# Cooperative Game Theory

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Previously ... on multi-agent systems.

### Extensive-Form Games

Any questions?

#### Subgame Perfect Equilibrium in finitely repeated games

	C	D	E
C	(3, 3)	(0,4)	(0,0)
D	(4,0)	(1, 1)	(0,0)
E	(0,0)	(0,0)	$(\frac{1}{2},\frac{1}{2})$

And now ...

## Simple Game

Consider the following coalitional game.

S	v(S)
{1}	5
{2}	10
{3}	20
$\{1, 2\}$	50
$\{1, 3\}$	70
$\{2, 3\}$	50
$\{1, 2, 3\}$	90

Find a payoff vector x for the grand coalition that will be in the core.

Calculate Shapley value for the members of the grand coalition.

#### Core

Give an example of a coalitional game with an empty core.

Give an example of a superadditive coalitional game with an empty core.

Find all core payoff vectors for the grand coalition for the previous example.

Design a generic algorithm for computing the core for coalitional game (N, v) and grand coalition.

What do you think is the complexity of the problem of computing the core (or deciding whether it is empty)?

## Airport

The maintenance costs of airport runways are charged to the airlines landing planes. Light planes require shorter runways than heavy planes, and this raises the question of how to determine a fair allocation of maintenance costs among airlines with different types of planes. Define a cost game (N,c), where N is the set of all planes, labeled  $A,B,\ldots,H$  that land at the airport on daily basis. Each plane requires the entire length of the runway up to (and including) the interval on which it is located in the figure. The weekly maintenance costs of each runway segment appear at the bottom of the figure.

Prove that if the Shapley value of this game is used to determine the allocation of costs, then the maintenance cost of each runway segment is borne equally by the planes using that segment.