

Windowing of line segments in 2D – conclusions

Construction: all variants $O(n \log n)$

	Search	Memory
1. Axis parallel		
i. Line (*sorted lists*)	O(k + log n)	O(n)
ii. Segment (*range trees*)	O(k + log^2 n)	O(n log n)
iii. Segment (*priority s. tr.*)	O(k + log n)	O(n)
2. In general position		
– *segment tree + BST*	O(k + log^2 n)	O(n log n)



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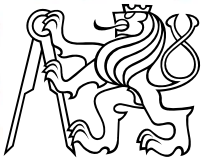
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