

Normal-Form Games

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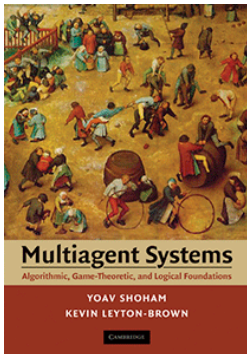
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Previously ... on multi-agent systems (tutorials and lectures).

- 1 Agent architectures,
- 2 Reactive Agents
- 3 BDI, logics

... and now for game theory.

Source for studying: www.masfoundations.org



Why Game Theory – Practical Impact

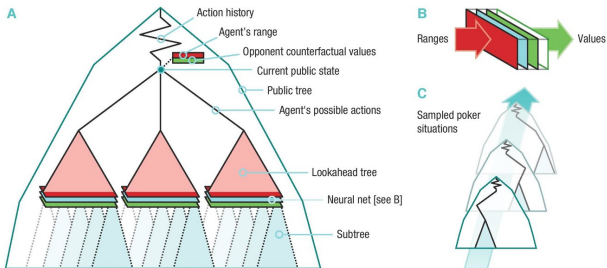
Game theoretic models are particularly useful for security applications in the physical world as well as in computer networks.



Why Game Theory – Deepstack

DeepStack: essentially solving 2-player heads-up no-limit Texas hold'em poker.

- Finds approximate NE for the game
- Large game (about 10^{160} states), uses neural networks for heuristics



Few Reminders from the Lecture

Definition (Normal Form Game)

We call a triplet $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ a normal-form game, where

\mathcal{N} is a finite set of players

\mathcal{A}_i is a finite set of actions (pure strategies; hence, we also use \mathcal{S}_i in some definitions) for player i

u_i is a utility function of player i that assigns the reward for joint action $a \in \mathcal{A}$, $a = (a_1, a_2, \dots, a_{\mathcal{N}})$ to player i

We typically restrict to 2 players games ($\mathcal{N} = \{1, 2\}$). We distinguish:

- **zero-sum games** where $u_1(a_1, a_2) + u_2(a_1, a_2) = 0$ for all $a_i \in \mathcal{A}_i$
- **general-sum games** where $u_1(a_1, a_2) + u_2(a_1, a_2) \neq c$ for any $c \in \mathbb{R}$ and all $a_i \in \mathcal{A}_i$

How to create a game?

Student is at the lecture and he/she wants to go home by tram 22. There are 10 people in the tram and another 10 people are waiting at the tram stop, the goal of the tram is to carry as many people as possible, there is no-one exiting at this stop. The tram stops either (A) right next to the beginning of the tram stop, (B) further away (in the middle of the tram stop), or (C) does not stop at all. The tram stops and opens the door only once. Simultaneously, the student has to decide where at the stop he/she is going to wait. If the student guesses the position of the tram correctly, he/she gets utility 1. If the student guesses the position of the tram incorrectly, he/she gets utility $\frac{1}{2}$ as he/she needs to go through the crowd of people to catch the tram. If the student misses the tram, his/her reward is -1 .

Task 1: Formalize the game.

How to create a game?

Student is at the lecture and he/she wants to go home by tram 22. There are 10 people in the tram and another 10 people are waiting at the tram stop, the goal of the tram is to carry as many people as possible, there is no-one exiting at this stop. The tram stops either (A) right next to the beginning of the tram stop, (B) further away (in the middle of the tram stop), or (C) does not stop at all. The tram stops and opens the door only once. Simultaneously, the student has to decide where at the stop he/she is going to wait. If the student guesses the position of the tram correctly, he/she gets utility 1. If the student guesses the position of the tram incorrectly, he/she gets utility $\frac{1}{2}$ as he/she needs to go through the crowd of people to catch the tram. If the student misses the tram, his/her reward is -1 .

Task 2: Imagine that the number of waiting people is a stochastic variable – there is a 50% chance that 10 people are waiting, and there is a 50% chance that no one else (besides the student) is waiting. How does the formal representation of the game change?

Best Response and Equilibria

Definition (Best Response)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game and let $BR_i(s_{-i}) \subseteq \mathcal{S}_i$ such that $s_i^* \in BR_i(s_{-i})$ iff $\forall s_i \in \mathcal{S}_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$.

Definition (Nash Equilibrium)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. Strategy profile $s = \langle s_1, \dots, s_n \rangle$ is a Nash equilibrium iff $\forall i \in \mathcal{N}, s_i \in BR_i(s_{-i})$.

Practical Tasks

Consider the following game

	L	M	R
U	1, 3	4, 2	-1, 2
C	1, 0	2, -2	0, -1
D	1, 2	-1, 1	3, 3

Task 3: Can you find all pareto optimal outcomes?

Task 4: Can you find all pure Nash equilibria?

Task 5: Can you find all (weakly) dominated pure strategies?
Apply iterative removal of (weakly) dominated pure strategies.

Practical Tasks

Task 6: Design a game where a pure Nash equilibrium outcome will be removed in a process of iterative removal of (weakly) dominated pure strategies.

Mixed Strategies

Definition (Mixed Strategies)

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A}, u)$ be a normal-form game. Then the set of *mixed strategies* \mathcal{S}_i for player i is the set of all probability distributions over \mathcal{A}_i ; $\mathcal{S}_i = \Delta(\mathcal{A}_i)$.

Player selects a pure strategy according to the probability distribution.

We use \mathcal{S}_{-i} to denote strategies of all other players except player i .

We extend the utility function to correspond to *expected utility*:

$$u_i(s) = \sum_{a \in A} u_i(a) \prod_{j \in \mathcal{N}} s_j(a_j)$$

We can extend existing concepts (dominance, best response, ...) to mixed strategies.

Practical Tasks

Consider the following game

	L	R
U	3, 2	1, 3
D	1, 0	2, -2

Task 7: Can you find a mixed Nash equilibrium?